XXI IMEKO World Congress "Measurement in Research and Industry"
August 30 – September 4, 2015, Prague, Czech Republic

DYNAMIC MEASUREMENTS ERROR CORRECTION ON THE BASIS OF NEURAL NETWORK INVERSE MODEL OF A SENSOR

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Abstract – The neural network inverse model of a sensor with the filtration of the sequentially recovered signal is considered. This model allows it to effectively correct the dynamic measurements error due to the deep mathematical processing of measurement data. The result of the experimental data processing of the dynamic temperature measurements validates the efficiency of the proposed model and the algorithm of the dynamic measurements error correction.

Keywords: dynamic measurements error, neural network model, inverse sensor model, recovery of sensor input signal, dynamic measurements data processing.

1. INTRODUCTION

The automatic control theory approach allows it to effectively improve the dynamic measurements accuracy [1, 2]. Along with it, the artificial neural network (ANN) approach to the creation of the dynamic models of measuring systems and algorithms for the data processing of dynamic measurements is one of the promising ways of the intelligent measuring systems development.

2. NEURAL NETWORK DIRECT MODEL OF A SENSOR

Suppose a primary measuring transducer (sensor) is described by the transfer function (TF) as follows (where U and Y are the sensor input and output signals respectively; T_{1j} and T_{2i} are time constants; ξ_{1j} and ξ_{2i} are damping coefficients; $i=1,2,\ldots,m$, $j=1,2,\ldots,n$ ($m \le n$); K_0 is the static gain; p is the complex number frequency):

$$W_s(p)=\frac{Y(p)}{U(p)}= = K_0\,\frac{\prod\limits_{i=1}^{k}\left(T_{2i}^2 p^2+2\xi_{2i}T_{2i}\,p+1\right)\prod\limits_{i=k+1}^{m}\left(T_{2i}\,p+1\right)}{\prod\limits_{j=1}^{l}\left(T_{1j}^2 p^2+2\xi_{1j}T_{1j}\,p+1\right)\prod\limits_{j=q+1}^{n}\left(T_{1j}\,p+1\right)} \tag{1}$$

The continuous TF (1) of the sensor can be represented in the following way (where $b_i=b_i\left(T_{2i},\xi_{2i},K_0\right)$ for $i=1,2,\ldots,m$ are coefficients depending on the parameters of the sensor TF (1) numerator and the static gain; $a_i=a_i\left(T_{1j},\xi_{1j}\right)$ for $j=1,2,\ldots,n$ are coefficients depending on the parameters of the sensor TF (1) denominator):

$$W_s(p)=\frac{Y(p)}{U(p)}= =\frac{b_m\cdot p^m+b_{m-1}\cdot p^{m-1}+\ldots+b_1\cdot p+b_0}{p^n+a_{n-1}\cdot p^{n-1}+\ldots+a_1\cdot p+a_0} \tag{2}$$

The discrete analogue of the continuous TF (2) can be represented as follows (where $U(z)$ and $Y(z)$ are the z-transformation of the sensor input and output signals, respectively; $\beta_i=\beta_i\left(b_0,\ldots,b_m,a_0,\ldots,a_{n-1},T\right)$ for $i=0,1,\ldots,n$ and $\alpha_j=\alpha_j\left(b_0,\ldots,b_m,a_0,\ldots,a_{n-1},T\right)$ for $j=1,2,\ldots,n$ are coefficients depending on the parameters of the sensor TF (2) and the sampling period T):

$$W_s(z)=\frac{Y(z)}{U(z)}= =\frac{\beta_0+\beta_1\cdot z^{-1}+\beta_2\cdot z^{-2}+\ldots+\beta_n\cdot z^{-n}}{1-\alpha_1\cdot z^{-1}-\alpha_2\cdot z^{-2}-\ldots-\alpha_n\cdot z^{-n}} \tag{3}$$

The difference equation corresponding to the discrete TF (3) of the sensor is as follows (where $u(k)$ and $y(k)$ are samples of the sensor input and output signals, respectively, at discrete times $t_k=k\cdot T$ for $k=0,1,2,\ldots$):

$$y(k)-\sum_{i=1}^{n}\alpha_i\cdot y(k-i)=\sum_{j=0}^{n}\beta_j\cdot u(k-j) \tag{4}$$

The relationship between the output and input of the discrete sensor model is described by the following recurrence equation derived from the previous one:

$$y(k)=\sum_{i=1}^{n}\alpha_i\cdot y(k-i)+\sum_{j=0}^{n}\beta_j\cdot u(k-j) \tag{5}$$

The parameters of the discrete model (3) can be determined on the basis of the ANN direct sensor model shown in fig. 1. This model is the recurrent ANN consisting of a single neuron with the linear activation function $f_a(net)$ and the zero bias b_0. The structure of this model is fully consistent with the recurrence equation (5).

