

From bell sound to architectural function: Quantitative vibroacoustic metrology applied to the Norman bell tower of Salerno Cathedral (Italy)

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ABSTRACT

This study formulates and assesses a measurement–model workflow for physical metrology applied to architectural heritage, using the Norman bell tower of Salerno Cathedral as a case study. Dominant spectral components of the 13th-century bell were identified through individually calibrated acoustic measurements, while a finite-element model provided the reference modal frequencies of the belfry-cell air volume. The two datasets were compared using perceptual frequency proximity based on the Equivalent Rectangular Bandwidth and restricted harmonic relations expressed in cents. Then, a Monte Carlo null model was used to determine whether the observed correspondence exceeded random spectral alignment. Sensitivity analyses addressed the effects of radiating aperture conditions and temperature-dependent variations in the speed of sound. Although the open-boundary model introduced mode-dependent frequency shifts, leakage, and modal reorganization, these effects did not materially alter the final interpretation. The study contributes an inspectable and transferable metrological chain linking measured quantities, model-derived outputs, influence factors, and statistical decision criteria for investigating sonic functions across the material and intangible dimensions of architectural heritage.

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Keywords: vibroacoustic metrology; acoustic heritage; bell tower; bell–cavity coupling; acoustic modal analysis; medieval archaeology; Salerno

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1. INTRODUCTION

Cultural heritage is increasingly understood as the result of an interaction between material evidence and intangible functions, including practices, technical skills, sensory experiences, and forms of empirical knowledge that may no longer be directly documented [1], [2]. Architectural heritage, in particular, cannot be fully interpreted only through its visible form, construction materials, or stylistic features. In many cases, buildings also preserve traces of functions that were activated through use, perception, and environmental interaction. Among these, sound represents one of the most difficult dimensions to investigate, because historical sound events are transient and cannot be directly recovered. However, the physical structures that generated, hosted, filtered, or projected sound may still be

analysed through measurable acoustic and vibroacoustic quantities.

This consideration is especially relevant for bell towers. Medieval bells were musical or liturgical objects, and instruments of collective communication. Their sound marked religious time, regulated civic life, transmitted warnings, structured social practices, and contributed to the perception of territorial identity [3]. In this sense, the bell tower was not merely a vertical architectural landmark or a support structure for the bells. It was also the built interface through which bell sound was elevated, spatially distributed, and made audible across the surrounding environment. More generally, built spaces may act as active acoustic environments, shaping how sound is experienced, oriented, and socially interpreted [4]. Therefore, the acoustic function of a bell tower depends on the architectural cavity in which the bell is housed, on the position of the source within

that cavity, and on the geometry of the openings through which sound radiates.

Despite this, the acoustic interaction between bells and belfry cells has often remained secondary with respect to structural, historical, stylistic, or material analyses of bell towers. From a metrological point of view, this represents an important gap. If the architectural volume contributes to shaping the emitted sound, then the bell and the belfry cell should be treated as a coupled vibroacoustic system. The relevant question is not limited to identifying the frequencies emitted by the bell but extends to the compatibility between the bell spectrum and the resonance properties of the cavity, the spatial dependence of modal excitation, and the directional behaviour imposed by the aperture system. These aspects require a quantitative procedure combining measurement, modelling, and statistical verification. This perspective is consistent with broader studies showing that architectural form and musical or acoustic behaviour may intersect in historically meaningful ways [5].

The Norman bell tower of Salerno Cathedral provides a significant case study for this approach. The tower belongs to the complex architectural and cultural context of Southern Italy during the Norman period, where Latin, Byzantine, Islamic, and local construction traditions interacted within a highly stratified Mediterranean environment. The monument rises beside the cathedral *quadriportico* and consists of a sequence of superimposed stages surmounted by an upper belfry cell. This upper architectural element, where the bells are housed, is particularly relevant, because its geometry defines both the enclosed air volume and the aperture system through which sound can be radiated outside. The oldest bell, preserved in the tower, offers a suitable source for investigating whether medieval bell sound and architectural form may have interacted in a physically meaningful way.

Previous research on the Norman bell tower of Salerno Cathedral showed that the bell and its architectural enclosure can behave as a coupled sound-producing and sound-transforming system [6]. In this interpretation, the belfry cell is not a passive container: its geometry and modal response interact with the bell spectrum and can modify the spectral and spatial organisation of the emitted sound. Thus, the combined bell–belfry system may be regarded as functionally analogous to a single musical instrument, whose purpose was not primarily musical performance, but the effective transmission of coded acoustic signals for liturgical, civic, and territorial communication. The experimental dataset, geometrical reconstruction, and principal acoustic findings supporting this interpretation have already been published [6]. Then, this study uses the Salerno case as a validated test environment for formulating a physical-metrology workflow capable of describing how such a conclusion is obtained from measurable quantities. The objective is to organise the passage from acoustic measurements and architectural geometry to model-derived quantities, comparison metrics, and statistical decision criteria within a reproducible and inspectable measurement–model chain.

The workflow begins with two independently obtained datasets: the dominant spectral components of the historical bell, identified using individually calibrated and comparatively accessible measurement instrumentation, and the modal frequencies of the belfry-cell air volume, derived from finite-element modelling of the measured geometry. Their relationship is quantified through perceptual frequency proximity, logarithmic detuning, and a restricted set of proportional mappings. A stochastic null model then establishes whether the

observed compatibility exceeds the level expected from spectral coincidence alone. Geometrical uncertainty, temperature-dependent variations in the speed of sound, spectral peak identification, mesh discretisation, and aperture boundary conditions are treated as influence factors whose effects can be propagated through the procedure.

Then, the metrological result is the outcome of a controlled sequence, connecting input quantities, modelling assumptions, intermediate outputs and final decision quantities. The sensitivity analyses introduced here are used to determine whether changes in the numerical representation affect only individual modal values or materially alter the final interpretation. This distinction is essential for separating the robustness of the physical inference from the idealisations inevitably introduced by the model.

The contribution of the work lies in defining a transferable procedure for physical metrology applied to cultural heritage. Once established, the workflow can support the characterisation of other bell–tower systems, comparative analyses among different architectural typologies and historical periods, and the improvement of soundscape models describing territorial communication networks based on bells and other historical sound sources, whose historical and archaeological plausibility has been previously investigated and validated [7], [8]. In this perspective, the Salerno bell tower is not only the object of the investigation, but also a reference case for developing quantitative methods capable of linking the material properties of architecture with the intangible functions of historical sound.

