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From observed successions to quantified time: formalizing the basic steps of chronological reasoning

Bruno Desachy[1]

[1] *université Paris-1 and UMR 7041 ArScAn (équipe archéologies environnementales), France, bruno.desachy@paris1-univ.fr*

***Abstract* – This paper presents a way to make more rigorous and explicit the passage from the stratigraphic relative chronology to the quantified time. It uses a simple computerizable inequations processing, integrating the stratigraphic order relationships and based on quantified chronological variables limited by inaccuracy intervals. Questions about the chronological units grouping, the differences between stratigraphic time and historic time, and the processing of uncertainties, are discussed.**

I. INTRODUCTION

Archaeological dating techiques have made great progress for the past decades. On the field, following the works of E.C. Harris [1], the stratigraphic analysis became more rigorous with a precise definition of discrete units of time (also characterized by a spatial location and a functional, cultural or social interpretation) and a precise definition of chronological order relationships observed from material interfaces, allowing to get complete relative chonologies. On the other hand, it is well known that the possibilities to get "absolute" (more exactly: quantified) indications of time about archaeological remains have been in constant progress for the past sixty years, thanks to the laboratory dating techniques.

However, this double progress of relative chronologies analyse and dating techniques still leave a dark zone in the chronological reasoning : as some authors said (e.g. [2]), archaeological discourses are widely implicit and sometimes ambiguous about dating (*i. e.* positioning in the quantified time) the stratigraphic units. Actually, the quantified time indications from laboratory techniques, as well as those provided by more classical (historical, numismatic, etc.) chronometric sources, are not direct solutions to the problem of relative chronological units dating; they are only parameters of this problem.

This paper, derived from a work in progress about formalisation of stratigraphic data processing and field chronological reasoning [3][4][5], presents some elements specifically devoted to this transfer of the stratigraphic units from a relative to a quantified chronological frame.

II. FROM STRATIGRAPHIC SEQUENCE TO QUANTIFIED TIME

Let us consider a basic well known Harris' example (fig. 1) that summarizes the stratigraphic analysis process.

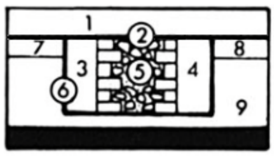

Fig. 1. Simplified example of stratification (from [1] p.87 fig.28). 2 (horizontal cut) and 6 (foundation trench) are not deposits but erosion units.

On the field, interfaces are observed, recorded and identified as a set of units and order relationships. This set may be simplified by removing the "redundant" relationships (*i.e.* transitively deductible relationships – transitivity is one of the mathematical order properties; regarding stratrigraphic relationships, it corresponds to the Harris' so called "fourth law of archaeological stratigraphy"). A graph ("Harris matrix") displays this simplified partially ordered set (fig. 2).

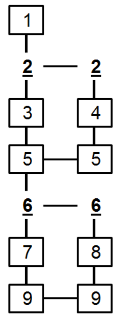

Fig. 2. Stratigraphic graph corresponding to the fig. 1 (automatically generated by a computerized application – *le Stratifiant* – mentioned below)

Different formalized approaches and computerized tools already exist for the stratigraphic data processing. The way we chose [6] includes an algorithm derived from graph theory and its applications in operational research (more precisely, derived from the Critical Path and *graphe MPM* methods, which were developed in the

1950s in order to design industrial production processes). It calculates relative time distances between the units of the considered relative chronology, and provides a graph corresponding to a Harris Matrix.

A second step of our approach is to adapt the quantified frame of time used in the industrial applications of graph theory mentioned above. This frame includes only four main variables. Three concern each single chronological unit : the beginning B_i of a chronological unit i, the end E_i and the duration D_i of this unit. A fourth variable is necessary: the duration D_{ij} allocated to the order relationship (if it exists) between a unit i and a unit j (which may be zero if the succession is immediate). Theses variables are linked by basic equalities:

$$E_i = B_i + D_i \quad (1)$$

$$B_j = E_i + D_{ij} \quad (2)$$

In a sequence $i < j < k$, the relationship $i < k$, transitively deductible, doesn't appear on the graph but it has a duration D_{ik}, so that:

$$D_{ik} = D_{ij} + D_i + D_{jk} \quad (3)$$

This simple frame is necessary but not sufficient to deal with archaeological data, because of their gaps. Our quantified time indications, with very few exceptions, do not directly provide values to the variables. They just can be used as limits of inaccuracy intervals including each basic variable [5]: [at the earliest, at the latest] intervals for the moments (beginnings and ends), [at the shortest, at the longest] intervals for the durations (fig. 3).