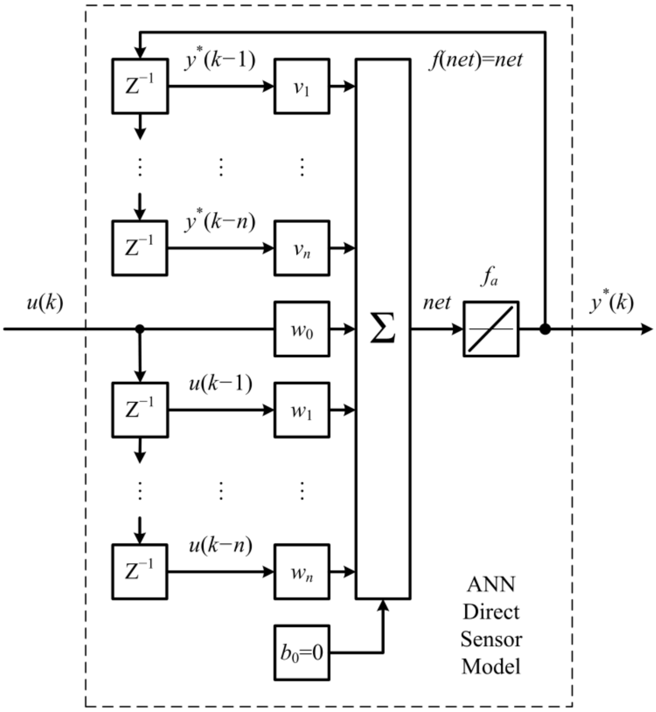

Fig. 1. Block diagram of the ANN direct sensor model

The recurrence equation that determines the relationship between input and output of the ANN direct sensor model is as follows (where $u(k)$ and $y^*(k)$ are samples of the sensor input signal and the output signal of the ANN direct sensor model, respectively, at discrete times $t_k = k \cdot T$ for $k = 0, 1, 2, \dots;$ w_i for $i = 0, 1, \dots, n$ and v_j for $j = 1, 2, \dots, n$ are adjustable coefficients (weights) of the ANN direct sensor model):

$$\begin{aligned} y^*(k) &= f_a(net) = net = \\ &= \sum_{i=1}^{n} v_i \cdot y^*(k-i) + \sum_{j=0}^{n} w_j \cdot u(k-j) \end{aligned} \tag{6}$$

By means of an appropriate procedure of the input and target training sets formation, that reflects the relationship between the input and output of the discrete sensor model (3), the weights of the ANN direct sensor model can be adjusted during the training process so that the samples of the ANN sensor model output will be equal to the respective samples of the continuous sensor model (1) output for a given level of the accuracy. The indicated possibility follows from the linearity and the conformity of the discrete and the ANN direct models of the sensor. Indeed, if $y^*(k) = y(k)$ for $k = 0, 1, 2, \dots,$ then from the equations (5) and (6) the following equality can be obtained:

$$\begin{aligned} &\sum_{i=1}^{n} \alpha_i \cdot y(k-i) + \sum_{j=0}^{n} \beta_j \cdot u(k-j) = \\ &= \sum_{i=1}^{n} v_i \cdot y(k-i) + \sum_{j=0}^{n} w_j \cdot u(k-j) \end{aligned} \tag{7}$$

Provided the sensor input is nonzero, the last equality becomes the identity only when $\beta_i = w_i$ for $i = 0, 1, \dots, n$ and $\alpha_j = v_j$ for $j = 1, 2, \dots, n$.