2. MATERIALS AND METHODS

2.1. Case study and geometrical characterisation of the belfry cell

The investigated case study is the Norman bell tower of Salerno Cathedral, located in Southern Italy. The monument belongs to the medieval architectural context that developed in Salerno after the Norman conquest, when the city of Salerno became an important political, religious, and cultural centre for the Normans of the South. The tower rises beside the cathedral *quadriportico* and represents one of the most significant vertical architectural elements of the complex. Its construction is associated with the 12th century and with the broader Mediterranean context in which Norman, Latin, Byzantine, and local building traditions interacted.

The bell tower (Figure 1) has a total height of approximately 52 m and a square base of about 10 m per side. Its architectural structure is composed of four superimposed stages, surmounted by an upper belfry cell. The lower parts of the tower are mainly characterized by massive masonry, while the upper stages are progressively lightened by openings. The most distinctive element for the present study is the final architectural unit, consisting of a cylindrical drum covered by a hemispherical dome. This upper cell is the space in which the bells are housed and therefore represents the most relevant portion of the tower for investigating the relationship between architecture and sound radiation.

The belfry cell is geometrically simple but acoustically significant. It is composed of a cylindrical cavity with an internal diameter of 5.64 m, corresponding to an internal radius of 2.82 m. The cylindrical part has a height of 6.41 m, while the hemispherical dome has a radius of 2.82 m. The total internal height of the belfry cell, measured from the lower reference level to the top of the dome, is 8.92 m. The wall thickness of the drum is approximately 0.69 m.



Figure 1. The bell tower of the Cathedral of Salerno (Italy)

In its original configuration, twelve vaulted openings were distributed around the perimeter of the cylindrical drum. These openings represent the main acoustic connection between the internal cavity and the external environment. Each opening has an internal width of 0.86 m and an external width of 1.03 m. The lateral part of the opening has a height of 1.96 m, while the total height, including the vaulted portion, is 2.39 m. The radius of the vaulted upper part is 0.43 m. From an acoustic point of view, this configuration defines an aperture system arranged around a nearly cylindrical radiating surface. Therefore, the belfry cell can be studied as an architectural cavity coupled to a periodic set of radiating openings.

The tower currently contains eight bells, dating from different historical periods. For the present analysis, the 13th century bell was selected because it is the oldest bell preserved in the tower, and because its chronology is closest to the medieval construction phase of the bell tower. The bell is housed in the upper belfry cell, and it is currently installed at a height of approximately 2.67 m. This positioning is coherent with the representation of the bell tower and the inside bells, represented in the book by Pietro da Eboli *Liber ad honorem Augusti* (also known as *Carmen de Rebus Siculis* or *Carmen de motibus Siculis*) [9], a work dedicated to Henry VI, which reports that the bell tower contained three bells inside. This position makes it suitable for investigating whether the acoustic emission of the bell may interact with the modal properties of the architectural cavity and with the directional radiation imposed by the openings.

The first step of the metrological workflow consisted in translating this architectural configuration into a simplified but physically meaningful acoustic geometry. The belfry cell was idealized as a cylindrical cavity surmounted by a hemispherical dome. This representation preserves the main features controlling the vibroacoustic behaviour of the system: radial extension, vertical development, upper curved termination, and lateral aperture distribution. The internal cavity was described by its principal geometric parameters, namely internal radius, cylindrical height, dome radius, total internal height, and opening geometry. Then, these parameters were used both for the finite-element acoustic model and for the simplified analytical radiation model.

2.2. Acoustic measurements and spectral characterization of the bell sound

The acoustic measurement campaign, deriving from a previous study [10], was designed to obtain a reproducible spectral description of the medieval bell selected as the sound source of the investigated bell–belfry system. The objective was not to perform a complete psychoacoustic characterization of the bell, but to extract a stable set of dominant tonal components suitable for comparison with the acoustic modal properties of the belfry cell. For this reason, the measurement procedure focused on frequency identification, signal stability, and robustness of peak detection.

The selected bell was recorded both inside and outside the belfry cell. The internal recording was acquired in the proximity of the bell, while the external recording was acquired in the far field, within the cathedral *quadriportico*. This double acquisition was introduced to distinguish the spectral components directly measurable close to the source from those still detectable after radiation through the architectural system. In this way, the measurement configuration provided both a near-field description of the bell emission and an external reference useful for checking the persistence of the main tonal components.

The signals were acquired using calibrated USB measurement microphones and recorded as uncompressed audio files. The adopted sampling frequency was selected to cover the audible interval relevant to the bell partials analysed in this work. The microphone calibration factor provided by the manufacturer was used for signal scaling. The recorded audio files were then processed in MATLAB. Before spectral analysis, the recorded signals were converted into monophonic waveforms. A background-noise estimate was used to improve the robustness of the spectral characterization. The denoising step was kept deliberately conservative, because the aim was to avoid altering the intrinsic spectral structure of the bell sound. No narrow band-pass filtering was imposed before peak extraction, in order not to bias the analysis toward preselected frequency intervals. This choice is relevant from a metrological point of view, because the spectral dataset used in the following sections must derive from the measured signal and not from a frequency range imposed a priori. The power spectral density of the bell signal was estimated using a Welch-based procedure [11]. In this approach, the signal is divided into partially overlapping time windows; the periodogram of each window is computed and the resulting estimates are averaged. This method reduces the variance of the spectral estimate with respect to a single-periodogram approach and is therefore suitable for identifying stable tonal components in measured acoustic signals. Windowing was used to reduce spectral leakage, following standard principles of harmonic analysis of finite-length records

[12]. Spectral smoothing was subsequently applied to reduce local fluctuations while preserving the dominant peaks.

The dominant peaks were automatically identified within the analysed frequency range and then inspected to retain the stable components consistently emerging from the measured spectrum. The near-field recording provided the main spectral dataset for the bell source. The corresponding far-field recording was used as a check on the persistence of the same components after radiation from the belfry cell. The retained peak set was then used as the experimental acoustic fingerprint of the bell in the subsequent stages of the workflow.

The second spectral component was adopted as the reference component for the ratio calculation. This choice was made because it corresponds to the dominant perceptual reference of the bell spectrum. The resulting ratios were used only as descriptive quantities, since bells do not behave as ideal harmonic oscillators with integer multiples of a fundamental frequency. Their vibrational and acoustic behaviour is governed by complex modal patterns, rather than by the harmonic series typical of strings or air columns. Therefore, the spectral characterization was not reduced to a search for integer harmonics. Instead, the measured peak set was retained as an experimental fingerprint and used directly in the subsequent compatibility analysis with the belfry-cell acoustic modes.

