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Integration of Monte Carlo simulation for uncertainty evaluation into intra-laboratory comparison for reference standards consistency assessment

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ABSTRACT
Accredited calibration and test laboratories are required to participate periodically in inter-laboratory comparisons and proficiency testing schemes, as part of their quality assurance procedures, for ensuring the validity of the data, which is provided to their clients. In this paper, an intra-laboratory comparison, as an additional tool for quality assurance, is presented. The proposed concept is practically realized in an accredited calibration laboratory for electrical quantities instruments, in the domain of high-resistance reproduction and measurements. Reference standards of the highest accuracy class available, which are traceable to BIPM intrinsic standards through different national metrology institutes, are used, covering the range above 100 MΩ. A methodology for the calculation of the E_n criterion, in accordance with the ISO/IEC 17043 guidelines, is deployed. The measurement uncertainty is evaluated according to the principles presented in the Guide to the expression of uncertainty in measurement (GUM), as well as by using the Monte Carlo simulation concept of distribution propagation. By regarding the different principles for resistance measurement, implemented in the selected instrumentation and the two methods for uncertainty calculation, this intra-laboratory comparison offers a quantitative assessment of the consistency and reliability of the selected reference standards, thereby enhancing confidence and credibility of the measurement results provided by the laboratory.

Section: RESEARCH PAPER

Keywords: intra-laboratory comparison; measurement uncertainty; Monte Carlo simulation; quality assurance

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1. INTRODUCTION

Calibration laboratories, accredited according to the international standard ISO/IEC 17025:2017 [1], are required periodically to participate in inter-laboratory comparisons (ILCs) and proficiency testing (PT) schemes [2]. In such a manner, a comparability between the measurement results from the protocols performed in multiple laboratories with the same or similar scope of accreditation, is enabled. According to these participations, which are included in the quality assurance procedures [3], the laboratories provide an enhanced level of confidence, regarding the validity of the data that is provided to their clients. Several publications covering the results of conducted ILCs and PT schemes are available from recent years in the domain of electrical metrology [4]–[9]. In these papers, the participations of both commercial laboratories and the National Metrology Institutes (NMIs) are regarded.

According to [1], laboratories are committed to introduce other measures for quality assurance, as well [3]. An example of such a measure is a comparison between calibration results, performed by several different operators, using the same equipment, in the same measuring conditions, during the shortest possible time period. Another option for maintaining the quality of the measurement data is a periodic check-up of the laboratory's equipment with lower accuracy class instruments or testing artefacts, with known characteristics. In this paper, an upgraded approach to quality assurance will be presented, referred to as intra-laboratory comparison. Similar to the concept of inter-laboratory comparison, it is based on weighing up the performance of two or more artefacts that are in the possession of a single laboratory. The meters that are deployed in the intra-laboratory comparison are supposed to have the same or similar measurement ranges. In such a manner, the consistency and the reliability of these meters is assessed, by introducing a numerical evaluation criterion [10].

In the paper, a practical realization of the selected concept will be presented. The real-time measurements and the evaluation of errors and uncertainties are conducted in an accredited calibration laboratory [1], regarding instruments and reference standards (RSs) for electrical quantities [11]: the Laboratory of Electrical Measurements (LEM) [3]. The laboratory is part of the Faculty of Electrical Engineering and Information Technologies (FEEIT) at Ss. Cyril and Methodius University in Skopje (UKIM), and it maintains international traceability to BIPM [12] intrinsic RSs. The laboratory possesses a documented history of participation in both ILCs and PT schemes [13]–[16] in the most of its accreditation scope.

Since the beginnings in 2015, LEM has been an accredited laboratory for the calibration of instrumentation for both DC and low-frequency AC voltages and currents, electrical resistance, electrical power and energy. In 2023, its accreditation scope was extended, in terms of both the introduction of new electrical quantities, such as inductance, and the improvement of the calibration and measurement capabilities (CMC), from the perspective of the range covered within the previously mentioned quantities. The extension of the accreditation scope and the introduction of new calibration protocols have already been presented in several scientific papers [17]–[22]. For the purposes of this work, the procurement of a high-resistance decade resistor and its introduction into the laboratory's list of equipment is important. The resistor enables the reproduction of electrical resistance values higher than 100 MΩ, up to 1 TΩ, and may withstand test voltages up to 5 kV. This enabled the calibration of a completely new scope of electrical equipment, most notably insulation testers, dielectric withstand testers, ESD stations, etc.