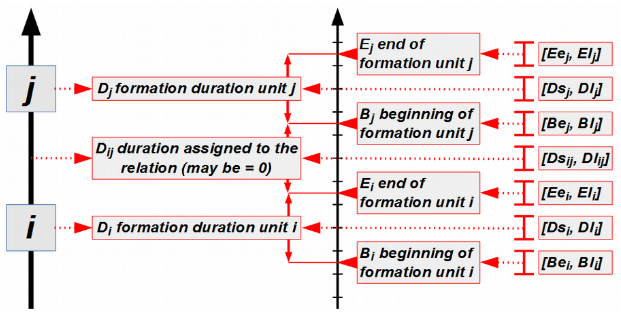

Fig. 3. Stratigraphic ordered time (that archaelogists can record and process), absolute quantified time (that archaelogists search) and inaccuracy intervals in quantified time (that archaelogists can know and process)

these intervals of inaccuracy with their limits (***Be***: beginning at the earliest, ***Bl***: beginning at the latest, Ee: end at the earliest, ***El***: end at the latest, ***Ds***: duration at the shortest, ***Dl***: duration at the longest) may be written in the form of inequalities; so that, for a unit i:

$$Be_i \leq B_i \leq Bl_i \quad (4)$$

$$Ee_i \leq E_i \leq El_i \quad (5)$$

$$De_i \leq D_i \leq Dl_i \quad (6)$$

and for an order relationship $i < j$:

$$De_{ij} \leq E_{ij} \leq Dl_{ij} \quad (7)$$

Then, stratigraphic order relationships, basic equalities of quantified time (1), (2), (3) and quantified inaccuracy intervals (4), (5), (6), (7) may be integrated in a whole system of inequations, with its unknown values (the basic variables of quantified time for each unit and relationship) and its valued parameters (the limits of inaccuracy intervals, provided by dating indication or by default values previously defined to include the whole period concerned). The main inequations are, for a unit i:

$$[Be_i, Bl_i] \text{ solution of : } \quad (8)$$
$$Ee_i - Dl_i \leq D_i \leq El_i - Ds_i$$

$$[Ee_i, El_i] \text{ solution of : } \quad (9)$$
$$Be_i + Ds_i \leq E_i \leq Bl_i + Dl_i$$

$$[Ds_i, Dl_i] \text{ solution of : } \quad (10)$$
$$Ee_i - Bl_i \leq D_i \leq El_i - Be_i$$

for an order relationship $i < j$:

$$[Ee_i, El_i] \text{ solution of : } \quad (11)$$
$$Be_j - Dl_{ij} \leq E_i \leq Bl_j - Ds_{ij}$$

$$[Be_j, Bl_j] \text{ solution of : } \quad (12)$$
$$Ee_i + Ds_{ij} \leq B_j \leq El_i + Dl_{ij}$$

$$[Ds_{ij}, Dl_{ij}] \text{ solution of : } \quad (13)$$
$$Be_j - El_i \leq D_{ij} \leq Bl_j - Ee_i$$

and for an order sequence $i < j < k$:

$$[Ds_j, Dl_j] \text{ solution of : } \quad (14)$$
$$Ds_{ik} - (Dl_{ij} + Dl_{jk}) \leq D_j \leq Dl_{ik} - (Ds_{ij} + Ds_{jk})$$

The simple algebraic processing of this inequations system results in the best possible reduction of the inaccuracy intervals. It gives, for each unit, a slot of "possible time", between the beginning at the earliest ***Be*** and the end at the latest ***El*** (fig. 4). It is important to note that if we do not have accurate indications of beginning at the latest and end at the earliest (such as ***Bl*** > ***Ee***), the unit position will be uncertain at each moment in this "possible time"; because at any moment, the formation of the unit may be already finished or not yet started. The only certainty is negative: the formation of the unit is impossible before and after the "possible time".

