Fringe-projection profilometry for recovering 2.5D shape of ancient coins

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ABSTRACT

The form of relief is undoubtedly one of the most topical subjects in the field of cultural heritage. Physical access to historic and artistic products can be limited by a number of factors. For example, access to collections of ancient coins is difficult, especially for students. Indeed, coin digital archives containing high-quality three-dimensional models and that can be accessed remotely are of great interest. The use of projected fringes for the measurement of surface profiles is a well-developed technique. In this paper, we present a surface-profile measurement system for small cultural heritage objects where it is important not only to detect the shape with a high degree of accuracy but also to capture and archive any signs of ageing. The equipment presented in this paper is simple, reliable and cheap. In addition, some examples are presented to demonstrate the potential of the proposed scheme for recovering 2.5D shapes of cultural heritage objects.

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**Keywords:** Temporal phase unwrapping; Profilometry; Cultural heritage; 2.5D reconstruction

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1. Introduction

Given its importance in many fields, ranging from quality control to medicine, from robotics to solid modelling, surveys of surface profiles by means of non-contact optical systems is the subject of extensive studies [1]:

* In the field of cultural heritage, 3D acquisition is used primarily for documentation, comparative studies [2], [3], remote examination [4], [5] and degradation monitoring [6] as well as to create virtual museums [7, 8].
* In medicine, 3D scanners are useful for obtaining accurate anthropometric body segment parameters for the biomechanical analysis of human motion [9] and for the accurate acquisition of patient dentition [10].
* In the field of forensics, 3D reconstructions are helpful for the analysis of writing [11], [12], the virtual reconstruction of crime scenes [13], firearm examination proficiency tests [14] and face recognition [15], [16]
* In quality control, 3D scanning can be used for production control and tool wear analysis [17]-[20]. Furthermore, it can be used for underwater 3D reconstruction [21], [22].

In the field of cultural heritage conservation, both the determination of surface finish (surface texture) and its variations over time are important. In fact, in order to establish the actual vulnerability of cultural assets, one of the most important parameters to determine is the loss of material or, more generally, the determination of microstructural changes in the exposed surface (currently, for these determinations, indirect methods are used). Therefore, the objective of the present research is to develop a non-contact optoelectronic system capable of analysing, at high resolution, the surface profile of artistic finds. The basic methodology to be used for the development of the system is the one known as grating projection or structured light [23]-[25]. This method, supported by optoelectronic signal-processing techniques, allows the development of a system characterised by its high precision, small footprint and relatively low production cost.

1. THE RANGE FINDER

Projected-fringe profilometry is one of the most effective methods of measuring the 2.5D surface profiles of rough engineering surfaces [26]-[28]. The fringe can be generated by projecting parallel light through optical grating slides with various projecting patterns, such as sinusoidal-like modulation. The traditional phase-shifting method, which involves physically moving a grating or a reference mirror, has two problems, namely, inefficiency due to mechanical movement and unavoidable phase-shifting errors incurred during the phase-shifting process. Therefore, a digital micro-mirror device has been proposed to generate flexible structured-light patterns with highly precise phase shifting.

This paper presents a surface-profile measurement method for micro-components based on the fringe-projection, phase-shifting technique and temporal phase unwrapping. Linear sinusoidal fringe patterns are projected onto an artwork surface by a digital fringe projector. The projected sinusoidal fringes will be deformed by surface height variations. By detecting the deformed sinusoidal fringes in combination with phase stepping and temporal phase unwrapping, high-precision 2.5D surface measurements can be efficiently acquired.

As shown in Figure 1, the system is a telecentric phase-shifting projected fringe profilometry tool for small objects [29]. Our tool consists of three main modules: a digital fringe projector, a charged-coupled device (CCD) camera and a computer.



Figure 1. Basic setup for measuring surface profiles with projected fringes.

Telecentric projection is realised by means of a commercially available digital light processing video projector (1920 × 1200 resolution), in which the projection lens is replaced by a telecentric lens. The optical telecentric imaging consists of a B/W CCD camera with a resolution of 1280 × 1020 and a telecentric lens. The measurement field size is about 120 mm × 100 mm and the depth of field reaches 10 mm, which is usually sufficient to measure the depth of a small, complex 3D object (i.e. an ancient coin).