The functional block diagram of the ANN direct sensor model training is shown in fig. 2. The procedure of the ANN direct sensor model training (i.e. its weights adjustment) consists in minimization of the training error represented as the aggregate standard deviation for the all $N+1$ samples of the input training set $h_u(k)$ between the target $h_y(k)$ and actual $h_y^*(k)$ outputs of the ANN direct sensor model:

$$E = \frac{1}{N} \sum_{k=0}^{N} \left(h_y(k) - h_y^*(k) \right)^2 \tag{8}$$

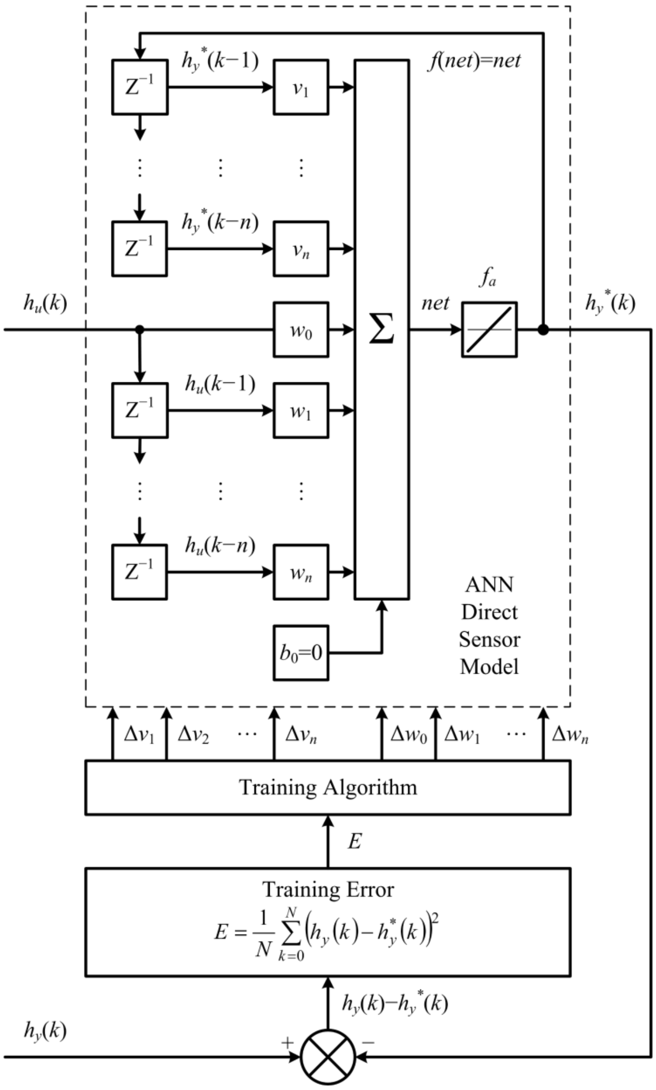

Fig. 2. Functional block diagram of the ANN direct sensor model training

3. NEURAL NETWORK INVERSE MODEL OF A SENSOR

The discussed in the previous section approach to the ANN direct sensor model creation for the solution to the problem of the sensor described by the TF (1) dynamic measurement error correction. Then, this problem is formulated as the problem of the dynamically distorted sensor input signal recovery on the basis of the respective samples of its output signal.

Considering the indicated formulation it is necessary on the basis of the ANN direct sensor model and the functional block diagram of its training to create the ANN inverse sensor model and, similarly, the functional block diagram of its training. This ANN inverse sensor model should provide the recovery of the dynamically distorted sensor input signal, i.e. implement the inverse relationship between the sensor input and output.

3.1. General representation

The discrete model of the sensor described by the TF (3) is considered to obtain the structure of the ANN inverse sensor model. This TF is represented in the inverse form as follows (where $\mu_0 = -\frac{1}{\beta_0}$, $\mu_i = -\frac{\alpha_i}{\beta_0}$ and $\lambda_i = -\frac{\beta_i}{\beta_0}$ for $i = 1, 2, \ldots, n$):

