

# Research on volume determination of mass standards with a new acoustic method

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## ABSTRACT

Acoustic volume measuring method is a promising non-contact method for volume determination of mass weights. To improve the measuring accuracy of volume determination with non-contact acoustic method, an acoustic measuring system with two measuring chambers is newly designed to compensate the non-linearly measuring errors. The volumes of mass standards ranging from 200 g to 5 kg are tested to evaluate the non-linearity errors of measurement.

**Keywords:** Mass standard; Volume measurement; Acoustic method;

## 1. INTRODUCTION

For the mass measurement of a weight, ABBA cycle is used on a mass comparator in which A is the reference (standard) mass weight and B is the test mass weight, and the mass comparison is usually carried out in the air based on the difference of gravitational force caused by the standard or test weight on the mass comparator, and the air buoyancy can contribute a big uncertainty especially in the high accuracy mass measurement such as the prototype level or E<sub>1</sub> class level. Thus the mass standard's volume needs to be precisely determined for the air buoyancy correction.[1]

There are many measurement technologies such as hydrostatic method, dimension measuring method and acoustic method. Although the measuring accuracy cannot be as the same level as hydrostatic method (usually with relative combined uncertainty low to  $1 \times 10^{-6}$ ), acoustic method is a promising volume measuring method for its non-contact to mass standards during the whole volume measuring procedure, especially for the weights with non-regular shape and 3D curved surface. Both the standard weights and test weights don't need to be immersed in any liquid.

M. Ueki et al. firstly developed an acoustic measuring system to determine the volume of mass weights ranging from 1 g to 10 kg in National Metrology Institute of Japan (NMIJ). [2-6] For the weights with nominal value ranging from 100 g to 10 kg, a relative uncertainty of  $1 \times 10^{-3}$  ( $k=2$ ) is achieved. For weights ranging from 1 g to 100 g, the measuring combined standard uncertainty is below  $0.0021 \text{ cm}^3$ . [2-5] An acoustic volume measuring system also has also been designed in National Institute of Metrology China (NIM) to extend the measurement range of nominal value of mass weight up to 20 kg.[7] However, because of the use of the approximate formula equation to get the high measuring accuracy, the ratio of shape and volume of reference weight needs to be similar to the test weight. Otherwise there will be a big non-linearly measurement errors introduced to the measuring process.

To investigate the non-linearity contribution in the acoustic measuring method, an acoustic measuring system with two measuring chambers is designed by National Institute of Metrology China. The volumes of mass standards ranging from 200 g to 5 kg are tested to evaluate the non-linearity errors of acoustic measuring process.

## 2. EXPERIMENTAL APPARATUS AND MEASURING PROCEDURE

### 2.1 EXPERIMENTAL APPARATUS

Acoustic method is based on gas compressibility laws. In state of gas is adiabatic, the air pressure,  $P$ , has a constant relation with the volume of air,  $V$ , as expressed in (1):

$$P \times V^\gamma = \text{cons} \quad (1)$$

Here,  $\gamma$  is ratio of specific heats, which is 1.40 at atmospheric pressure and room temperature. The newly designed measuring apparatus made of aluminum alloy with two measuring chambers is showed in Figure 1.

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A Sinusoidal drive signal from a signal generator is applied to the loudspeaker between the two measuring chambers. This will alternately generate compression wave with inverse phase in the left chamber and the right container.

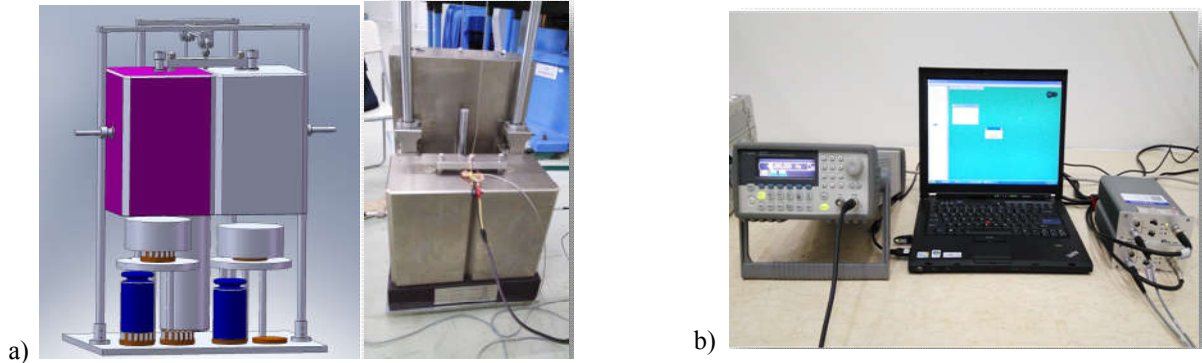


Figure 1: Schematic and pictures of the new measuring apparatus with two measuring chambers