From a metrological perspective, the output of this subsection is the experimental frequency set describing the acoustic source. This set constitutes the input for the following stages of the workflow: comparison with the finite-element cavity eigenfrequencies, perceptual frequency-distance assessment, stochastic testing of non-random alignment, and evaluation of aperture-mediated radiation.

2.3. Finite-element acoustic model of the belfry cell

The acoustic modal behaviour of the belfry cell was investigated through a finite-element model of the internal air volume implemented through the software ABAQUS. The purpose of this model was to identify the natural frequencies and pressure mode shapes of the architectural cavity, independently from the direct acoustic emission of the bell. The procedure is derived from a previous literature work [13]. This step provides the modal reference set against which the measured bell spectrum is compared in the subsequent frequency-compatibility analysis.

The belfry cell was modelled as an air-filled cavity whose geometry reproduces the main measured features of the upper architectural volume: a cylindrical drum surmounted by a hemispherical dome. The model was limited to the internal air domain, while the masonry body was not explicitly included as a deformable structural medium. This choice was made because the objective was to characterize the acoustic resonance scaffold of the enclosed volume, rather than to perform a fully coupled vibroacoustic analysis of masonry and air. The governing equation was the homogeneous acoustic wave equation expressed in terms of acoustic pressure. Under the assumptions of small-amplitude pressure fluctuations, homogeneous and isotropic air, and absence of mean flow, the pressure field p satisfies the classical acoustic wave equation:

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0, \quad (1)$$

where c is the speed of sound in air. For harmonic motion, the pressure field can be written as:

$$p(\mathbf{x}, t) = P(\mathbf{x}) e^{i\omega t}, \quad (2)$$

leading to the Helmholtz eigenvalue problem:

$$\nabla^2 P + k^2 P = 0, \quad (3)$$

where $P(\mathbf{x})$ is the spatial pressure distribution, $k = \omega/c$ is the acoustic wavenumber, ω is the angular frequency, and the corresponding natural frequency is:

$$f = \frac{\omega}{2\pi}. \quad (4)$$

The masonry boundaries were treated as acoustically rigid. Under this condition, the normal component of particle velocity at the wall is assumed to be zero. In pressure formulation, this corresponds to a Neumann-type boundary condition:

$$\frac{\partial P}{\partial n} = 0, \quad (5)$$

where $\mathbf{n}$ is the outward normal direction to the boundary. This approximation is commonly adopted for massive enclosures when the acoustic response of the air volume is investigated separately from wall vibration, especially when the wall displacement is expected to be small compared with the acoustic wavelengths of interest.

The lateral openings were represented as closed boundaries in the finite-element modal model in order to define the cavity volume. This standard modelling step is required to obtain a discrete set of cavity eigenfrequencies and corresponding pressure mode shapes. It does not imply that the real belfry cell behaves as a perfectly closed cavity during bell ringing. Rather, it defines a controlled reference configuration in which the intrinsic modal structure of the internal air volume can be identified before the radiation through the openings is analysed separately. This separation between cavity resonance and aperture-mediated radiation is useful from a metrological perspective, because it allows the contribution of each physical mechanism to be isolated and then recombined in the subsequent stages of the analysis.

The air was modelled as a linear acoustic medium with standard atmospheric properties. Its density was set as $\rho = 1.2 \text{ kg m}^{-3}$, while the speed of sound was set as $c = 343 \text{ m s}^{-1}$, corresponding approximately to air at 20 °C. The corresponding bulk modulus was computed as $K = \rho c^2$. It is worth noting that, for fixed geometry and boundary conditions, the acoustic eigenfrequencies scale linearly with the speed of sound. Therefore, a variation Δc would produce the same relative variation in the computed frequencies, according to:

$$\frac{\Delta f_n}{f_n} = \frac{\Delta c}{c_0}. \quad (6)$$

Thermal gradients, viscous dissipation, mean flow, and acoustic-structural coupling were neglected. These simplifications are consistent with the purpose of the model, which is not to predict absolute sound pressure levels inside the real open tower, but to determine the reference modal frequencies of the architectural air volume.

The air domain was discretized using three-dimensional acoustic finite elements. The mesh resolution was selected according to the shortest wavelength associated with the maximum analysed frequency. The characteristic element size h was chosen to satisfy the condition:

$$h \leq \frac{\lambda_{\min}}{6}, \quad (7)$$

which can be equivalently expressed as:

$$h \leq \frac{c}{6 f_{\max}}, \quad (8)$$

where $\lambda_{\min}$ is the shortest wavelength considered and $f_{\max}$ is the maximum frequency included in the modal analysis. This criterion ensures a sufficient spatial resolution of the acoustic pressure field and limits numerical dispersion in the finite-element solution.

The eigenvalue analysis produced a discrete set of natural frequencies and pressure mode shapes of the belfry-cell cavity. These eigenfrequencies were then used as the modal dataset of the architectural volume. In the subsequent analysis, this dataset is compared with the measured bell spectral peaks to assess whether the source spectrum and the cavity resonance structure exhibit direct or harmonically mediated compatibility.

To quantify the model-form uncertainty introduced by the temporary closure of the twelve belfry openings, an additional three-dimensional finite element analysis was performed by explicitly implementing a model in MATLAB environment, which included the belfry windows openings. The basic cavity envelope, consisting of the cylindrical drum and the hemispherical dome, was generated from the measured meridional profile as a solid of revolution. The twelve vaulted openings were then introduced as air-filled tapered ducts extending through the wall thickness and equally distributed at angular intervals defined according to the geometrical data reported in Section 2.1. Although the basic cavity envelope is axisymmetric, the discrete aperture arrangement interrupts continuous rotational symmetry; consequently, the complete three-dimensional air domain was analysed.

Two configurations were evaluated using the same geometrical discretisation. In the first configuration, the external mouths of the twelve ducts were assigned a rigid acoustic condition, corresponding to zero normal particle velocity, equivalent to the model implemented in ABAQUS. In the open configuration, the masonry surfaces remained acoustically rigid, whereas a first-order local Sommerfeld radiation condition was imposed at the external mouth of each aperture [14]:

$$\frac{\partial p}{\partial n} = i k p, \quad (9)$$

where p is the complex acoustic-pressure amplitude, n is the outward normal direction, and $k = \omega/c$ is the acoustic wavenumber. This condition approximates an outgoing wave and reduces artificial reflection at the termination of the computational domain. This solution was a controlled boundary-condition sensitivity model, rather than an exact representation of the complete exterior acoustic field. In fact, first-order Sommerfeld-type conditions provide a computationally efficient local approximation for truncated acoustic domains, having a sufficient accuracy in relation to the incidence direction and on the distance between the radiating source and the artificial boundary [15].