The intra-laboratory comparison will encompass the aforementioned decade resistor and other RSs that have measurement capabilities for high resistance recording. The participation list includes two high-resolution multimeters, as well as a multifunction calibrator with a high-resistance measurement adapter. The implemented measurement equipment, alongside with the constraints for the selection of measurement points, are presented in Section 2. For the evaluation of measurement uncertainty, two complementary approaches are going to be adopted. At first, there is the traditional statistical method for uncertainty propagation, based on the concept provided in the Guide to the expression of uncertainty in measurements (GUM) [23]. The realization of the intra-laboratory comparison by using the GUM-based [23] approach has already been presented in [24]. Additionally, the distribution propagation concept, based on Monte Carlo simulation [25], will also be used. The implementation of the Monte Carlo distribution propagation for uncertainty evaluation in the domain of electrical calibrations has been covered in [26]–[27]. Both concepts regarding the measurement or reproduction of high electrical resistance with the introduced equipment will be discussed in Section 3 of the paper. For the quantitative analysis of the measurement results within the intra-laboratory comparison, the E_n criterion, introduced in the international standard ISO/IEC 17043:2023 [28], will be adopted. The results of the intra-laboratory comparison and their discussion are going to be presented in Section 4. Section 5 of the paper is going to cover the sensitivity analysis, i.e. it will regard eventual variations in the intra-laboratory comparison results, due to the underestimation of some influencing factors. In the end, the conclusions of the work, as well as the pathway for future research, will also be provided.

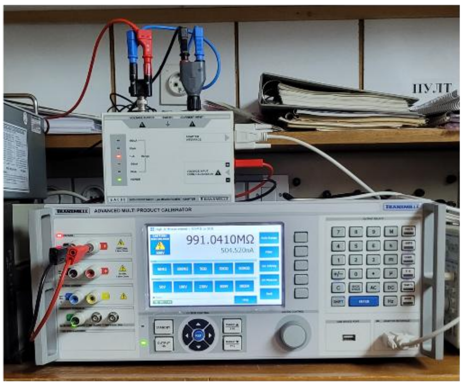
2. REFERENCE STANDARDS AND CONSTRAINTS IN THE SELECTION OF MEASUREMENT POINTS

The four RSs, which are selected to participate in the intra-laboratory comparison, are illustrated in Figure 1. The decade resistor, IET Labs, Inc. HRRS-Q-4-1M-5KV [29], shown in Figure 1a), is the laboratory's primary RS in the domain of high resistance calibrations, in the range between 100 MΩ and 1 TΩ. It maintains international traceability to BIPM [12] intrinsic RSs, through the national standards of the USA, via the National Institute of Standards and Technology (NIST) [30]. In the following discussion, it will be labelled as RS No.1. The second instrument is the laboratory's primary RS in the domain of DC and low frequency AC voltages and currents, as well as electrical resistance between 0.1 Ω and 100 MΩ, an 8 ½ digit multimeter, Agilent 3458A [31], illustrated in Figure 1b). In the following, it will be labelled as RS No.2. Even though this meter is used as a primary RS for electrical resistance up to 100 MΩ, it has a

a)

b)

c)



d)

Figure 1. The LEM reference standards deployed in the intra-laboratory comparison: a) RS No.1, b) RS No.2, c) RS No.3, d) RS No.4.

maximal measurement range of 1 GΩ. The laboratory's secondary (working) RS, in the domain of the aforementioned electrical quantities, a 6 ½ digit multimeter, FLUKE 8846A [32], presented in Figure 1c), is also included in the intra-laboratory comparison. This unit is further labelled as RS No.3. Its maximal resistance range also equals 1 GΩ. Both multimeters are traceable to BIPM [12] intrinsic RSs via the National Metrology Institute (NMI) of the Republic of Serbia, The Directory of Measures and Precious Metals [33]. The last artefact regarded in the intra-laboratory comparison is a multifunction calibrator, Transmille 4015 [34], with a high resistance/low current measuring adapter, EA008 [35], both of them illustrated in Figure 1d), labelled as RS No.4. The calibrator is used as the laboratory's primary RS for frequencies up to 600 MHz and inductance, and as a secondary (working) standard for the other electrical quantities. It is traceable to BIPM [12], through the national RSs of the United Kingdom, via the National Physical Laboratory (NPL) [36].