Two Sound pressure sensors (also called microphones) are used to separately measure the pressure changes, that is,  $\Delta P_1$  in the left chamber and  $\Delta P_2$  in the right chamber respectively, as shown in (2) and (3), where  $P_0$  is the air pressure in the chamber. The output signals from two microphones,  $e_1$  and  $e_2$ , are converted into digital signals and sent to a computer for sound pressure calculation. The result sound pressure  $\Delta P_x$  is measured. The ratio of the pressures  $R_n$  can be calculated as  $\Delta P_1/\Delta P_2$ . [3]

$$\frac{\Delta P_1}{P_0} = \gamma \frac{\Delta V}{V_{01}} \quad (2)$$

$$\frac{\Delta P_2}{P_0} = \gamma \frac{\Delta V}{V_{02}} \quad (3)$$

## 2.2 MEASURING PROCEDURE

When measuring the volume with acoustic method, it is assumed that air changes adiabatically in the two measurement chambers. [2-7] However, air near the surface of the test weight or reference weight and the wall of the containers changes isothermally.[3] Thus during the measurement, the actual displaced volume in the chamber by test weight,  $V_{t0}$  and reference weight,  $V_{r0}$  can be expressed with (4) and (5).

$$V_{t0} = V_t - dS_t \quad (4)$$

$$V_{r0} = V_r - dS_r \quad (5)$$

where  $S_t$  and  $S_r$  are the surface area of the test weight and reference weight, and  $d$  is the thickness of air isothermal layer.[5]

To evaluate the effect of surface area to the volume measurement, the effect of surface area to measuring is not considered firstly, which means  $V_{t0} = V_t$ . And based on the sequence of Figure 2, Equation (6-8) can be concluded. And thus the volume of test weight,  $V_t$  can be calculated with (9). Equation (9) is used to evaluate the non-linearity error caused by the effect of surface area. [3]

$$R_1 = \frac{\Delta P_1}{\Delta P_2} = \frac{V_{02} - V_t}{V_{01} - V_r} \quad (6)$$

$$R_2 = \frac{\Delta P_1}{\Delta P_2} = \frac{V_{02} - V_r}{V_{01} - V_t} \quad (7)$$

$$R_3 = \frac{\Delta P_1}{\Delta P_2} = \frac{V_{02} - V_t - V_r}{V_{01}} \quad (8)$$

$$V_t = V_r \times \frac{(R_2 - R_3)(1 + R_1)}{(R_1 - R_3)(1 + R_2)} \quad (9)$$

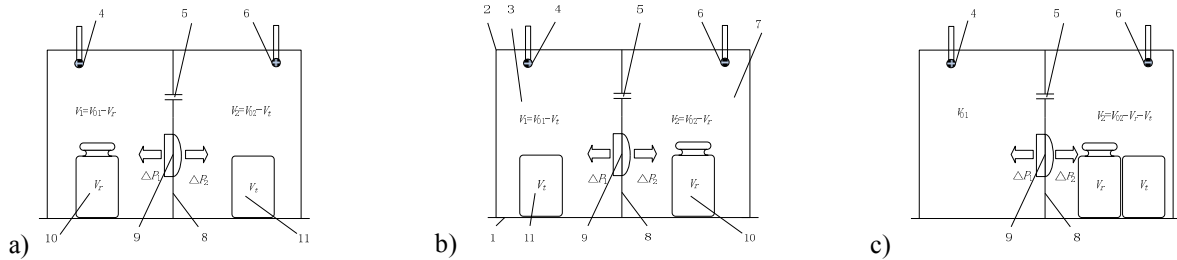


Figure 2: The schematic of procedures (a, b, c) of measuring process, in which 1: Bottom of measuring chamber, 2: Side walls of measuring chamber, 3: Left measuring chamber, 4: Sound pressure sensor 1, 5: Connecting tube, 6: Sound pressure sensor 2, 7: Right measuring chamber, 8: Separating wall, 9: Loudspeaker, 10: Reference weight, 11: Test weight.