$$\begin{aligned} W_s^{-1}(z) &= \frac{U(z)}{Y(z)} = \\ &= \left(\frac{\beta_0 + \beta_1 \cdot z^{-1} + \beta_2 \cdot z^{-2} + \ldots + \beta_n \cdot z^{-n}}{1 - \alpha_1 \cdot z^{-1} - \alpha_2 \cdot z^{-2} - \ldots - \alpha_n \cdot z^{-n}} \right)^{-1} = \\ &= \frac{1 - \alpha_1 \cdot z^{-1} - \alpha_2 \cdot z^{-2} - \ldots - \alpha_n \cdot z^{-n}}{\beta_0 + \beta_1 \cdot z^{-1} + \beta_2 \cdot z^{-2} + \ldots + \beta_n \cdot z^{-n}} = \\ &= \frac{\frac{1}{\beta_0} - \frac{\alpha_1}{\beta_0} \cdot z^{-1} - \frac{\alpha_2}{\beta_0} \cdot z^{-2} - \ldots - \frac{\alpha_n}{\beta_0} \cdot z^{-n}}{1 + \frac{\beta_1}{\beta_0} \cdot z^{-1} + \frac{\beta_2}{\beta_0} \cdot z^{-2} + \ldots + \frac{\beta_n}{\beta_0} \cdot z^{-n}} = \\ &= \frac{\mu_0 + \mu_1 \cdot z^{-1} + \mu_2 \cdot z^{-2} + \ldots + \mu_n \cdot z^{-n}}{1 - \lambda_1 \cdot z^{-1} - \lambda_2 \cdot z^{-2} - \ldots - \lambda_n \cdot z^{-n}} \end{aligned} \quad (9)$$

The difference equation corresponding to the inverse discrete TF (9) of the sensor is as follows (where $u(k)$ and $y(k)$ are samples of the sensor input and output signals, respectively, at discrete times $t_k = k \cdot T$ for $k = 0, 1, 2, \ldots$):

$$u(k) - \sum_{i=1}^{n} \lambda_i \cdot u(k-i) = \sum_{j=0}^{n} \mu_j \cdot y(k-j) \quad (10)$$

The relationship between the input and output of the inverse discrete sensor model is as the following recurrence equation derived from the previous one:

$$u(k) = \sum_{i=1}^{n} \lambda_i \cdot u(k-i) + \sum_{j=0}^{n} \mu_j \cdot y(k-j) \quad (11)$$

The structure of the equation (11) for the inverse discrete sensor model is the same as the structure of the equation (5) for the direct discrete sensor model. Therefore the structure of the ANN inverse sensor model will also be the same as the structure of the ANN direct sensor model.

The block diagram of the ANN inverse sensor model is shown in fig. 3. This model is the recurrent ANN consisting of a single neuron with the linear activation function $f_a(net)$ and the zero bias b_0. The structure of this model is fully consistent with the recurrence equation (11).

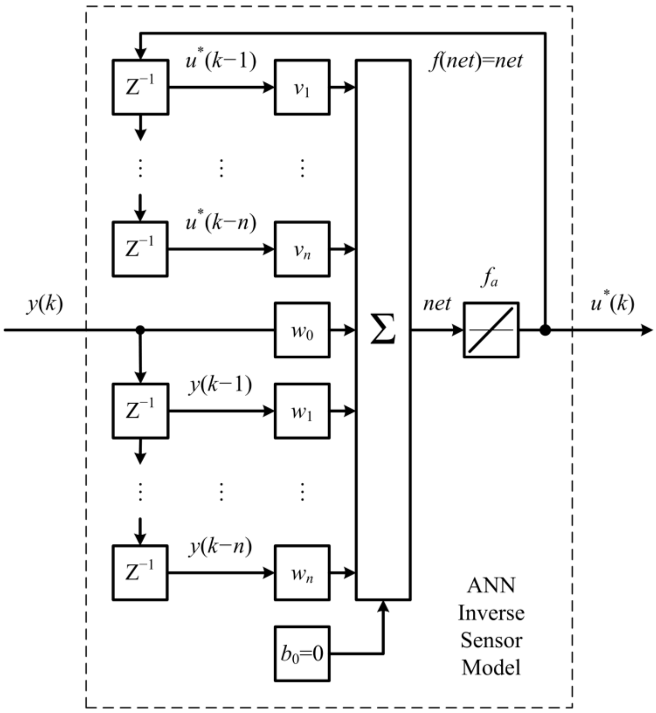

Fig. 3. Block diagram of the ANN inverse sensor model

The recurrence equation that determines the relationship between the input and output of the ANN inverse sensor model is as follows (where $y(k)$ and $u^*(k)$ are samples of the sensor output signal and the output signal of the ANN inverse sensor model, respectively, at discrete times $t_k = k \cdot T$ for $k = 0, 1, 2, \ldots$; w_i for $i = 0, 1, \ldots, n$ and v_j for $j = 1, 2, \ldots, n$ are weights of the ANN inverse sensor model):