The optical axis of the projector is perpendicular to the reference plane and intersects with that of the camera at the reference plane. The line connecting the exit pupil of the projector and the entrance pupil of the camera is parallel to the reference plane.

Figure 2 shows the constructed system.

Immagine che contiene interni, tavolo, torta, sedendo

Descrizione generata automaticamente

Figure 2. The layout of the fringe-projection system.

Structured light with sinusoidal fringes is projected onto the object surface. Due to the depth variation of the surface, the structured light is phase modulated, leading to a deformed spatial carrier fringe pattern in which the topographic information of the object surface has been encoded.

Without loss of generality, it is assumed that the fringes are in the x–y plane and perpendicular to the y-axis. The optical geometry of the phase-measuring techniques is shown in Figure 3.



Figure 3. Optical geometry for fringe projection. In the realised system, we have θ ≈ 45°; field of view x ≈ 100 mm, y ≈ 120 mm; max z ≈ 10 mm; minimum period of projected fringe ; minimum period of the fringe seen by the CCD .

The intensity distribution of the fringe-pattern image can be expressed as [30]-[33]

where denotes an arbitrary point in the image,is the background intensity, is the intensity modulation amplitude, is the period of the fringe seen by the CCD camera on the reference plane and is a phase change related to the 2.5D profile of the measured object. In other words, phase contains the information about the surface height/depth variations.

The relationship between the phase distribution and the profile information of the measured object can be achieved by [34]-[37]

where , and are the parameters related to the measurement system, which can be determined by a calibration process in advance [38]-[40].

To calculate the system constants , and we use the least-squares method.

As shown in Figure 4, during the calibration procedure, the reference plane is shifted to position along the direction of the object height. Each new position has height with respect to the zero position ().



Figure 4. The calibration process of the system constant.

To determine the system constants , and , we define the error function as

where is the number of shifts of the standard reference plane. In the least-squares sense, the system constants , and can be directly estimated by minimising the sum of square . The minimum error is at the point where the partial derivatives of with respect to , and are all zero. Therefore, we have [35]

Thus, if the phase map of planes and the relative positions are known, the system constants can be obtained from Equation 4.

For the phase calculation, it is possible to use the fast Fourier transform algorithm [41]-[43] or phase-shifting technique [23].

Generally, phase can be retrieved using the phase-shifting method. With this technique, a set of phase-shifted sinusoidal fringe patterns is projected onto the object’s surface. The intensity values, captured by the CCD camera, can be expressed as [44]-[47]

where represents the phase-shift index

The values of phase can be calculated by

Since there are three unknowns, , and in Equation 5, at least three images, , and should be used to enable the calculation of .

In Equation 5, we considered the intensity of the sinusoidal fringes. However, the nonlinearity of the luminance caused by the gamma effect of the digital camera and projector introduces significant harmonics that lead to errors in the phase determination [48]-[51].

Considering the harmonics, and indicating with the highest significant harmonic order of the captured fringes, Equation 5 can be rewritten as [52]

Equation (7) involves unknowns:

and ;

consequently, phase-shifted fringe patterns are required to solve these unknowns.

Therefore, if we consider a set of uniform phase-shifted sinusoidal fringe patterns, the values of the phase can be retrieved by means of

Equation 8 indicates that an uniform phase-shifted scheme can be employed to retrieve the phase accurately from the fringe patterns with nonlinear harmonics up to the th order.

Generally, the highest significant harmonic order, in practice, is below eight.

Commonly, phase can be distributed over an interval greater than . Unfortunately, the arctan function, using the numerator and denominator signs, returns values of only in the range from to . In other words, the phase is rolled up in a interval. Superscript denotes a wrapped phase value.

To recover the correct phase value, it is necessary to use an unwrapping phase procedure. This results in an unwrapped phase map, which is given by Equation 3:

where , the number of turns, is an integer that may be positive or negative.