Assuming a harmonic time dependence of the form $\exp(-i\omega t)$, the finite-element discretisation of the open-domain problem resulted in the quadratic eigenvalue equation

$$(K - i k C - k^2 M) \phi = 0, \quad (10)$$

where K and M are respectively the acoustic stiffness and mass matrices, C is the surface matrix associated with the radiating aperture mouths, and ϕ is the nodal acoustic-pressure vector. The quadratic problem was linearised and solved in terms of complex wavenumbers. Quadratic and nonlinear eigenvalue formulations of this type are commonly used, when boundary absorption, radiation or other dissipative mechanisms make the acoustic modal problem non-self-adjoint [16].

For each open-system quasi-mode, the complex frequency was calculated as

$$\tilde{f}_n = \frac{c k_n}{2 \pi} = f_{n,r} + i f_{n,i}. \quad (11)$$

The real part $f_{n,r}$ represents the oscillation frequency, whereas the imaginary part $f_{n,i}$ describes the radiation-related decay. Thus, open acoustic resonators are characterised by complex eigenfrequencies, rather than by the purely real spectrum obtained for a perfectly closed cavity [17]. A radiation-related quality factor was calculated as

$$Q_n = \frac{f_{n,r}}{-2 f_{n,i}}, \quad (12)$$

with negative values of $f_{n,i}$ corresponding to temporally decaying modes under the adopted harmonic convention.

The modes obtained from the sealed and open configurations were not associated solely according to their frequency order. Instead, their pressure distributions were compared over the common internal air domain using a mass-weighted complex Modal Assurance Criterion (MAC):

$$MAC_{ij} = \frac{|\phi_{i,sealed}^H M \phi_{j,open}|^2}{(\phi_{i,sealed}^H M \phi_{i,sealed})(\phi_{j,open}^H M \phi_{j,open})}, \quad (13)$$

where the superscript H denotes the complex-conjugate transpose. The MAC was used to identify the open quasimode retaining the closest spatial pressure distribution to each sealed-cavity mode, independently of possible changes in modal ordering.

For each paired mode, the frequency variation induced by the radiating apertures was expressed as

$$\delta_{f,n} = \frac{f_{n,open} - f_{n,sealed}}{f_{n,sealed}} \cdot 100 \%. \quad (14)$$

The real parts of the open-system frequencies were subsequently propagated through the harmonic-compatibility and Monte Carlo procedures described in Sections 2.4 and 2.5. This final step was used to determine whether the boundary-condition-induced frequency variations affected only the intermediate finite-element quantities or also modified the final bell-cavity compatibility result.

Therefore, from a methodological point of view, the finite-element model provides the physical bridge between architectural geometry and acoustic function. The geometry of the belfry cell is transformed into a measurable modal signature, expressed in terms of resonance frequencies and spatial pressure distributions. This modal signature is not interpreted alone but becomes meaningful when compared with the measured acoustic fingerprint of the bell and with the radiation properties of the openings.

2.4. Spectral compatibility between bell emission and cavity modes

Once the acoustic fingerprint of the bell and the modal signature of the belfry-cell cavity had been obtained, the next step consisted of evaluating whether the two frequency sets exhibited a measurable relationship. The purpose of this analysis was to determine whether the dominant spectral components of the bell could be associated with the acoustic resonances of the architectural cavity either by direct perceptual proximity or by simple harmonic relations. The comparison was therefore formulated as a frequency-domain compatibility problem between an experimentally measured source spectrum and a numerically identified cavity spectrum. Let $\mathcal{B} = \{f_{b,1}, f_{b,2}, \dots, f_{b,N}\}$ be the set of dominant spectral components extracted from the bell recordings, and $\mathcal{C} = \{f_{c,1}, f_{c,2}, \dots, f_{c,M}\}$ be the set of eigenfrequencies obtained from the finite-element acoustic model of the belfry-cell air volume. The analysis was performed under two complementary assumptions. The first assumption tested whether a bell component and a cavity mode were sufficiently close in perceptual frequency space. The second assumption tested whether their relationship could be expressed through simple consonant ratios and octave shifts.

The first metric, here referred to as direct perceptual proximity, was based on the Equivalent Rectangular Bandwidth (ERB), which provides a standard approximation of the bandwidth of auditory filters and is commonly used to describe frequency resolution in complex sounds [18]. For a frequency f , expressed in hertz, the ERB was computed as:

$$ERB(f) = 24.7 \left(4.37 \frac{f}{1000} + 1 \right). \quad (15)$$

For each pair composed of a bell component f_b and a cavity frequency f_c , the normalized perceptual distance was defined as:

$$d_{ERB}(f_b, f_c) = \frac{|f_b - f_c|}{ERB(f_b)}. \quad (16)$$

This quantity expresses the frequency separation in units related to auditory resolution. Therefore, the same absolute frequency difference is not interpreted in the same way across the whole spectrum but is weighted according to the perceptual bandwidth associated with the analysed frequency region. Then, the normalized distance was transformed into a continuous similarity value:

$$S_{ERB}(f_b, f_c) = e^{-\frac{d_{ERB}(f_b, f_c)^2}{2\sigma_{ERB}^2}}, \quad (17)$$

where σ_{ERB} controls the tolerance in auditory-bandwidth units. This formulation avoids a binary classification of matches and allows partial frequency proximity to contribute to the compatibility score.

For each cavity mode, the direct perceptual compatibility was defined as the best ERB-based match with the measured bell spectrum:

$$S_j^{\text{dir}} = \max_i S_{ERB}(f_{b,i}, f_{c,j}). \quad (18)$$

This score evaluates whether a cavity resonance is directly close to at least one measured bell component. It does not introduce any harmonic or proportional transformation.

The second metric, here referred to as harmonic compatibility, was introduced to test whether the two spectra could be related through simple proportional relations. This step

is important because a functional relation between a resonant architectural cavity and a bell spectrum does not necessarily require equality between frequencies. In musical and acoustic systems, octave equivalence and simple integer ratios may produce meaningful correspondence even when the absolute frequencies differ. In the present context, these relations were not used to infer intentional design, but only as a restricted and testable family of physically plausible frequency mappings.

A discrete set of scale factors was defined as:

$$\alpha = r 2^k, \quad (19)$$

where r is a simple consonant ratio and k is an integer octave shift. The ratio r was selected from a finite set of low-order proportional relations, while 2^k accounts for octave transposition. Unlike an arbitrary continuous scaling, this formulation restricts the search to interpretable harmonic relations.