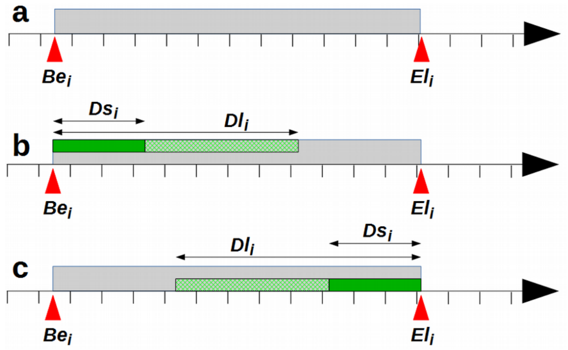

Fig. 4. Possible formation time of a unit ***i*** (a) ; with duration indications: possible positions of ***i*** at the earliest (b) or at the latest (c)

In favorable cases (if $\boldsymbol{Bl} < \boldsymbol{Ee}$ *i.e.* the latest date for the beginning of the unit is before the earliest date for the end), a slot of "certain time" exists, in whom it is certain that the unit was in the making (fig. 5).

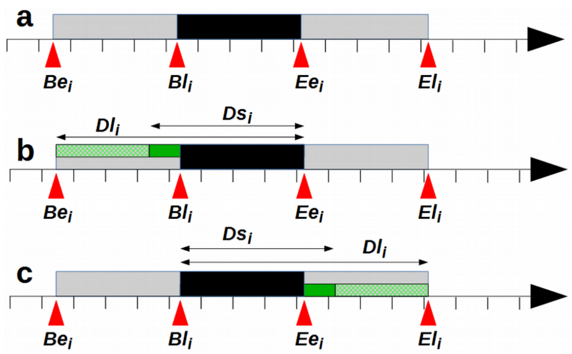

Fig. 5. certain formation time for a unit ***i*** , if $\boldsymbol{Bl_i} < \boldsymbol{Ee_i}$ (a) ; with duration indications: possible positions of ***i*** at the earliest (b) and at the latest (c)

Let us return to the Harris' example and let us suppose we have a document – an old photography dated 1860 – that shows the place without the wall corresponding to the units 3 to 6. It means that this wall was already destroyed in 1860. This is a *terminus ante quem* (*TAQ*), usual notion which corresponds here to the end at the latest of the destruction cut (unit 2). This limit ($\boldsymbol{El_2}$ = 1860) allows to reduce the possible time for the unit 2 and the stratigraphically previous units.
If we suppose now that a coin minted in 1600 was found in the unit 3, this coin provides a *terminus post quem* (*TPQ*) for this unit. According to the well known (but sometimes forgotten) rule, it provides not a direct indication of the unit 3 position in quantified time, but only a limit at the earliest for this position. But what limit exactly: the beginning or the end of the unit formation ? Here, we have to note that the usual *TPQ* notion is often ambiguous in archaeological reports and papers. Actually, the most recent object found in a unit ***i*** (usual definition ot the *TPQ*) corresponds to the earliest date for its formation end ($\boldsymbol{Ee_i}$), and *not* to its formation beginning (because an object made *after* the formation beginning of a unit may be deposited in this unit, if the formation duration is long enough). Consequently, a single $\boldsymbol{Ee_i}$ value (with no other indications) does not allow to reduce the "possible time" ([$\boldsymbol{Be_i}$, $\boldsymbol{El_i}$]) for a unit ***i.*** In other words, contrary to what is often said, a *TPQ* does not precise the position of the unit which contains it. However, it improves the "possible time" of the stratigraphically later units. It results from the inequations above that if $\boldsymbol{i} < \boldsymbol{j}$, then $\boldsymbol{Ee_i} \leq \boldsymbol{Be_j}$: the beginning of the later unit is of course later than (or equal to) the end of the previous unit).
To precise the position of the unit 3, we need explicit duration indications. Let us assume that observations indicate a short formation duration for the whole sequence formed by the units 3, 4, 5 and 6 (for instance because the foundation trench 6 is not collapsed, so it must have been filled quickly): certainly less than a year. So we have a ***Dl*** value for the unit 3. With these two known limits ($\boldsymbol{Ee_3}$ = 1600, $\boldsymbol{Dl_3}$ = 1) we get a beginning at the earliest ($\boldsymbol{Be_3}$ = 1600 -1 = 1599) which reduces the "possible time" for the unit 3 (*cf.* inequation (8) above). This duration estimate – necessary to get the beginning at the earliest from a *TPQ* provided by material finds included in the units – is generally elided in many dating statements, when a *TPQ* is used to give directly an "absolute beginning" to the unit.
Using the inequations (11) to (13) to integrate the stratigraphic relationships and the limit known for the other units (here, the *TAQ* applied to the unit 2), we get finally a [1599, 1860] "possible time" interval for the unit 3, by the means of a simple but totally mathematical and computerizable way.
The result may be displayed by a graph on a quantitative timescale (fig. 6), complementary to the stratigraphic graph. We added to our example a case of "certain time" interval, assuming that the unit 1 is a contemporary floor whose the date of construction (1980) is known with total certainty. In this case, the two endpoints of each interval have the same value (the inaccuracy interval range is zero), and so there is no "possible time", only a "certain time" for this unit 1.
This little example is enough to show that our inequations system is no more than a generalization and a systematization of the inaccuracy intervals principle, which is already applied by archaeologists in the form of the [*TPQ*, *TAQ*] interval. It also shows that this generalization and this systematization are necessary, because the only [*TPQ*, *TAQ*] interval is not sufficient to make totally explicit (and so, computerizable) the basic dating of stratigraphic units; particularly to take in account the formation durations.