$$\begin{aligned} u^*(k) &= f_a(net) = net = \\ &= \sum_{i=1}^{n} v_i \cdot u^*(k-i) + \sum_{j=0}^{n} w_j \cdot y(k-j) \end{aligned} \quad (12)$$

The criterion for the ANN inverse sensor model training as in the case of the ANN direct sensor model training is the minimum of the training error represented as the standard deviation between the target and actual outputs of the ANN inverse sensor model. Obviously, both in the criterion and in the functional block diagram of the ANN inverse sensor model training it is necessary to swap the input and target training sets towards the criterion and the functional block diagram of the ANN direct sensor model training.

3.2. Cascade representation

In order to avoid possible problems with the stability of the inverse model discussed, the cascade representation of the ANN inverse sensor model in the form of first and second order sections is proposed. This approach is based on the representation of the direct sensor model described by the TF (1) in the following form of s_1 first-order and s_2 second order cascades with the real coefficients:

$$W_s(p) = \prod_{i=1}^{s_1} W_{1i}(p) \cdot \prod_{j=1}^{s_2} W_{2j}(p) \tag{13}$$

The functional block diagram of the ANN inverse model of the sensor in the cascade representation is shown in fig. 4 (where $C_1[i]$ are sections implementing the inverse model of the first-order cascades $W_{1i}(p)$ for $i = 1, 2, \ldots, s_1$ and $C_2[j]$ are sections implementing the inverse model of the second-order cascades $W_{2j}(p)$ for $j = 1, 2, \ldots, s_2$).

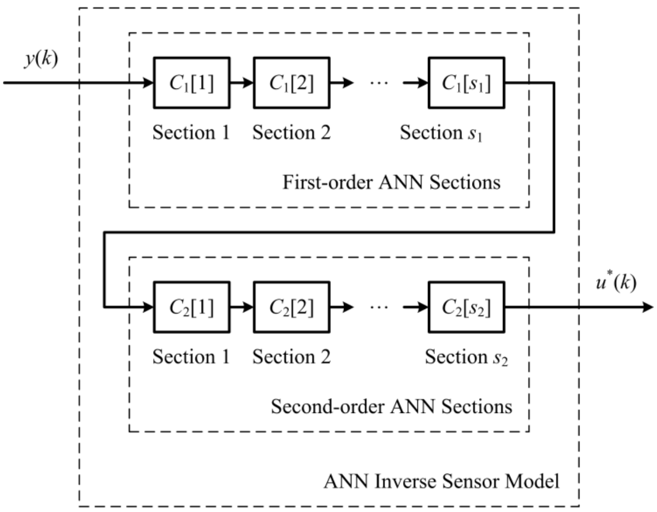

Fig.4. Functional block diagram of the ANN inverse model of the sensor in the cascade representation

Thus, the structure of the cascade ANN inverse model of the sensor corresponds to the structure of the sensor in the cascade representation. Therefore, each section is the ANN inverse sensor model of the respective cascade in the structure of the sensor described by the TF (13).

3.3. Sequential representation

The recovery of the input signal of the sensor with TF (1) is implemented by its measured output signal processing on the basis of the ANN inverse sensor model. This model is presented as the sequential connection of the correcting filter and the identical first-order sections [3]. Every such section is the ANN inverse model of a unit with the following first-order TF:

$$W_1(p) = \frac{1}{pT_1 + 1} \tag{14}$$

The value of the time constant T_1 in the TF (14) is set equal to such a value among the time constants T_{1j} in the TF (1), that provides the proximity of the step responses of units with the TFs (1) and (14). The TF $W_{cf}(p)$ of the correcting filter is the inverse TF of the sensor, which is supplemented with a certain number $q = n - m$ of the TFs (14) to ensure the stability of the inverse model. The TF $W_{cf}(p)$ of the correcting filter is as follows:

$$\begin{aligned} W_{cf}(p) &= W_s^{-1}(p) \cdot W_1^q(p) = \\ &= W_s^{-1}(p) \cdot \left(\frac{1}{pT_1 + 1} \right)^q = \\ &= \frac{1}{K_0} \cdot \frac{\prod_{j=1}^{l} \left(T_{1j}^2 p^2 + 2\xi_{1j} T_{1j} p + 1 \right) \prod_{j=q+1}^{n} \left(T_{1j} p + 1 \right)}{\prod_{i=1}^{k} \left(T_{2i}^2 p^2 + 2\xi_{2i} T_{2i} p + 1 \right) \prod_{i=k+1}^{m} \left(T_{2i} p + 1 \right)} \times \\ &\times \frac{1}{\left(pT_1 + 1 \right)^{n-m}} \end{aligned} \tag{15}$$

The recovery of the dynamically distorted input signal of the sensor on the basis of its ANN inverse model can be accompanied by the significant increase of the additive noise at the sensor output, as well as the internal noise of the ANN inverse sensor model.

For the correct recovery of the input signal of the sensor it is expedient to expand the ANN inverse sensor model, taking into account the presence of the additive noise at the sensor output. This expansion can be implemented as the additional low-pass filtration of the recovered signal by means of an increase of the order of the sequential sections in the structure of the ANN inverse sensor model.

The functional block diagram of the ANN inverse sensor model is shown in fig. 5, where d -order section $C_d[T_1]$ is the ANN inverse model of the unit with first-order TF (14).

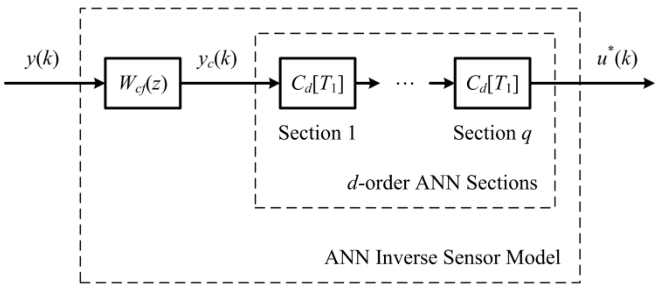

Fig.5. Functional block diagram of the ANN inverse model of the sensor in the sequential representation

The proposed sequential representation of the inverse sensor model allows it to implement the structure of the section $C_d[T_1]$ as a nonrecursive ANN filter. The basic advantage of this structure is its guaranteed stability.

Therefore, it guarantees stability of the ANN inverse model of the sensor as a whole and provides simultaneous and stable the sensor input signal recovery and filtration.

3.4. Dynamic model of the first-order section

The block diagram of the ANN section $C_d[T_1]$ with the filtration of the recovered signal is shown in fig. 6.

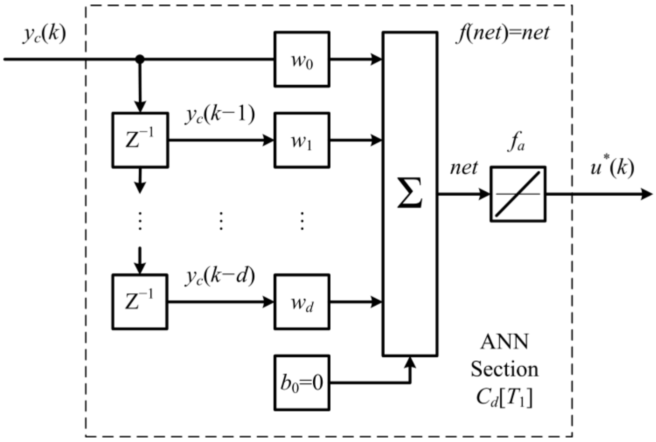

Fig. 6. Block diagram of the ANN section $C_d[T_1]$

The discrete TF of the section $C_d[T_1]$ is as follows (where $w_0, w_1, w_2, \ldots, w_d$ are adjustable coefficients (weights) of the section):

$$W_{cd}(z) = w_0 + w_1 \cdot z^{-1} + \ldots + w_d \cdot z^{-d} \tag{16}$$

The elimination of the additive noises amplified during the sensor input signal recovery is implemented due to the presence of the internal filter in the structure of the ANN section $C_d[T_1]$. The discrete TF of the internal filter in the structure of the ANN section $C_d[T_1]$ is described by the following equation (where $W_1(z)$ is the discrete analogue of the continuous TF (14)):