For a bell component f_b , a cavity frequency f_c , and a scale factor α , the deviation from the exact harmonic relation was expressed in cents:

$$\Delta_{\text{cents}}(f_b, f_c, \alpha) = 1200 \log_2 \left(\frac{f_b}{\alpha f_c} \right). \quad (20)$$

The use of cents provides a logarithmic measure of frequency deviation, consistent with the way pitch intervals are usually represented in musical acoustics. A null value of Δ_{cents} corresponds to an exact proportional relation between f_b and αf_c , while increasing absolute values indicate increasing detuning from the tested harmonic mapping.

Then, the harmonic similarity score was defined as:

$$S_{\text{harm}}(f_b, f_c, \alpha) = e^{-\frac{\Delta_{\text{cents}}(f_b, f_c, \alpha)^2}{2\sigma_c^2}}, \quad (21)$$

where σ_c is the tolerance expressed in cents. For each cavity frequency, the best harmonic correspondence with the measured bell spectrum was retained:

$$S_j^{\text{harm}} = \max_{i,r,k} S_{\text{harm}}(f_{b,i}, f_{c,j}, r 2^k). \quad (22)$$

This procedure identifies whether a cavity resonance can be brought into correspondence with the bell spectrum through one of the admitted proportional mappings. Since only the best discrete match is retained, the analysis does not impose that all resonances follow the same relation, but it allows the dominant compatible relations to emerge from the data.

The two compatibility scores answer different questions. The ERB-based score evaluates whether the bell and cavity frequencies are directly close from the standpoint of auditory frequency resolution. The cents-based harmonic score evaluates whether a frequency relation becomes meaningful after applying a simple proportional mapping. Therefore, the two metrics were kept conceptually distinct. The direct score is primarily perceptual, whereas the harmonic score is primarily relational and proportional.

For each tested configuration, a global compatibility index was obtained by averaging the modal scores over the analysed cavity modes:

$$Q^{\text{dir}} = \frac{1}{M} \sum_{j=1}^M S_j^{\text{dir}}, \quad (23)$$

and:

$$Q^{\text{harm}} = \frac{1}{M} \sum_{j=1}^M S_j^{\text{harm}}. \quad (24)$$

The comparison between Q^{dir} and Q^{harm} was used to evaluate whether the bell–cavity relationship is better described by direct frequency proximity or by harmonically mediated compatibility. The modal scores were also retained as frequency-dependent weights for the subsequent stages of the analysis, where the spectral relationship was combined with source position and aperture-mediated radiation.

From a metrological perspective, this step transforms the qualitative question of whether the bell “matches” the belfry cell into a reproducible quantitative operation. The input data are two independently obtained frequency sets, one measured and one modelled; the metrics are explicitly defined; and the resulting compatibility scores can be subsequently tested against random spectral configurations. This provides the basis for the statistical verification described in the following subsection.

2.5. Statistical assessment of spectral compatibility

The frequency-domain compatibility analysis described in the previous subsection provides a quantitative score for the relation between the measured bell spectrum and the computed cavity modes. However, the existence of a numerical score is not sufficient to establish whether the observed relation is meaningful. In any finite frequency interval, some degree of apparent correspondence may occur simply because several spectral components and several cavity modes are present within the same bandwidth. Therefore, a statistical assessment was introduced to test whether the observed compatibility exceeds what would be expected from random frequency alignment.

The statistical test was formulated as a Monte Carlo null model. The null hypothesis assumes that the measured bell components and the cavity eigenfrequencies are not physically related, and that any observed compatibility arises only from the number of frequencies considered and from the analysed frequency range. To reproduce these constraints, synthetic bell spectra were generated by randomly sampling the same number of components as the measured bell spectrum within the same logarithmic frequency interval. In this way, the null model preserves the cardinality and spectral span of the experimental dataset, while removing any specific association with the cavity modes.

The procedure was applied consistently to the compatibility metric under examination. For each random realization, the synthetic bell spectrum was compared with the same finite-element cavity frequency set using the same scoring procedure defined in Section 2.4. Therefore, when direct perceptual compatibility was tested, the ERB-based score was recomputed for each synthetic spectrum. When harmonic compatibility was tested, the same discrete set of proportional mappings and octave shifts was explored, and the cents-based score was recomputed in the same way as for the measured data. This ensured that the measured and random spectra were evaluated under identical rules.

Let Q_{obs} be the global compatibility index obtained from the measured bell spectrum and the computed cavity modes. Depending on the tested hypothesis, Q_{obs} corresponds either to Q^{dir} or to Q^{harm} . For each Monte Carlo realization l , a synthetic spectrum was generated and the corresponding compatibility index Q_l was calculated. The set of values $\{Q_1, Q_2, \dots, Q_{N_{\text{MC}}}\}$

defines the null distribution of compatibility scores expected under random association. The statistical significance of the observed score was evaluated by computing the empirical p-value:

$$p = \frac{1 + \sum_{l=1}^{N_{\text{MC}}} I(Q_l \geq Q_{\text{obs}})}{1 + N_{\text{MC}}}, \quad (25)$$

where N_{MC} is the number of Monte Carlo realizations and $I(\cdot)$ is the indicator function. The numerator counts how many random spectra produce a compatibility index equal to or greater than the observed value. The addition of one to both numerator and denominator avoids zero p-values and provides a conservative finite-sample estimate.

A small p-value indicates that the observed compatibility is located in the upper tail of the null distribution and is therefore unlikely to be explained by random alignment alone. Conversely, a large p-value indicates that similar compatibility values are frequently obtained from random spectra satisfying the same basic constraints. This interpretation is important because the statistical test does not establish intentional design, nor does it prove a causal relation between bell making and architectural geometry. It only evaluates whether the measured bell–cavity frequency relation is statistically distinguishable from chance under the adopted null model.

From a metrological perspective, this step is essential, because it introduces an explicit decision criterion into the interpretation of spectral compatibility. The relationship between bell emission and cavity resonance is not judged only by visual inspection of frequency tables or by qualitative agreement between peaks. Instead, the observed compatibility is compared with a reproducible reference distribution generated under controlled assumptions. This makes the interpretation falsifiable and allows the strength of the spectral relation to be expressed through a quantitative uncertainty-related measure.

The p-value obtained from this test was subsequently used to decide whether the compatibility score could be retained as a meaningful frequency-domain descriptor in the following analyses. Only compatibility relations that exceeded the random-alignment benchmark were considered relevant for the source-position and aperture-radiation evaluations. In this way, the statistical assessment acts as a filter between spectral comparison and functional interpretation.