### 3. MEASURING RESULTS AND UNCERTAINTY ANALYSIS

#### 3.1 MEASURING RESULTS

According to (9), the amplitude ratio  $R$  is the key parameter for the acoustic volume measurement. As the driving signal to the loudspeaker as showed in Figure 3, the amplitude and frequency of sinusoidal signal should be carefully chosen to achieve best measurement of sound pressure and the amplitude ratio  $R$ .

Figure 4 shows the relationship between  $R$  and amplifier & frequency of sinusoidal signal using two measuring chambers. For each amplitude and frequency, 100 samples of sound pressure are acquired. It can be seen that to obtain an accuracy of  $R$  better than  $1 \times 10^{-4}$ , the amplitude of sinusoidal drive signal should be between 1.6 V and 1.9 V, and the frequency should be 43 Hz. The parameters used for the mass volume measurement are shown in Table 1.



Figure 3: schematic of the connection of signal generator to the Loudspeaker

Table 1. Parameters used for the volume measurement

Parameters	Configuration of sinusoidal signal
Gain of left chamber (dB)	$\times 20$
Gain of right chamber (dB)	$\times 20$
Sinusoidal signal Frequency (Hz)	43
Sinusoidal signal Voltage (V)	1.6~1.9

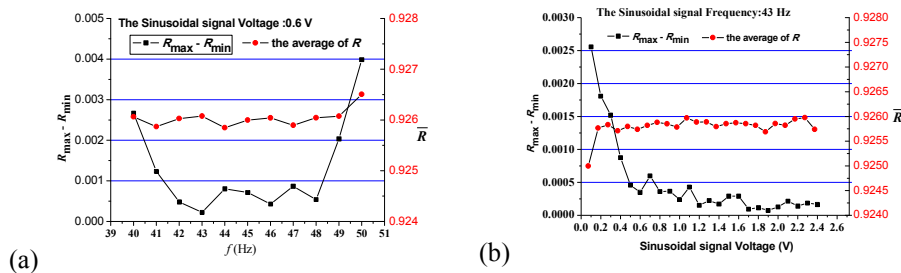


Figure 4. Relationship between  $R$  and the amplifier or frequency of sinusoidal signal.

According to (9), as showed in Figure 5 to evaluate the non-linearity errors, nominal mass weights of 100 g is used as the reference weight, and nominal mass weights ranging from 200 g to 5 kg are measured with 3 step showed in Figure 2. The non-linearity error is expressed as the deviation of measurement value by acoustic method to the volume measuring value by hydrostatic method. With the same reference weight, the non-linearity error of measured volume of test weight increases with its nominal value.



Figure 5. The reference weight of 100 g and test weights ranging from 200 g to 5 kg (a) and the measuring results (b).

Also as shown in Figure 6, a test weight with nominal value of 2 kg is used as the test weight, weights ranging from 100 g to 1 kg are used as the reference weight. The non-linearity error of measured volume of 2 kg weight decreases when the nominal value of reference weight increases.

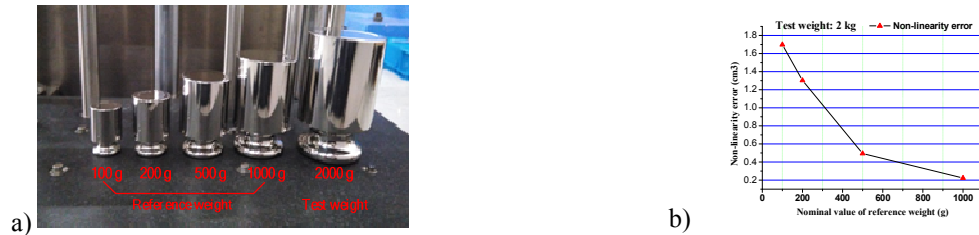


Figure 6. The reference weights ranging from 100 g to 1 kg and test weight of 2 kg (a) and the measuring results (b).