By using the four RSs, three different options, for establishing and maintaining an unbroken traceability chain in the area of high resistance calibrations, are enabled. Both multimeters' [31]–[32] measuring algorithm is based on Ohm's law. The calibrator [34] with the measuring adapter [35] uses high DC voltage test signals, i.e. a measuring principle similar to the one implemented in insulation testers. The decade resistor [29] is a solid-state artefact, a physical realization of the measured quantity [11]. This implies that the intra-laboratory comparison is not conducted solely for providing enhanced confidence in the measurement data, but also for increasing the quality assurance of the laboratory's calibration artefacts performance. The obtained results provide an affirmation on the possibility for maintaining an unbroken traceability chain, in the domain of high resistance, with instruments based on different measuring principles, as well.

The measurement points are supposed to be chosen in accordance with the constraints provided by the measuring/ sourcing range of each RS. The lowest resistance that can be reproduced with RS No.1 [29] equals 100 MΩ, while the highest surpasses 1 TΩ. On the other hand, both RS No.2 [31] and RS No.3 [32] have measurement ranges between 10 Ω and 1 GΩ. This implies that the intra-laboratory comparison is meant to be realized in a very narrow range, between 100 MΩ and 1 GΩ. The RS No.4 measurement configuration [34]–[35] possesses a wider selection of measurement ranges, which are dependent on the applied test voltage. In such a way, it does not have any significant impact on the selection of measurement points. Based on the introduced constraints, the intra-laboratory comparison is chosen to be realized in three measurement points, related to resistance values of 100 MΩ, 500 MΩ, and 1 GΩ. One set of measurements is going to be conducted with both digital multimeters [31]–[32]. Regarding the calibrator [34] and its adapter [35], three sets of measurements will be performed, each one corresponding to a different test voltage. For the particular experiment, test voltages of 100 V, 500 V, and 1000 V are selected. The measurements with all the participating instruments are conducted within a 24-hour interval.

In Table 1, the best one-year specifications of the four RSs are presented. The specifications are given as a percentage accuracy of the sourced/measured value. Table 1 shows that the decade resistor [29] demonstrates the highest accuracy of all the introduced artefacts in the selected range, therefore, it is going to serve as a pilot instrument of the intra-laboratory comparison. The accuracy presented for the 8 ½ digit multimeter [31] is variable, considering that measurements in the previously introduced points require alteration between its measuring ranges, the one of 100 MΩ and the other of 1 GΩ. The same conclusion is valid for the 6 ½ digit instrument [32], which demonstrates the lowest accuracy among all RSs. The accuracy of the calibrator [34] and its adapter [35] is rather constant in terms of the measured value, no matter the applied test voltage. However, some variations exist due to the alterations of the measurement ranges, which will be addressed in the discussion that follows.

Table 1. Best one-year specifications of the reference standards in the range between 100 MΩ and 1 GΩ.

Reference standard	Best 1 year's specification
RS No.1	± 0.1 %
RS No.2	± 0.1 % to ± 1 %
RS No.3	± 0.8 % to ± 2 %
RS No.4	± 0.5 %

As already mentioned in the introduction of the paper, the E_n criterion, introduced in ISO/IEC 17043:2023 [28], is chosen for numerical evaluation in the intra-laboratory comparison. The E_n value, in a single measurement point, is calculated as follows:

$$E_\mathrm{n} = \frac{R_\mathrm{par} - R_\mathrm{ref}}{\sqrt{U^2_{95\,\%,\mathrm{par}} + U^2_{95\,\%,\mathrm{ref}}}}\,, \tag{1}$$

where R_par is the resistance recorded with one of the digital multimeters [31]–[32] or the multifunction calibrator [34]–[35], and R_ref is the reference resistance, set up on the decade resistor [29]. Regarding the measurement uncertainties, $U_{95\,\%,\mathrm{par}}$ is the uncertainty attributed to the measured resistance with any of the participating instruments, while $U_{95\,\%,\mathrm{ref}}$ is the uncertainty attributed to the set-up value on the decade resistor [29]. The presented uncertainties, for both the evaluation concepts, GUM [23] or Monte Carlo [25], are expressed for a coverage probability of 95 %. According to [28], if the calculated E_n criterion is between -1 and 1, the result of the intra-laboratory comparison is marked as PASS, otherwise it stands as FAIL.