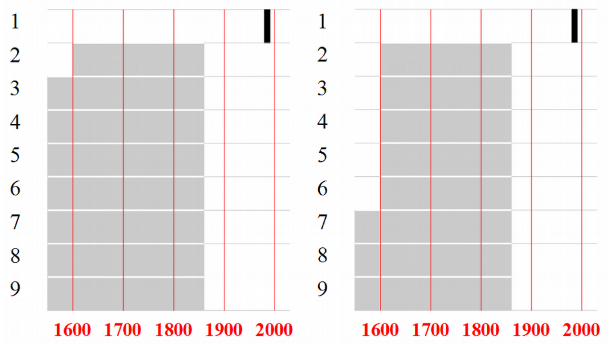

Fig. 6. Harris' example : "possible time" (in grey) calculated onlly with TPQ-TAQ intervals (left), and with duration intervals (right). In black : "certain time" of the unit 1 whose the formation date is accurately known.

III. CHANGING SCALE OF TIME : SUBSET RELATION

Grouping the smallest observed stratigraphic units is a task that must be performed in order to get a synthetic view of the site evolution. If the archaeologist continue considering the same nature of time – the stratigraphic time of the site formation steps (and macro steps), this grouping may be simply formalized by a subset relation between macro unit and included units, and can be taken in account in our inequations system.

From a simple first principle – the "possible time" of the included units is limited by the "possible time" of the macro (inclusive) unit – we can deduce the basic relations between the variables of an inclusive macro unit ***i*** and its included units ***j(i)***:

$$B_i \le B_{j(i)} \quad (15)$$
$$E_{j(i)} \le E_i \quad (16)$$
$$D_{j(i)} \le D_i \quad (17)$$

From those basic inequalities, result the inequalities linking the intervals limits of ***i*** and ***j(i)*** :

$$Be_i \le B_i \le B_{j(i)} \le Bl_{j(i)} \quad (18)$$
$$Ee_{j(i)} \le E_{j(i)} \le E_i \le El_i \quad (19)$$
$$Ds_{j(i)} \le D_{j(i)} \le D_i \le Dl_i \quad (20)$$

More developed inequalities and inequations may be deduced, integrating the internal and external order relationships of the inclusive macro unit. Without getting into details, these developments are based on two principles :

- the formation duration ot the macro unit can't be less than the critical path (minimum irreductible total duration) of the partially ordered set formed by the included units and relationships;
- the macro unit inherits from the included units their order relationships with the other (non included) units.