3. RESULTS AND DISCUSSION

3.1. Results

The finite-element acoustic modal analysis of the belfry-cell air volume provided the eigenfrequency set reported in Table 1. The computed modes cover the low-frequency range of the enclosed architectural cavity and constitute the modal dataset used for the frequency-domain comparison with the measured bell spectrum.

The possible model improvement, where radiating aperture conditions were introduced, did not produce a uniform translation of the closed-cavity spectrum. Instead, the response was strongly mode dependent. Across the paired modes, the median absolute frequency variation was approximately 1.29 %, with 14 pairs remaining within 2 % of their closed-cavity reference values. When only modal associations with MAC values greater than 0.8 were considered, the median absolute shift decreased to approximately 0.83 %, with a maximum value of 2.11 %. Modes showing the strongest spatial correspondence

Table 1. Computed eigenfrequencies of the belfry-cell air cavity.

Mode	Frequency in Hz	Mode	Frequency in Hz	Mode	Frequency in Hz	Mode	Frequency in Hz
1	13.0587	12	47.7393	23	74.0675	34	84.6691
2	13.0589	13	53.7907	24	78.1256	35	85.5141
3	21.8372	14	53.8200	25	78.1522	36	85.5194
4	28.3338	15	62.6414	26	81.6500	37	91.2808
5	34.1829	16	62.6448	27	81.7393	38	91.3119
6	34.1845	17	63.1151	28	81.8231	39	93.7868
7	40.8746	18	64.9438	29	82.4390	40	93.8072
8	41.9835	19	64.9468	30	82.4614	41	97.4440
9	41.9838	20	66.1576	31	82.9529	42	98.0953
10	43.0744	21	66.1994	32	82.9552	43	98.2393
11	47.7098	22	74.0665	33	83.7353	44	98.7972

between the two configurations were therefore comparatively stable in frequency. Conversely, larger shifts were generally associated with lower MAC values or stronger radiation losses. The modes around 63.1 Hz and 84.6 Hz retained closely related pressure distributions and exhibited frequency shifts below 0.9 %, with radiation-related quality factors exceeding 60 and 120, respectively.

More importantly, propagating the open-boundary frequencies through the subsequent bell–cavity compatibility analysis did not change the identified harmonic relation or the overall statistical interpretation. Therefore, the FEM frequency shifts produced by the radiating apertures did not materially affect the final results of the study.

The acoustic measurements of the 13th century bell provided the near-field and far-field spectral components reported in Table 2. The ratios were computed with respect to the second component of each measured spectrum.

The direct perceptual comparison between the measured bell spectrum and the computed cavity modes did not provide the highest compatibility condition. The best frequency-domain correspondence was obtained under the harmonic-compatibility hypothesis, for an octave-type relation with a four-octave shift, corresponding to a frequency multiplier equal to 16. Table 3 reports the measured bell peaks up to approximately 2000 Hz, the corresponding shifted cavity frequencies, the original cavity frequencies, the detuning in cents, and the tolerance outcome.

The Monte Carlo statistical test was applied to the harmonic-compatibility score obtained under the best frequency-domain

Table 2. Dominant spectral components of the 13th century bell measured in near-field and far-field conditions.

Component	Near-field peak in Hz	Near-field ratio	Far-field peak in Hz	Far-field ratio
1	210.9	0.5	209.9	0.5
2	460.0	1.0	460.3	1.0
3	544.9	1.2	543.7	1.2
4	656.3	1.4	656.8	1.4
5	914.1	2.0	915.2	2.0
6	1086.9	2.4	1087.4	2.4
7	1344.7	2.9	1345.8	2.9
8	1834.0	4.0	1833.0	4.0
9	2370.1	5.2	2368.7	5.1
10	2941.4	6.4	2942.0	6.4
11	3539.1	7.7	3526.1	7.7
12	4136.7	9.0	4129.0	9.0

Table 3. Frequency-domain correspondence between measured bell components and shifted belfry-cell cavity eigenfrequencies under the best harmonic-compatibility condition.

Bell peak in Hz	Shifted cavity frequency in Hz	Cavity frequency in Hz	Detuning in cents	Within tolerance
210.3	208.9	13.1	11.2	Yes
457.8	453.3	28.3	16.9	Yes
545.7	546.9	34.2	3.9	Yes
655.3	654.0	40.9	3.5	Yes
913.6	861.1	53.8	102.4	No
1043.7	1039.1	64.9	7.6	Yes
1086.6	1059.2	66.2	44.2	Yes
1146.2	1185.1	74.1	57.7	No
1346.2	1339.8	83.7	8.3	Yes
1474.1	1461.0	91.3	15.5	Yes
1726.3	1580.8	98.8	152.5	No
1835.3	1580.8	98.8	258.5	No

relation. The best correspondence was associated with an octave-type relation involving a four-octave shift, corresponding to a frequency multiplier equal to 16. The empirical probability of obtaining an equal or higher compatibility score from random spectral realizations was $p = 0.034$. This value identifies the position of the measured bell–cavity compatibility score within the null distribution generated from synthetic spectra having the same cardinality and frequency span as the measured bell spectrum.

3.2. Discussion

The results show that the relationship between the 13th century bell and the belfry-cell cavity is not adequately described by direct frequency coincidence alone. The strongest correspondence emerges when the cavity eigenfrequencies are interpreted through an octave-type scaling with a four-octave shift, corresponding to a frequency multiplier equal to 16. As reported in Table 3, this transformation brings several low-frequency cavity modes into close correspondence with measured bell components. From a physical point of view, this supports the interpretation of the belfry cell as an architectural air volume whose modal scaffold is compatible with the spectral structure of the bell.

The distinction between direct perceptual proximity and harmonic compatibility is central to the metrological interpretation of the results. The direct ERB-based comparison evaluates whether bell components and cavity modes are close within auditory frequency-resolution units. The harmonic comparison, instead, evaluates whether the two frequency sets can be related through simple proportional mappings. These two criteria do not express the same physical condition. The first concerns immediate perceptual proximity between frequencies; the second concerns proportional frequency organization between two spectra occupying different frequency ranges. The fact that the relevant match appears under the harmonic-compatibility condition indicates that the bell–cavity relationship is not a trivial overlap between measured and computed frequency lists, but a structured relation mediated by octave scaling.