1. Phase unwrapping

Phase unwrapping is the restoration of the original phase values of an image, given the observed wrapped values. While, theoretically, this problem can be easily solved, this is not the case for ‘experimental’ wrapped phases. Indeed, when a complex object is measured, abrupt and irregular changes in the measured surface may result in local shadows.

With the use of computer-generated fringe patterns, however, the phase in the projected fringe pattern can be easily programmed. Therefore, temporal phase unwrapping techniques can be used for recovered phase maps [53]-[55]. The temporal phase combines phase shifting with a change in fringe pitch (change in wavelength or distance between fringes). The strength of the method is that the problem with phase rolling never occurs and that it results in an absolute phase measurement.

Temporal phase unwrapping offers several potential advantages over the spatial method [56]:

* errors do not propagate to the whole phase map
* fringe order errors can be easily detected and filtered out ‘a posteriori’ since they are multiples of
* it does not require user intervention to indicate specimen edges or to provide the phase value at a particular point
* it is very fast and does not require additional memory
* the implementation is extremely simple.

However, temporal phase unwrapping has the disadvantage of working well only with the hypothesis of static objects. In other words, throughout the process, it is critical that the measured quantities be of temporal invariance.

The idea of temporal phase unwrapping is to use a sequence of fringe maps with a change in fringe pitch (see Figure 5). In other words, frequency repeats single phase shifting times with periods of the fringe at .

In the fringe pattern, the relative phases are .

For subsequent time values (), the number of fringes is also set equal to so that the phase range increases to ().

There is a linear relationship between the phase and the number of the projected fringe. If one phase jump is projected, we have one fringe of wrapped phase map; when projecting two fringes, the wrapped phase map contains exactly two phase jumps. In temporal phase unwrapping, the phase difference is calculated from successive frames. This technique is inherently robust.



Figure 5. Sequence of fringe maps projected onto an object surface. For each value, phase-stepped images are acquired.

Wrapped phase measurements for each used fringe period are acquired sequentially in time.

By means of Equation 8, for each , we obtain a wrapped phase.

Initially, a single fringe () is projected so that the fringe phase range is only from to .

Therefore,

For subsequent time values (), the number of fringes is also set equal to so that the phase range increases to (). With this procedure, for each value, phase-stepped images are acquired.

The unwrapped phase distributions of the higher frequency fringes can be readily calculated without a general phase unwrapping. The algorithm can be expressed as follows [55], [56]:

where denotes rounding to the nearest integer.

Using an iterative process, the unwrapped phase can be computed by summing the wrapped phase difference between the successive phase maps.

Figure 6 shows an example of a sequence of fringe maps achieved with our system.

In the presence of noise, as generally occurs, a better approach to the estimation of the unwrapped phase is a fitting, in the least-squares sense of the line defined by Equation 9, to the experimental data [57]:

Since the algorithm involves only one single governing equation, it is very simple and easy to use.



Figure 6. Example of fringe maps used in our system.

1. EXPERIMENTAL RESULTS AND DISCUSSION

First, some validation experiments were conducted. Figure 7 shows the recovered 2.5D plot from a metallic specimen (2.5 cm × 2.5 cm) with a step of 50 μm in height. The specimen was realised by means of a grinding machine. A comparison between the conventional direct contact measurement and the optical results shows that the maximum discrepancy is about 2 μm.

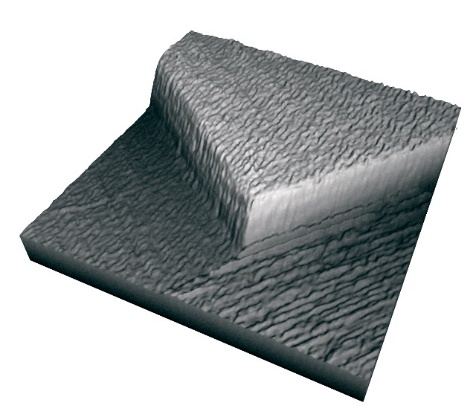


Figure 7. Three-dimensional reconstruction of specimen with 50 μm calibrated steps.