3. MEASUREMENT UNCERTAINTY EVALUATION ACCORDING TO GUM AND MONTE CARLO

In the following discussion, the influencing factors that contribute to the uncertainty budget, regarding both the measured resistance with any of the participating RSs [31]–[32], [34]–[35] and the set-up value with the reference resistor [29], are going to be mathematically evaluated. The single uncertainty components are evaluated differently, based on the available data in the specifications and datasheets of the calibration artefacts and the available calibration certificates. Two uncertainty components are calculated in the same way, no matter the regarded instrument.

The first uncertainty component is calculated, according to the principles provided in GUM [23], as Type A uncertainty [37]. Namely, in a single measurement point, n recordings are conducted for statistical random variations of the measured quantity to be considered. The measured resistance is then calculated as arithmetic mean from the n recordings performed:

$$R_\mathrm{M} = \frac{1}{n}\sum_{i=1}^{n} R_i\,, \tag{2}$$

where R_i are the single resistance readings. The Type A uncertainty is calculated as standard deviation of the mean, assuming t-distribution [23], [37]:

$$u_{\mathrm{A}} = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(R_i - R_{\mathrm{M}})^2}\,. \tag{3}$$

The second uncertainty component, which is commonly attributed to the measurements performed with any of the instruments, is related to the meter's finite resolution. The resolution, r, varies for different instruments and for different measuring ranges selected, however, the related uncertainty is calculated in the same manner, assuming rectangular (uniform) distribution [37]:

$$u_{\mathrm{res}} = \frac{r}{2\cdot\sqrt{3}}\,. \tag{4}$$

In equation (4), the division with a factor of $\sqrt{3}$ is used for obtaining the standard uncertainty from the distribution boundaries, $\pm\, r/2$ [23].

3.1. Case study – RS No.1

In the previous discussion, it was highlighted that the Type A and the resolution-related uncertainties are common components, for all the participating instruments. However, that is not the case with the decade resistor [29], as it is a solid-state artefact, intended for the reproduction of the particular quantity, i.e. no actual recordings are conducted with this unit. The mathematical model for the illustration of the influential factors that affect the set-up value is given as follows:

$$R_{\mathrm{ref}} = R_{\mathrm{n}} + \delta_{\mathrm{acc}} + \delta_{\mathrm{st}} + \delta_{\mathrm{temp}} + \delta_{\mathrm{cal}}\,. \tag{5}$$

In equation (5), R_{n} is the nominal set-up value on the decade resistor, and the other components have the following meaning:

- δ_{acc}: error due to the accuracy of the resistor,
- δ_{st}: error due to its long-term stability,
- δ_{temp}: error due to temperature influence on the resistor's performance,
- δ_{cal}: calibration error.

All these errors are taken as having a mean value of zero [23], [37], and the standard uncertainties associated with them are mathematically evaluated as:

$$u_{\mathrm{acc}} = \frac{U_{\mathrm{ACC,\%}}}{100}\cdot\frac{R_{\mathrm{n}}}{\sqrt{3}}\,, \tag{6}$$

where $U_{\mathrm{ACC,\%}}$ is the accuracy of the RS No.1, expressed in a relative form, according to its specification [29];

$$u_{\mathrm{st}} = \frac{U_{\mathrm{ST,\%}}\cdot y}{100}\cdot\frac{R_{\mathrm{n}}}{\sqrt{3}}\,, \tag{7}$$

where $U_{\mathrm{ST,\%}}$ is the long-term stability of RS No.1, expressed in a relative form, according to its specification [29], and y is the number of years that have passed since its last calibration;

$$u_{\mathrm{temp}} = \frac{U_{\mathrm{TEMP,\%}}\cdot\Delta t}{100}\cdot\frac{R_{\mathrm{n}}}{\sqrt{3}}\,, \tag{8}$$