In the harris' example, it is for instance possible to group the sequence from the unit 6 to the units 3 and 4, in a more synthetic step (of construction). We previously assumed that this macro unit has a whole duration at longest (1 year); it inherits its other limits from intervals and relationships of its included units, so that the "possible time" for this macro unit is [1599, 1860].

IV. INFERRING HISTORIC TIME FROM STRATIGRAPHIC TIME

Beyond the construction of the stratigraphic chronology, including dating and grouping, there is another stage of chronological synthesis, which implies a change of nature of the studied time.

At this new stage, the considered time is no more only the formation time of the material remains; it includes the whole "operating life" of theses remains. For instance, in the Harris' example, it includes not only the construction duration of a wall that the unit 2 to 6 reflect directly, but the duration of use of this wall, as a part of inhabited or used structure, until its destruction. This duration of remains use or "cultural life" correponds to the "systemic context" notion developed by M. Schiffer [6], opposed to the "archaeological context" of the buried remains wich no more interact with an alive society. The stratigraphic time, limited to the formation of field units, does not take in account this remains use time. M. Carver [7] in particular has criticized the "Harris matrix" from this point of view.

Indeed, this extension of the chronological vision is necessary to approach the historic time of societies. *De facto*, from the stratigraphic units and sequence, archaeologists recognize material historic entities implicitly or explicitly provided with a use or "life" duration (*e.g.* "structure", "house", "temple", "street"…).

It must be noted that this passage from the stratigraphic time to the historic time is not a simple grouping based on subset relations. It is a more complex n-n relation between the stratigraphic units and the historic material entities to whom they belong.

Indeed, these two temporalities – stratigraphic time and "cultural life" time – cannot be confused. A material remain may stay in "systemic context" longer than another one more recently formed. Furthermore, the materiality of a stratigraphic unit may remain in use or may be reused through several successive states of a historic entity (what is formalized in particular in the *OH-FET* urban evolution analysis model [8]). Moreover, the stratigraphic time totally exludes cyclical phenomena: the same unit is never formed twice, and it cannot be both after and before another unit (or it is a logical fault). But the "cultural life" time admits cyclical phenomena: the same remain or object may stay in "systemic context"

after its formation for a first “cultural life”, then fall into “archaeological context”, and then come back into “systemic context” (*e.g.* an excavated and restored archaeological site).
However, from our point of view, the same frame may be used to localize in the quantified time the stratigraphic units and the historic entities provided with a use time ; except that the formation duration becomes a “life” or “use” duration.
To illustrate this, we added to the Harris' example a case or reverse dating, from an historic entity to stratigraphic units (fig. 7). We assume that the unit 3, 4, 5 and 6 belong to a house (entity *B*) which appears on two old maps, dated 1710 and 1790. These documents directly give a “certain time” for this house as historic entity, and consequently they provide endpoints for the “possible time” of the stratigraphic units (construction and destruction) related to this historic entity. In the same example, concerning the unit 1 (contemporary floor), we can consider not only its formation duration, but its use duration (from 1980 to 2015, if we assume that the excavations started this year) : then we consider not a stratigraphic unit, but a historic material entity (entity *A*)

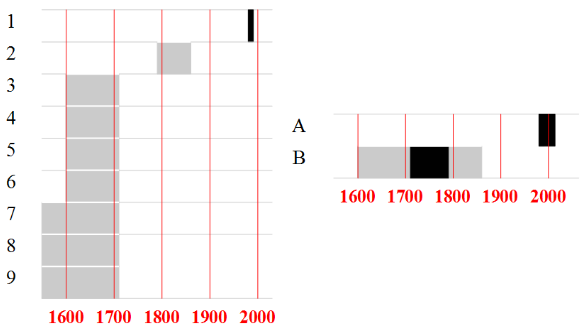

Fig. 7. Stratigraphic time (left) and historic time (right) ; the historically dated house *B* has a “certain use” time provided by historic documentation ; the “possible time” of the related stratigraphic units of construction and destruction is consequently limited.