Subsequently, we performed a test on a telephone token. Figure 8 shows the recovered 2.5D plot of an Italian telephone token, in use in Italy from 1959 until 2001.



Figure 8. Recovered 2.5D Italian telephone token (diameter ø = 24 mm).

Dimensional measurements of the throat profile were obtained to check the quality of the 2.5D reconstruction. An example of a profile is shown in Figure 9.



Figure 9. Depth profile plot of the telephone token throat.

Again, the discrepancies between the optical measurement and direct contact measurement are less than 2 μm.

Considering the characteristics of our system, it is well suited to 2.5D coin reconstruction. In general, coins can be analysed in two and three dimensions. The advantage of 3D coin data is that it allows a more detailed and reliable analysis due to an exact description of the coin’s surface, which is a great benefit for any automated classification or recognition task [59]-[61].

To check the usability of the instrumentation for the 3D survey, we surveyed the surface topography of some coins.

We initially acquired the relief of an old Italian 10 Lire coin used before 2000. Figure 10 shows the recovered 2.5D plot; it can be noted that the three-dimensional details are correctly reconstructed.



Figure 10. Recovered 3D plot of a coin: ‘10 Italian lire’ (ear of corn; coin diameter ø = 23.3 mm).

Finally, the surface topography of a New Zealand one-cent piece is shown in Figure 11. On the reverse of the coin, a fern leaf is represented. In the 2.5D plot, all the details are correctly reconstructed.



Figure 11. Recovered 2.5D plot of a coin: 1 Cent, New Zealand (standard circulation 1967–1985; ‘fern leaf’ on the reverse; coin diameter ø = 17.4 mm).

1. pipeline for 3D colour model

For a correct and accurate acquisition of the surface, it is essential that the colour characteristics are also acquired.

In a typical 3D data elaboration pipeline, to obtain a complete 3D model of the acquired object from the raw data, the following steps should be followed:

* Acquisition of 2D colour information [62], [63].
* 3D geometric data acquisition.
* Data integration.
* Model conversion.

To test the usability of the proposed technique in the construction of a virtual gallery of ancient artifacts, we have digitised some ancient Roman coins.

Figure 12 shows the data-processing pipeline steps used to obtain a complete 3D model of a Roman sestertius of Agrippina Senior, who was born in 15 B.C. and died in exile in A.D. 33. This commemorative coin, bearing her name and portrait, was struck by her son, the Emperor Caligula (A.D. 37–41).



Figure 12. Ancient Roman coin (A.D. 37–41). (a) Photograph of the coin: obverse and reverse; (b) Shaded rendering of the coin geometry; (c) Rendering of the coin shape and colour model (diameter ø = 33 mm).

Figure 12(a) shows the colour photos of the coins (obverse and reverse), Figure 12(b) provides the shared view of the geometry with surface details and Figure 12(c) depicts the colour rendering of the coin.

It is important to note that our system is only capable of a correct 2.5D reconstruction. Therefore, the back of the coin has not been reconstructed.

1. Conclusions

Projected fringes for the measurement of surface profiles are extensively used in 2.5D optical metrology. However, low-cost, commercially available projectors are susceptible to gamma non-linearity and a lack of depth of field, which distorts the fringe patterns and introduces significant errors in the phase measurements. In this work, we have presented a simple optical method that can be applied to the measurement of micro-surface profiles. In the proposed setup, the number of phase shifts applied in the projected grating can be changed via the user interface, depending on the accuracy required.

The proposed device has the potential of the traditional fringe projection technique but with more versatility, offering the freedom to choose and control the fringe period. Experiments using real samples have shown that the 2.5D profiling measured using the proposed system is efficient and effective. Furthermore, the method relies on very simple and cheap equipment.

The proposed system uses only consumer-available technology (a video projector and a digital camera) and can be achieved for less than $1,000.

In other words, for less than $1,000, it is possible to achieve a system with precision and accuracy that is comparable to much more complex and expensive systems (> $5,000).

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