The use of ERB as a perceptual metric is methodologically relevant because bell sounds are complex, multi-component signals. Their perceptual relevance cannot be evaluated only through absolute frequency differences expressed in hertz. ERB provides a frequency-dependent descriptor of auditory filter

bandwidth and allows spectral separation to be expressed in relation to auditory resolution rather than only to physical frequency spacing [18]. This is important in the present study because the comparison concerns not isolated pure tones, but a complex bell spectrum interacting with the resonance scaffold of an architectural cavity. The ERB-based metric therefore provides a conservative perceptual baseline, while the cents-based harmonic metric evaluates proportional compatibility.

The Monte Carlo test further supports the interpretation of a non-random bell–cavity relationship. The empirical probability associated with the observed harmonic-compatibility score is $p = 0.034$. This value indicates that the measured correspondence lies in the upper tail of the null distribution obtained from random spectral configurations with the same cardinality and frequency span as the measured bell spectrum. In statistical terms, the test does not demonstrate causality or intentional design. Instead, it assesses whether the observed score is unlikely under the adopted null hypothesis. The use of empirical p-values derived from Monte Carlo procedures is a recognized way to evaluate whether an observed statistic is extreme with respect to a simulated null distribution, provided that the same statistic is computed for both observed and randomized data [19]. In this study, this condition is satisfied because each random spectrum was evaluated using the same compatibility procedure adopted for the measured bell spectrum.

The stability of the measured bell components in near-field and far-field conditions, reported in Table 2, is also relevant from a metrological standpoint. The near-field signal provides the most direct spectral description of the bell as an acoustic source, while the far-field signal confirms that the main components remain identifiable after propagation through the architectural environment. This supports the use of the retained peak set as an acoustic fingerprint of the bell. Since the compatibility analysis is based on these frequencies, their persistence across two different measurement conditions strengthens the robustness of the experimental input.

The cavity eigenfrequencies reported in Table 1 provide the complementary acoustic fingerprint of the architectural volume. The finite-element model identifies a dense low-frequency modal structure, with several close frequencies and modal clusters. The use of finite-element modelling for acoustic cavities is well established in computational room and enclosure acoustics, especially when the aim is to resolve modal behaviour in geometrically defined domains [20]. In the present framework, these low-frequency modes are not interpreted as directly audible tones corresponding to the bell sound. Rather, they define the resonance scaffold of the belfry-cell air volume, which can be compared with the audible bell spectrum through perceptual and harmonic criteria.

The main metrological contribution of the work lies in the conversion of a cultural-historical question into a chain of measurable and testable operations. The bell tower is not described as a resonator only by analogy with a musical instrument. Instead, the architectural geometry is converted into a computed modal dataset, the bell sound is converted into a measured spectral dataset, and the relationship between the two is evaluated through explicit compatibility metrics and a stochastic null model. This workflow is consistent with the increasing role of acoustic measurement and numerical modelling in the study of architectural and sonic heritage, where acoustic behaviour is treated as part of the cultural value of historical spaces [21]. More generally, recent work on acoustical heritage has emphasized that sound-related properties of

historical places require specific methodological frameworks because they combine physical, perceptual, and cultural dimensions [22].

Several limitations of the present study must be explicitly acknowledged. The first limitation concerns the acoustic model of the belfry cell. The finite-element analysis treats the internal air volume as an idealized cavity with rigid boundaries and computes the modal scaffold under simplified conditions. The masonry structure is not modelled as a deformable medium, and acoustic–structural coupling is not included. This simplification limits the possibility of predicting absolute sound pressure levels or detailed energy exchange between air and masonry. However, it does not invalidate the main result, because the objective of the model is not to reproduce the full vibroacoustic response of the tower, but to identify the reference eigenfrequency set of the enclosed air volume. The tested quantity is therefore frequency compatibility between two datasets: measured bell peaks and computed cavity modes. For this purpose, a simplified rigid-wall cavity model is appropriate as a first-order modal descriptor.

A further source of model-input uncertainty is the temperature dependence of the speed of sound. The nominal value $c_0 = 343 \text{ m s}^{-1}$ was adopted as a controlled reference condition, whereas the real belfry is an open environment in which air temperature may vary over daily and seasonal timescales. Around room temperature, the speed of sound changes by approximately 0.6 m s^{-1} per degree Celsius. Since the cavity eigenfrequencies are directly proportional to c , their relative temperature sensitivity is approximately 0.18% per degree Celsius. For example, a temperature deviation of 5°C above or below the nominal condition would produce an approximately 0.9% increase or decrease in the computed eigenfrequencies. This variation is limited at the level of the modal frequencies, but it may affect individual bell–cavity correspondences located close to the adopted detuning threshold. Temperature should therefore be regarded as an input quantity to be propagated through the compatibility analysis rather than as a source of independent modal restructuring. Under the same illustrative variation of 5°C , the corresponding frequency displacement is approximately ± 15 cents. Consequently, strongly aligned frequency pairs are expected to remain identifiable, whereas matches initially located near the tolerance boundary may change classification.

A second limitation concerns the temporary closure of the openings in the modal model. In reality, the belfry cell is not a closed cavity, and the openings allow sound radiation and energy leakage. This means that the computed eigenfrequencies should be interpreted as the intrinsic modal scaffold of the architectural volume, not as a complete description of the open radiating system. This limitation does not undermine the frequency-domain compatibility result, because the analysis does not rely on predicted reverberation times, sound pressure levels, or radiation efficiency. It only asks whether the characteristic frequency structure imposed by the geometry of the air volume is compatible with the measured spectrum of the bell. The opening system would affect damping and radiation, but the underlying geometric modal organization remains a meaningful reference for testing frequency relationships.

A third limitation is related to the use of a reduced frequency-domain framework. The analysis does not reconstruct the complete time-dependent acoustic field generated during bell ringing, nor does it simulate nonlinear source–cavity interaction. It also does not quantify directional radiation, spatial propagation in the surrounding urban space or subjective listening conditions.

This restriction is deliberate. The present study focuses only on the coupling hypothesis between the bell and the belfry-cell cavity, expressed as a frequency-domain relationship. Therefore, conclusions are limited to spectral compatibility and cannot be extended to claims about territorial audibility, soundscape reconstruction, or directional performance.

A fourth limitation concerns historical interpretation. The statistical result cannot be used to prove that medieval builders intentionally designed the belfry cell to match the bell spectrum. The p -value only indicates that the observed harmonic compatibility is unlikely under the adopted random-frequency null model. It does not distinguish among possible historical mechanisms, such as intentional design, empirical refinement, inherited craft rules, proportional reasoning, or later compatibility produced by bell replacement and installation history. This is why the interpretation must remain functional rather than intentional: the system is acoustically compatible, but the analysis does not determine why it became so.