where $U_{\mathrm{TEMP,\%}}$ is the temperature-related influence on the resistor's performance, expressed in a relative form, according to its specification [29], and Δt are the temperature variations in the environment where the measurements are conducted. Accuracy, long-term stability, and temperature-related standard uncertainties are calculated by assuming rectangular (uniform) distributions, as no additional information about these influencing factors is presented in [29]. Therefore, for the calculation of the standard uncertainty from the distribution boundaries, the division by a factor of $\sqrt{3}$ is adopted;

$$u_{\mathrm{cal}} = \frac{U_{\mathrm{CAL,\%}}}{100}\cdot\frac{R_{\mathrm{n}}}{2}\,, \tag{9}$$

where $U_{\mathrm{CAL,\%}}$ is the resistor's calibration uncertainty, taken from its calibration certificate, in which it is expressed in relative form. The uncertainty of calibration is usually comprised of multiple influencing factors, therefore, for the evaluation of u_{cal}, Gaussian distribution is adopted [23], [37]. In equation (9), $U_{\mathrm{CAL,\%}}$ is presented with a coverage probability of approximately 95 %.

In Table 2, the magnitudes of the uncertainty components are presented for the three measurement points. The accuracy-related uncertainty is the dominant component in all three cases. The long-term stability and the calibration-related components have the same order of magnitude value, while the temperature-related uncertainty is the lowest in the overall budget. If the GUM [23] concept is adopted, the following equation is used for the calculation of the standard combined uncertainty:

$$u_{\mathrm{C,ref}} = \sqrt{u_{\mathrm{acc}}^2 + u_{\mathrm{st}}^2 + u_{\mathrm{temp}}^2 + u_{\mathrm{cal}}^2}\,, \tag{10}$$

where all the influencing factors are treated as mutually uncorrelated. This is a usual approach in the domain of calibration of instruments for electrical quantities [38]. A correlation between u_{acc}, u_{st}, and u_{temp} may eventually be considered, as the three components emerge from the producer's findings, given in [29]. Based on the routine check-ups performed, the decade resistor showed excellent stability of the reproduced resistance over a several-year period, i.e. no significant implications on the accuracy were detected. As u_{temp} is significantly smaller than u_{acc}, the approach presented in (10) is then encouraged, emphasizing efficiency in the analysis.

According to GUM [23], if multiple uncertainties are regarded, the resultant distribution will tend towards Gaussian, no matter the distributions prescribed to the input influencing factors. The expanded combined uncertainty is evaluated by multiplying the value obtained according to (10), with a coverage factor, which for 95 % coverage interval, equals k = 1.96:

$$U_{95\,\%,\mathrm{ref}} = U_{\mathrm{C,ref}} = k\cdot u_{\mathrm{C,ref}} = 1.96\cdot u_{\mathrm{C,ref}}\,. \tag{11}$$

Data presented in Table 2 indicates that the dominant uncertainty component, in any measurement point, is evaluated by assuming uniform distribution. A situation where the input

Table 2. Standard uncertainties attributed to the set-up resistance on RS No.1.

Uncertainty component	Measurement point		
	100 MΩ	500 MΩ	1 GΩ
u_{acc}	0.058 MΩ	0.29 MΩ	0.0012 GΩ
u_{st}	0.017 MΩ	0.087 MΩ	0.00087 GΩ
u_{temp}	0.0043 MΩ	0.022 MΩ	0.000043 GΩ
u_{cal}	0.005 MΩ	0.025 MΩ	0.00015 GΩ

quantities are attributed to a distribution other than normal is an indication for the implementation of an alternative method for uncertainty evaluation, the Monte Carlo method of distribution propagation [25], [37]. The result of the Monte Carlo simulation is an illustration of the equivalent distribution, according to the previously introduced parameters (mean and standard deviation or distribution boundaries) of the input quantities.

The Monte Carlo distribution propagation method [25] is included for obtaining the resultant distribution of values, which the set-up resistance on the resistor [29] may actually possess. For its implementation, the model presented in (5) is used, alongside with the standard uncertainties, evaluated according to equations (6)–(9). The simulation is performed by using MATLAB®. In each measurement point, a total of $N = 1000000$ (one million) simulations are performed. According to [25], such a number of simulations is found sufficient, for the coverage interval of 95 % to be adequately obtained.

