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OPTIMIZING SENSITIVITY OF MASS COMPARATORS

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Abstract – In calibration of weight pieces, the evaluation of comparator sensitivity and its associated uncertainty component is essential. Yet, there is not much documented guidance on how to assess these values. This paper proposes a procedure for testing, evaluating and optimizing mass comparator sensitivity.

Keywords: Mass calibration, sensitivity, uncertainty, mass comparator, OIML R111

1. INTRODUCTION

When calibrating weight pieces, the difference between the unknown weight and the reference weight is calculated from the indications of the mass comparator display. This difference is then used to calculate the conventional mass of the unknown weight. For common industrial applications, weight pieces are calibrated in Conventional Mass [1] and electronic balances and mass comparators also indicate in this conventional unit.

The term sensitivity is defined in [2] as “quotient of the change in an indication of a measuring system and the corresponding change in a value of a quantity being measured” [2, 4.12]. We understand that according to this definition the equation for sensitivity of a mass comparator is

$$S = \frac{\Delta I}{\Delta m_c} \quad (1)$$

with change in indication ΔI and conventional mass of a sensitivity test weight Δm_c (and not the reciprocal value as is frequently used, e.g. in [3] and [7]).

The general assumption is that for electronic balances and mass comparators the difference ΔI of the indications is equal to the difference in conventional mass Δm_c. This corresponds to a sensitivity equal to 1.

However, there are sources that suggest that this is not always the case. In earlier days, it was apparently usual that comparators with an optical scale did not necessarily indicate correct mass differences. Therefore weighing cycles including a sensitivity weight (S), for example of the form “A – B – B+(S) – A+(S)” are described e.g. in [3, SOP4] and [1, C.4.1.2]. With these cycles, the sensitivity is determined in every single weighing cycle.

2 REVIEW OF THE LITERATURE

2.1 Testing Procedures for Sensitivity

However, according to some sources in the literature, a sensitivity determination is not necessary in every weighing cycle for modern electronic comparators:

- "A sensitivity weight is not required if the electronic mass comparator that is used has been tested (with supporting data available) to determine that the balance has sufficient accuracy…" [3, GMP 14]. The uncertainty of this assumption shall be included as an uncorrected systematic error in the uncertainty budget and acceptable limits are "2 %" [3, SOP8].
- Kochsiek et al. mention in the German edition of their book “Massebestimmung” [4] (translated to English: “mass determination”), that for a “frequently adjusted electromagnetically compensated balance”, sensitivity shall be assumed 1 and an uncertainty in S shall be assumed less than $5\,E-04$ or may even be neglected, especially when using auxiliary weights …”.[4]. However, it remains unclear *how* to adjust the comparator so that it fulfills the condition of being “adjusted”, nor are there any suggestions as to the accuracy and thus uncertainty of the adjustment procedures, nor as to how often an adjustment must take place to fulfil the requirement of being “frequently adjusted”. It is interesting to note that the cited numerical value ($5\,E-04$) is omitted in the (later) English edition of the same book [5].
- Chapter C.6.4.2 of [1] requires that an uncertainty component for sensitivity be included in the budget when calibrating weights, but no specific guidance is given on how to assess its value and especially on how to select a proper sensitivity test weight regarding its size and its calibration quality.

In general, there are two ways to treat sensitivity: One is to set a limit and *test* the sensitivity error against that limit. This limit value is considered in the uncertainty budget as an uncorrected error. The other one is to evaluate a number for sensitivity, correct balance readings with this value, calculate the uncertainty of it and have this contribute to the uncertainty budget.

Since mass calibrations involve a significant amount of calculations, it is common practice today to use at least software spreadsheets or even dedicated software for the calculations. If this is the case, the sensitivity value can easily be incorporated as a correction in the calculation of the mass differences and its uncertainty will then contribute to the uncertainty calculation.

In some commercially available calibration software (e.g. Scalesnet), a numerical value of sensitivity is determined in a separate test for each comparator. But for its uncertainty, the uncertainty value given in [4] is used for all cases. Where the mass difference Δm_c between the calibrated weight and the reference weight becomes large, this might have significant influence on the final combined calibration uncertainty of the weight piece under test. The objective of this paper is to answer the question whether this worst case estimation of a sensitivity uncertainty of $5\,E-04$ is justified.