These limitations are not weaknesses of the result, provided that the result is stated at the correct level. The study does not claim complete acoustic reconstruction, full causal explanation, or historical intentionality. It claims that the measured bell spectrum and the computed belfry-cell modal structure exhibit a statistically supported harmonic compatibility. This claim is directly supported by the measurement and modelling chain. The limitations mainly restrict the scope of interpretation, not the validity of the frequency-domain result.

This distinction is important for cultural heritage studies. In historical acoustics, there is often a risk of moving too quickly from qualitative analogy to functional interpretation. The present approach reduces this risk by separating measurable evidence from historical inference. The measurable evidence consists of: the spectral components of the bell, the cavity eigenfrequencies, the harmonic scaling relation, the detuning values, and the Monte Carlo probability. The historical inference is more cautious: the belfry cell may be interpreted as an architectural volume functionally compatible with the sound of the medieval bell. This interpretation is supported by the quantitative evidence, but it does not exceed it.

From the point of view of intangible heritage, this is a relevant outcome. The sonic function of the bell tower is not directly preserved as an audible medieval event, but part of its physical basis may still be accessible through the relationship between architectural form and acoustic response. Studies on church acoustics and acoustical heritage have shown that acoustic properties can be considered part of the cultural value of historical buildings, especially when they are linked to liturgical, musical, or social use. In the present case, the acoustic property under investigation is more specific: the compatibility between a bell source and the modal structure of the belfry cell that hosts it.

Overall, the results indicate that the Norman bell tower of Salerno Cathedral can be interpreted as a measurable vibroacoustic heritage system, in which the medieval bell and the belfry-cell air volume exhibit non-random acoustic compatibility. The contribution of the paper is therefore not only the analysis of a single monument, but the demonstration of a metrological procedure for investigating historical sonic function. By combining acoustic measurement, finite-element modal analysis, perceptual frequency metrics, and Monte Carlo testing, the study provides a reproducible framework for assessing bell-cavity coupling in cultural heritage contexts.

4. CONCLUSIONS

This study proposed a quantitative vibroacoustic metrological framework for investigating the relationship between bell sound and architectural function in the Norman bell tower of Salerno Cathedral. The work focused specifically on the coupling between the 13th century bell and the belfry-cell air cavity, avoiding broader claims on sound propagation, directivity, or territorial audibility. The central question was whether the measured spectrum of the bell and the computed modal structure of the belfry cell exhibit a physically meaningful and statistically non-random compatibility.

The results show that the bell-cavity relationship is not primarily explained by direct frequency coincidence. The strongest correspondence was obtained under a harmonic-compatibility condition based on an octave-type relation with a four-octave shift, corresponding to a frequency multiplier equal to 16. Several measured bell components were found to align with shifted cavity eigenfrequencies within the adopted tolerance. The Monte Carlo test returned an empirical probability of $p = 0.034$, indicating that the observed compatibility is unlikely to arise from random spectral alignment alone. This result does not prove intentional acoustic design, but it demonstrates that the medieval bell and the belfry-cell cavity form a physically coherent vibroacoustic system in the frequency domain.

The main real impact of the study is methodological. The paper demonstrates that hypotheses concerning intangible sonic functions in cultural heritage can be transformed into measurable and testable relationships. The acoustic role of the belfry cell is not inferred only from architectural analogy or historical plausibility, but from a reproducible workflow based on acoustic measurement, finite-element modal analysis, perceptual frequency metrics, and statistical testing. In this sense, the work strengthens the role of physical metrology as an interpretative discipline in heritage science, capable of producing evidence that is independent from, but complementary to, historical, archaeological, and architectural analysis.

A further real impact concerns the interpretation of medieval architectural heritage. The results suggest that the upper belfry cell of the Salerno Cathedral bell tower should not be regarded only as a structural container for bells or as a symbolic architectural termination. It can also be interpreted as an architectural air volume, whose modal properties are compatible with the spectral structure of the bell it houses. This shifts the reading of the monument from a purely visual and material perspective to a functional and vibroacoustic one, in which architectural form, sound production, and perceptual relevance become part of the same heritage system.

The potential impact of the work is broader. The proposed procedure can be applied to other historical bell towers, belfry cells, and sound-producing architectural systems. By comparing measured source spectra with modelled cavity modes and testing the statistical significance of the relationship, it becomes possible to classify different monuments according to their degree of bell-cavity compatibility. This could support comparative studies across historical periods, construction traditions and geographical areas, helping to identify whether similar acoustic-functional patterns recur in medieval architecture.

The study may also have practical implications for conservation and restoration. If the acoustic relationship between a bell and its architectural cavity is part of the functional value of a monument, then conservation strategies should not

consider only structural stability, material integrity, and visual appearance. They should also preserve, document and, where possible, monitor the acoustic conditions that allow the monument to perform its sonic function. This is particularly relevant when bells are replaced, relocated, silenced, restored, or mechanically modified, because such interventions may alter the vibroacoustic identity of the heritage system.

From a social and cultural perspective, the results contribute to the recognition of sound as a component of cultural memory. Bell sound historically structured religious time, civic life, and collective identity. Although the original medieval soundscape cannot be recovered directly, the physical basis of that sonic function may still be investigated through the surviving relationship between architectural geometry and acoustic response. The study therefore supports a more complete understanding of cultural heritage, where intangible functions are not treated as abstract narratives but as phenomena that may leave measurable traces in built form.

The study remains deliberately limited in scope. It does not reconstruct the outdoor soundscape of medieval Salerno, does not quantify directional radiation through the openings, and does not demonstrate historical intentionality. Its conclusion is more focused and more robust: the measured bell spectrum and the computed belfry-cell modal structure exhibit statistically supported harmonic compatibility. This result is sufficient to support the interpretation of the Salerno Cathedral bell tower as a measurable vibroacoustic heritage system, and to justify further investigations on bell–architecture coupling in historical monuments.

Overall, the work shows that quantitative vibroacoustic metrology can move beyond a purely technical role and become a tool for recovering functional meanings embedded in cultural heritage. By making the acoustic relationship between bell and belfry measurable, testable, and reproducible, the study opens a methodological path for investigating how historical architectural forms may have supported sonic practices, empirical knowledge, and intangible cultural functions.

AUTHORS' CONTRIBUTION

All the authors equally contributed to the present work. Fabrizio Barone: conceptualization, formal analysis, methodology, supervision, visualization, writing – original draft; Rosa Fiorillo: investigation, validation, writing – review & editing; Marco Casazza: methodology, data curation, investigation, writing – review & editing.

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