V. TAKING UNCERTAINTY IN ACCOUNT

As said above, dating inaccuracies result in uncertainties linked to the formation of a stratigraphic units. Likewise, the existence of a historic entity is uncertain at any moment of its “possible time”, and is never certain if it has no “certain time”. Finally, for each stratigraphic unit or historic entity, the chronological hypothesis are limited by its positions at the earliest and at the latest, taking in account its “certain time” or/and its duration indications (if it has it) (*cf.* fig. 4 and 5).
Adding to inaccuracies, doubts may also appear in the field observations, or about the documentation reliability. In previous works, we have developed a simple way, derived from modal logic, to deal with uncertain stratigraphic relationships [3][4]; it may be logically extended to the quantified frame of time, so that an intermediary “estimate” quantified time between the “possible” and the “certain” time may be obtained, if a status distinction between “certain” and “estimate” data is introduced into the system. A double system of intervals (certain and estimate) is then necessary. The principle is that the “estimate” intervals, more accurate but hypothetical, are limited by the certain endpoints. The advantage is a more flexible process, able to take in account uncertain observations or a qualitatively heterogeneous documentation; it is also a way to display a chosen chronological hypothesis.

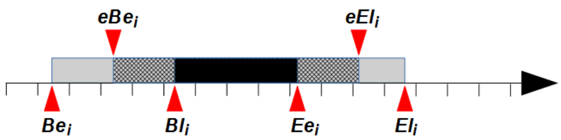

Fig. 8. “Estimate time” provided to an unit *i* (between an estimate beginning at the earliest and an estimate end at the latest) if estimate inaccuracy intervals have been valued besides the certain limits.

The graph (fig. 9) shows the dating of our example if we add this assuming : the coin dated 1600 found in the layer 3 is a primary deposit, so that even with its circulation duration, its deposit date is not very far from its minting date ; probably not more than 50 years. Consequently we can indicate an estimate end at the latest for the unit 3 ($\boldsymbol{eEl_3}$ = 1650). The process results in an “estimate time” – more accurate, but less certain than the “possible time” – applied to the related sequence.

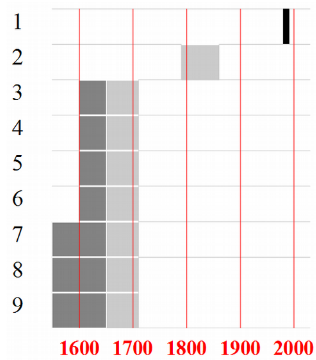

Fig. 9. possible time (grey) and “estimate time” (darker grey)

VI. COMPUTERIZED TOOLS

two computerized tools currently exist, as practical applications of the work presented here (graphs in the figures 2, 6, 7, 9 are provided by these tools):
- *Le Stratifiant* [3] is a stratigraphic data processing application which is able to make stratrigraphic graphs, to

detect logical faults and to process uncertain relationships. It is an add-on to the Microsoft Excel software. It is used by several field archaeologists, in particular in the *Institut National de Recherches Archéologiques Préventives* (*INRAP*);

- *Chronophage* is an application providing help to process the inaccuracy intervals of stratigraphic units (or historic entities). It contains formula to solve inequations and to reduce the intervals (from known endpoints and default values chosen by the user); it detects logical faults, gives the possible, certain and / or estimates times for the units, and provides quantified time graphs. It is an add-on to the free software LibreOffice/OpenOffice Calc. This application is still experimental; it is intended to be integrated in a future *le Stratifiant* release.

These tools are free. They will be soon available at : https://cours.univ-paris1.fr/course/view.php?id=1879

VII. CONCLUSION

This brief presentation leaves out many aspects that would be interesting to develop and discuss ; for instance the relation between the artefacts and finds dating and the stratigraphic units dating. However, this work, still in progress, is an attempt to explicit basic steps of the chronological reasonning, upstream to the advanced chronological modellings. Indeed, these basic steps, that are fundamental, remain often implicit and ambiguous. Formalizing them – as simply as possible – is a practical issue, in order to get computer-based aids ; it is also a methodological and epistemological issue, in order to make our chronological statements more rigorous and falsifiable.

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