2.2 Test Weights for Sensitivity Testing

There seems to be common understanding that sensitivity of mass comparators is tested with a “small” weight: Reference [3, SOP 2] mentions a “small weight”, [3, SOP 34] mentions a maximum of 0.5 % of balance capacity, while [3, GMP 10] mentions a maximum of 1 % of balance capacity. Obviously no standard method for the selection the sensitivity test weight is available.

3 ASSESSMENT OF SENSITIVITY

The VIM [2] defines adjustment of a measuring system as “set of operations carried out on a measuring system so that it provides prescribed indications corresponding to given values of a quantity to be measured”

Today’s mass comparators with electromagnetic force compensation are equipped with provisions for self-adjustment. These consist of one (or more) internal weight piece(s) and an algorithm which can be time and/or temperature controlled or manually triggered. Furthermore, mass calibration software allows for an additional adjustment of the reading by applying a sensitivity factor in the processing of the value that was read from the comparator output. In this light, we consider the calibration software being a part of the “measuring system” together with the comparator (see Figure 1).

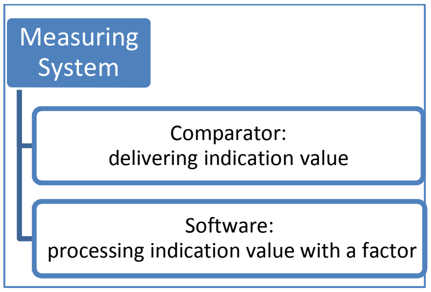

Figure 1: The measuring system.

The sensitivity factor that is applied by the software is gained from the following test procedure: The comparator is loaded with a pre-load (between zero and nominal load) which brings the comparator into a typical working range or working state. Then a test is carried out using an “ABBA” cycle which starts with the pre-load (“A”), then a calibrated sensitivity test weight is added (“B”), this step is repeated (“B”) and finally the test weight is removed (“A”). From the calibration value of the test weight and the calculated, buoyancy-corrected difference of indication “B”-“A”, the sensitivity value S is calculated according to (1).

Once this sensitivity value is determined, it must be used for any further processing of a reading of that comparator. We assume here that the self-adjustment procedure of the comparator is run about once a day, so that any climate-induced changes (air density, temperature) and their effect on the comparator sensitivity are negligible, this means that the sensitivity of the comparator stays “the same” over time, but will not necessarily be exactly 1.

4 UNCERTAINTY OF SENSITIVITY

Given the procedure above and (1) for the calculation of sensitivity, we find the following sources of uncertainty in the determination of S:

- Readability of the comparator u_{read},
- repeatability of the comparator u_{repeat},
- calibration uncertainty of the sensitivity test weight u_{weight}.

The uncertainty in S can thus easily be derived as:

$$u_S^2 = \left[\frac{1}{\Delta m_c} \times u_{read}\right]^2 + \left[\frac{1}{\Delta m_c} \times u_{repeat}\right]^2 + \left[-\frac{\Delta I}{{\Delta m_c}^2} \times u_{weight}\right]^2 \quad (2)$$

With the components

$$u_{read} = \frac{d}{2} \times \frac{1}{\sqrt{3}}$$

for an ABBA cycle (derived from the calculation of an ABBA difference) and with readability d,

$$u_{repeat} = \frac{s}{\sqrt{n}}$$

with the repeatability standard deviation s of the comparator and the number of cycles of test n

and

$$u_{weight} = \frac{U_{cal}}{2}$$

from the calibration certificate of the sensitivity test weight.

For simplification of the uncertainty calculation (only) we set $\Delta I = \Delta m_c$ so that (2) becomes:

$$u_S^2 = \left[\frac{1}{\Delta m_c} \times \frac{d}{2\times\sqrt{3}}\right]^2 + \left[\frac{1}{\Delta m_c} \times \frac{s}{\sqrt{n}}\right]^2 + \left[-\frac{1}{\Delta m_c} \times \frac{U_{cal}}{2}\right]^2 \quad (3)$$

Please note that since S is a relative number, its uncertainty u_S^2 is also a relative number while its uncertainty components are in mass units.