Based on the analysis in Figure 5 and Figure 6, it can be concluded that the more difference of the volume, surface or the ratio of volume and surface between the reference weight and the test weight, the more non-linearity of acoustic measuring method can be introduced. If the effect of weight surface can be compensated, the accuracy of acoustic volume method can be improved significantly.

### 3.2 UNCERTAINTY ANALYSIS

According to mathematic equation (9) of the measuring process, There are four main contribution factors which are the reference weight's volume,  $R_1$ ,  $R_2$  and  $R_3$ . The Uncertainty budget of the volume of 200 g weight using 100 g weight as reference weight is shown in Table 2. With the same uncertainty evaluation method, the volume uncertainty evaluation results of weights ranging from 5 kg to 200 kg are shown in Table 3, and the relative extended uncertainties are almost in the same level for the introduction of non-linearity error during the measuring process. The volume uncertainties of 2 kg weight using different reference weights are shown in Table 4. The Relative extended uncertainty decreases with the reference weight's nominal value which indicates that the non-linearity errors decrease in the same trend.

Table 2. Uncertainty budget of the volume of 200 g weight using 100 g weight as reference weight

Sources	Uncertainty	Factor	Uncertainty contribution
Volume of reference weight( $\text{cm}^3$ )	0.0005	1	0.0005
$R_1(\text{cm}^3)$	0.00001	11152 $\text{cm}^3$	0.11
$R_2(\text{cm}^3)$	0.00001	5545 $\text{cm}^3$	0.06
$R_3(\text{cm}^3)$	0.00001	16672 $\text{cm}^3$	0.17
Combined uncertainty( $\text{cm}^3$ )			0.21
Extended uncertainty( $\text{cm}^3$ ) ( $k=2$ )			0.42
Relative extended uncertainty( $\times 10^{-2}$ , $k=2$ )			1.7

Table 3. Uncertainty budget of test weights using 100 g weight as reference weight

Sources	5 kg	2 kg	1 kg	500 g	200 g
Volume of reference weight( $\text{cm}^3$ )	0.0005	0.0005	0.0005	0.0005	0.0005
$R_1(\text{cm}^3)$	3.14	1.2	0.53	0.27	0.11
$R_2(\text{cm}^3)$	0.06	0.06	0.05	0.05	0.06
$R_3(\text{cm}^3)$	3.20	1.17	0.59	0.32	0.17
Combined uncertainty( $\text{cm}^3$ )	4.49	1.62	0.79	0.42	0.21
Extended uncertainty( $\text{cm}^3$ )	8.97	3.24	1.59	0.84	0.42
Relative extended uncertainty( $k=2$ )	$1.4 \times 10^{-2}$	$1.3 \times 10^{-2}$	$1.2 \times 10^{-2}$	$1.3 \times 10^{-2}$	$1.7 \times 10^{-2}$

Table 4. Uncertainty budget of the volume 2 kg weight using different reference weights

Sources	Reference weights			
	1 kg	500 g	200 g	100 g
Volume of reference weight(cm <sup>3</sup> )	0.003	0.002	0.001	0.0005
$R_1$ (cm <sup>3</sup> )	0.11	0.22	0.55	1.18
$R_2$ (cm <sup>3</sup> )	0.06	0.06	0.06	0.06
$R_3$ (cm <sup>3</sup> )	0.16	0.27	0.60	1.17
Combined uncertainty(cm <sup>3</sup> )	0.21	0.36	0.8	1.6
Extended uncertainty(cm <sup>3</sup> )	0.42	0.71	1.6	3.2
Relative extended uncertainty( $k=2$ )	$1.6 \times 10^{-3}$	$2.8 \times 10^{-3}$	$6 \times 10^{-3}$	$1.2 \times 10^{-2}$

#### 4. CONCLUSION

To investigate the non-linearity contribution in the acoustic measuring method, an acoustic measuring system with two measuring chambers is newly designed. The optimization of measurement parameters was investigated by analyzing the relationship between  $R$  and the amplifier or frequency of sinusoidal signal. The volumes of mass standards ranging from 200 g to 5 kg are tested to evaluate the non-linearity errors of acoustic measuring method.

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