$$W_{fd}(z) = W_{cd}(z) \cdot W_1(z) \tag{17}$$

3.5. Training procedure and training sets composition

The block diagram of the ANN section $C_d[T_1]$ training is shown in fig. 7. The procedure of the ANN section training (i.e. its weights $w_0, w_1, w_2, \ldots, w_d$ adjustment) consists in the minimization of the training error represented as the aggregate standard deviation for all $N+1$ samples of the input training set $h_y(k)$ between the target $h_u(k)$ and actual $h_u^*(k)$ outputs of the ANN section $C_d[T_1]$:

$$E = \frac{1}{N} \sum_{k=0}^{N} \left(h_u(k) - h_u^*(k) \right)^2 \tag{18}$$

The algorithm of the training sets composition and their length evaluation for the ANN section $C_d[T_1]$ weights adjustment during the training procedure was proposed [3].

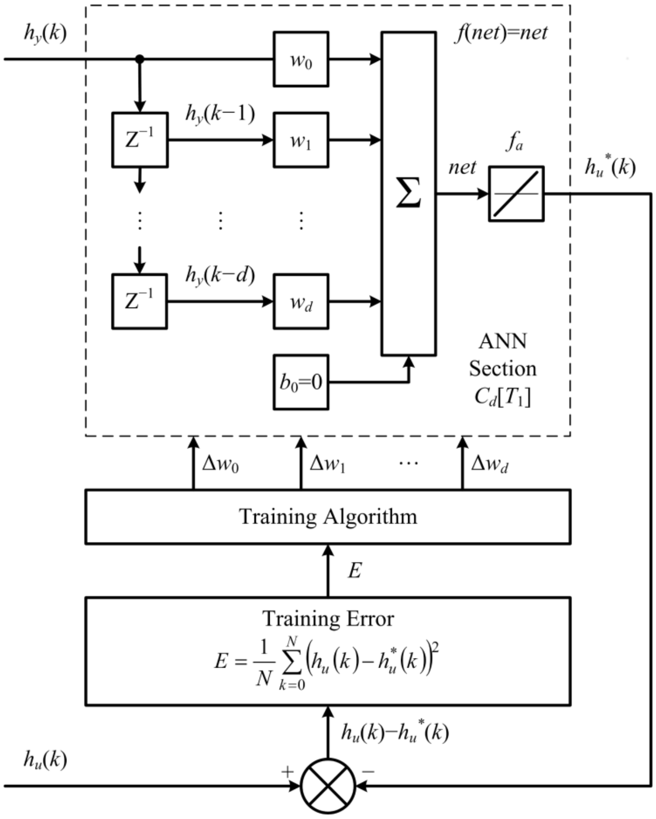

Fig. 7. Block diagram of the ANN section $C_d[T_1]$ training

In order to implement the possibility of the ANN section $C_d[T_1]$ internal filter bandpass regulation by means of the section order d adjusting the sinusoidal smoothing was applied. On its base, the samples of the target training set are composed according to the following equation (where T is the sampling period and $k = 0, 1, \ldots, N$):

$$h_u(k) = \begin{cases} \dfrac{1 + \sin\left(\dfrac{\pi}{2} \cdot T \cdot \left(k - \dfrac{d}{2} \right) \right)}{2}, & 0 \le k \le d \\ 1, & d < k \le N \end{cases} \tag{19}$$

Then, the input training set should be composed from the samples of the TF (14) step response as follows (where $t_k = k \cdot T$ are discrete times for $k = 0, 1, \ldots, N$):

$$h_y(k) = 1 - \exp(-t_k / T_1) = 1 - \exp(-k \cdot T / T_1) \tag{20}$$

Suppose, provided $\varepsilon \ll 1$, starting with time $T_h = N \cdot T$ all the successive samples of the TF (14) step response lie within the following range:

$$\Delta_h = (1 \pm \varepsilon) \tag{21}$$

Then, the training sets can be composed in accordance with the equations (19) and (20).