5 UNCERTAINTY OF CONVENTIONAL MASS

OIML R111 [1] requires an uncertainty contribution u_s (note lower case “s” in the subscript) to be estimated that, as a component of the balance uncertainty u_{ba}, accounts for the uncertainty of the sensitivity in the calculation of conventional mass. In our case, where a factor S is used to calculate the conventional mass difference according to (1), this uncertainty component of conventional mass is:

$$u_S(\Delta m_c)^2 = \left[-\frac{\Delta I}{S^2} \times \mathrm{u}_S\right]^2 \tag{4}$$

6 INITIAL CHOICE OF A SENSITIVITY TEST WEIGHT

As has been mentioned above, there is only little literature available on mass comparator sensitivity. The publication of R. Davis [6] was obviously written in the light of comparators with optical scales and a sensitivity assessment in every mass calibration cycle and therefore provides only a rough direction for today's questions. Lee Shih Mean's paper [7] focuses on a very special problem (the calibration of stainless steel against Pt-Ir standards and evaluating true mass) and thus has its own reasons for the choice of the weight size. But all sources agree in the general idea that the weight should be "small" and in the magnitudes of the weighing differences that will be obtained. The reason is probably, that this procedure tries to approximate an ideal differential sensitivity $\frac{\partial I}{\partial m_c}$ from the test with finite values $\frac{\Delta I}{\Delta m_c}$ thus avoiding any influence of non-linearities in the characteristic curve of the comparator.

As a first practical assumption, we chose calibrated test weights with a nominal value of about 100 times the readability d of the comparator, but not smaller than the smallest OIML-weight which is 1 mg. The weights are made of stainless steel to avoid any complications arising from buoyancy effects. As test objects we chose the manual comparators in the mass calibration laboratory of METTLER TOLEDO, accredited as SCS 0032. One of the XP6U comparators is operated with reduced readability by one display digit. (Table 1).

Table 1: Mass comparators and readabilities and test weights of "initial choice".

Comparator	XP64003L	XP10003S	XP5003S	XP2004S	AX1005	AX106	XP6U 1/10 d	XP6U
d (mg)	5	1	1	0.1	0.01	0.001	0.001	0.0001
Δm_c (mg)	500	100	100	10	1	1	1	1

We further assume for simplicity reasons that the test weights were calibrated with an uncertainty of one third of the MPE of class E_1 according to [1]. For each mass comparator used in our laboratory, we calculated the uncertainty of the sensitivity as given in (3).

This revealed some unexpected results. Figure 2 shows the sensitivity uncertainties (k=1) for each comparator type listed in Table 1:

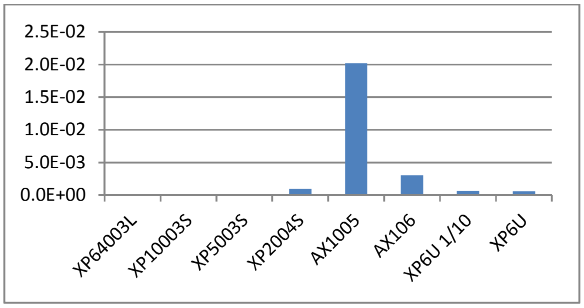

Figure 2: Sensitivity uncertainties based on first choice of the sensitivity test weight.

We note the following important findings:

- Although we applied the same basic idea for the choice of the sensitivity test weight, the differences of the sensitivity uncertainties between the comparators were of about 3 magnitudes, ranging from 3.3 E-05 to 2.0 E-02.
- The maximum uncertainty value observed (2.0 E-02) was significantly higher than the simplified value of 5 E-04 from the literature.

We conclude that a general assumption of an uncertainty of comparator sensitivity of 5 E-04 (as can be found in the literature) is not justified.

7 OPTIMIZED PROCEDURE AND TEST WEIGHT

Further investigation of the uncertainty budget of the sensitivity revealed that in most cases (and especially in the case of the high values identified above), the dominant contributor to the uncertainty budget was the influence of repeatability (which is the second component in (3)). The equation suggests that this contributor could be reduced by an increase in the number of weighing cycles used in the adjustment. However, this has little effect since it is not practical to use more than about 5 ABBA cycles.

Re-visiting equation (3), we find that the nominal value of the chosen sensitivity test weight influences all three uncertainty contributors. This opens the door to optimizing the sensitivity uncertainty by adjusting nominal values and number of cycles. Increasing the nominal weight value will lead to smaller sensitivity uncertainties. A massive increase would, however, violate the principle of "small" sensitivity test weights (as explained above), so we prefer to keep nominal values small. Apart from that, we will only use nominal values that are specified in [1].

With these two restrictions, we increased the nominal values in the sensitivity test weight with the aim to reduce uncertainties of all sensitivities to 1.0 E-05 or less. In order to keep the procedures easy to understand for all laboratory personnel, we kept the number of weighing cycles n for the sensitivity test to be one (1) ABBA only.

Increasing the nominal values, we iteratively find a nominal value for a sensitivity test weight for every comparator type that satisfies the condition to produce sensitivities with an uncertainty of 1.0 E-05 or less. The new values are shown in Figure 3. Please note the difference in y-axis scale compared to Figure 2.

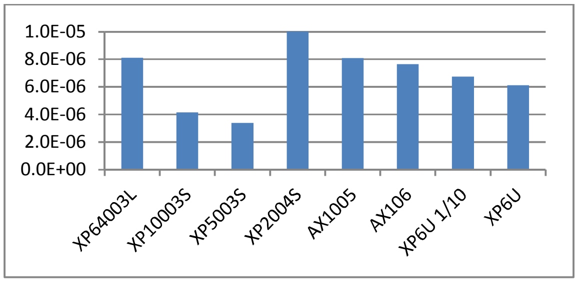

Figure 3: Sensitivity uncertainties with optimized procedure.

This result was achieved using the sensitivity test weights for the comparator models as shown in Table 2.

Table 2: Mass comparators and test weights for optimized sensitivity.

Comparator	XP64003L	XP10003S	XP5003S	XP2004S	AX1005	AX106	XP6U 1/10 d	XP6U
Δm_c (mg)	1000	500	500	100	50	20	10	10

8 GENERAL COOKBOOK PROCEDURE FOR ASSESSING OPTIMIZED COMPARATOR SENSITIVITY AND ITS UNCERTAINTY

The following procedure can be derived from the considerations above:

1. Set and define a maximum acceptable value for sensitivity uncertainty (e.g. 1.0 E-05).
2. Use (3) and iteratively increase the nominal weight value until the above condition is fulfilled for each comparator.
3. Execute sensitivity test and apply the value found for S for each comparator to all readings.
4. Use the value of u_s for the uncertainty estimation of mass calibrations according to [1].

9 DISCUSSION ON FURTHER VARIATIONS

9.1 Variation of sensitivity test weight accuracy

Except for microbalance comparators, the calibration uncertainty of the sensitivity test weight has little influence on the uncertainty in S. So using weights calibrated in E_2 quality instead of E_1 weights is possible without major disadvantage.

9.1 Variation of sensitivity test weight size for microbalances

For a target uncertainty value of 1 E-05, a microbalance of type XP6U with a readability of 0.0001 mg requires a sensitivity test weight of not smaller than 10 mg (see Table 2). Trying a smaller 1 mg E_1 weight instead, yields an uncertainty of 6 E-04 which is even higher than the cited literature value. Thus, using smaller weights on microbalances to assess sensitivity has no positive effect in the specific light of this investigation.

10 CONCLUSIONS

The sensitivity of mass comparators and its associated uncertainty are both values with important significance in the field of weight piece calibration. The literature does not provide much guidance neither on procedures to be used for assessing sensitivity and its uncertainty nor on the selection of suitable weights for sensitivity testing. We have presented a procedure for assessing and evaluating sensitivity and its associated uncertainty. By means of iterative application, a test weight can be selected so that relative sensitivity uncertainties of e.g. 1.0 E-05 or less are achieved.

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