The use of the sensor step response as the source signal for the input training set composition allows it to define the required length of the following training sets:

$$H_u = \left[h_u(0) \quad h_u(1) \quad h_u(2) \quad \ldots \quad h_u(N) \right] \tag{22}$$

$$H_y = \left[h_y(0) \quad h_y(1) \quad h_y(2) \quad \ldots \quad h_y(N) \right] \tag{23}$$

This consequence follows from the analysis of the equation (18) for the ANN section $C_d[T_1]$ training in terms of the problem considered.

The limiting value of the training error, where the length of the training sets approaches the infinity, is as follows:

$$\begin{aligned} E_0 &= \lim_{N\to\infty} E = \lim_{N\to\infty} E(w_0, w_1, w_2, \ldots, w_d) = \\ &= \lim_{N\to\infty}\left[\frac{1}{N}\sum_{k=0}^{N}\left(h_u(k) - \sum_{i=0}^{d} w_i \cdot h_u^*(k-i)\right)^2\right] = \\ &= \left(1 - \sum_{i=0}^{d} w_i\right)^2 \end{aligned} \tag{24}$$

Therefore, under conditions defined by the equation (21) the error of the ANN section $C_d[T_1]$ training will lie within the following range:

$$\begin{aligned} \Delta_E &= \frac{1}{N}\sum_{k=0}^{N}\left(\Delta_h - \Delta_h \cdot \sum_{i=0}^{d} w_i\right)^2 = \\ &= \frac{1}{N}\sum_{k=0}^{N}\left((1\pm\varepsilon) - (1\pm\varepsilon)\cdot\sum_{i=0}^{d} w_i\right)^2 = \\ &= \frac{1}{N}\sum_{k=0}^{N}\left(1 - \sum_{i=0}^{d} w_i\right)^2 \cdot (1\pm\varepsilon)^2 = \\ &= \left(1 - \sum_{j=0}^{d} w_i\right)^2 \cdot (1\pm\varepsilon)^2 = E_0 \cdot (1\pm\varepsilon)^2 \end{aligned} \tag{25}$$

Thus, the equation (25) shows the direct relationship between the deviation of the training error from its limiting value and the length $N+1$ of the training sets.

4. RESULTS OF EXPERIMENTAL DATA PROCESSING

The algorithm for the recovery of dynamically distorted signals on the basis of the proposed ANN inverse sensor model was developed. In order to validate experimentally the efficiency of these model and algorithm the dynamic measurement of the temperature was made. The step response of the thermoelectric transducer (thermocouple) «Metran-281» by heating it from 0 °C to 800 °C was obtained.

The result of the experimental data processing at d = 66 in the form of the plots of the thermocouple measured output $y(t)$ and the thermocouple recovered input $u^*(t)$ is shown in fig. 8.

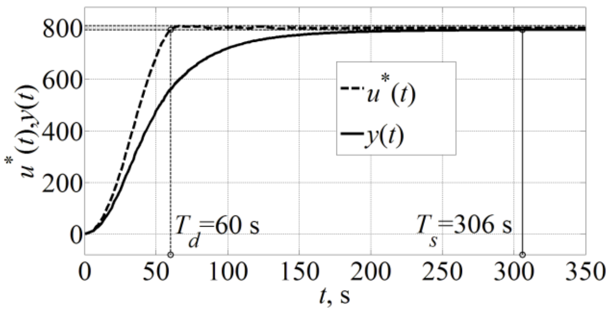

Fig.8. Result of experimental data processing

The obtained result shows that the dynamic measurement error after the correction decreased by 40 % in comparison with the initial state without any additional correction. In addition, the time of the dynamic temperature measurement decreased from T_s = 306 s to T_d = 60 s, that is more than 5 times. These evaluations validate the efficiency of the proposed model and the algorithm of the dynamic measurements error correction.

5. CONCLUSIONS

The neural network approach to the recovery of dynamically distorted signals allows it to effectively correct the dynamic measurements error caused by the inertia of the sensor and the additive noise at its output.

The considered neural network inverse model of a sensor with filtration of sequentially recovered signal allows it to effectively improve the sensor dynamic behaviour due to the deep mathematical processing of measurement data.

The results of the experimental data processing validate the considerable improvement of the sensor dynamic behaviour due to the application of the proposed model and the algorithm to the dynamic measurement of the temperature.

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