In Figure 2, the results of the Monte Carlo simulation are presented. The equivalent distributions in the three measurement points do not resemble the pattern of a normal distribution. The distribution may be assumed to be trapezoidal for 100 MΩ and 500 MΩ, and it tends toward triangular for the 1 GΩ measurement point. This implies that the uncertainty, presented for a coverage interval of 95 %, will be different if evaluated according to the GUM [23] and Monte Carlo [25] concepts. This may later result in a different outcome of the intra-laboratory comparison.

3.2. Case study – RS No.2

The mathematical model for representing the influencing factors that affect the measured resistance with the 8 ½ digit multimeter, [31], is given as follows:

$$R_{\text{par}} = R_{\text{M}} + \delta_{\text{res}} + \delta_{\text{acc}} + \delta_{\text{temp}} + \delta_{\text{cal}}\,. \tag{12}$$

In equation (12), R_{M} is the mean resistance, calculated according to (2), by making $n = 10$ recordings in each measurement point, δ_{res} is an error due to the limited resolution of the instrument, and all other errors, δ_{acc}, δ_{temp} and δ_{cal}, have the same meaning as described in subsection 3.1. The number of single recordings was chosen in accordance with the calibration working instructions of LEM [3]. Based on the experience gained in the laboratory, a further increase in n would result in only marginal improvements of the data set regarding precision. This is further backed up by the high-accuracy performance and the stability of the introduced RSs, based on their history of periodic calibrations. The Type A uncertainty is then evaluated according to (3), from the performed single recordings. The resolution uncertainty is calculated according to (4).

The uncertainty that emerges from the accuracy specifications of RS No.2 is calculated as prescribed in its datasheet [31]:

$$u_{\text{acc}} = \frac{A_{\text{meas},\%} \cdot R_{\text{M}} + B_{\text{ran},\%} \cdot R_{\text{ran}}}{100} \cdot \frac{1}{\sqrt{3}}\,, \tag{13}$$

where $A_{\text{meas},\%}$ is a component of the accuracy limits, given as a percentage of the measured value, while $B_{\text{ran},\%}$ is a component of the accuracy limits presented as a percentage of the measurement range, R_{ran}. The standard uncertainty is evaluated by assuming a uniform distribution, as no additional information about the accuracy is presented in [31], hence, the division by √3.

The uncertainty component that represents the temperature fluctuations' influence on the RS's performance is evaluated as:

$$u_{\text{temp}} = \frac{(T_{\text{ran},\%} \cdot R_{\text{ran}})}{100} \cdot \frac{1}{\sqrt{3}}\,, \tag{14}$$

where $T_{\text{ran},\%}$ is the corresponding influence given as a percentage of the measurement range, R_{ran}, and it is valid for a specific temperature range, according to [31]. Once again, a rectangular distribution is adopted.

The last uncertainty component, referring to the level-up calibration of the RS No.2 [31], is calculated as:

$$u_{\text{cal}} = \frac{U_{\text{CAL}}}{2}\,, \tag{15}$$

where U_{CAL} is the expanded calibration uncertainty, expressed in absolute form, obtained from the multimeter's calibration certificate. U_{CAL} is presented by assuming Gaussian distribution, with a coverage interval of approximately 95 %, hence, the division by 2, as stated in the calibration certificate.

The values of the single uncertainty components are presented in Table 3. The multimeter's accuracy and calibration uncertainty dominantly shape the overall budget in all three measurement points. These two components have an equal order of magnitude value, u_{acc} being almost double the value of u_{cal}. They are followed by the estimate of the statistical variations of the real-time measurements. The remaining components may even be neglected, due to their insignificant contribution to the budget.

If the GUM [23] approach is adopted, the standard combined uncertainty is calculated as follows:

$$u_{\text{C,par}} = \sqrt{u_{\text{A}}^2 + u_{\text{res}}^2 + u_{\text{acc}}^2 + u_{\text{temp}}^2 + u_{\text{cal}}^2}\,, \tag{16}$$

by assuming uncorrelated input components [38]. The accuracy- and the temperature-related components may be regarded with some degree of correlation, as they both emerge from the

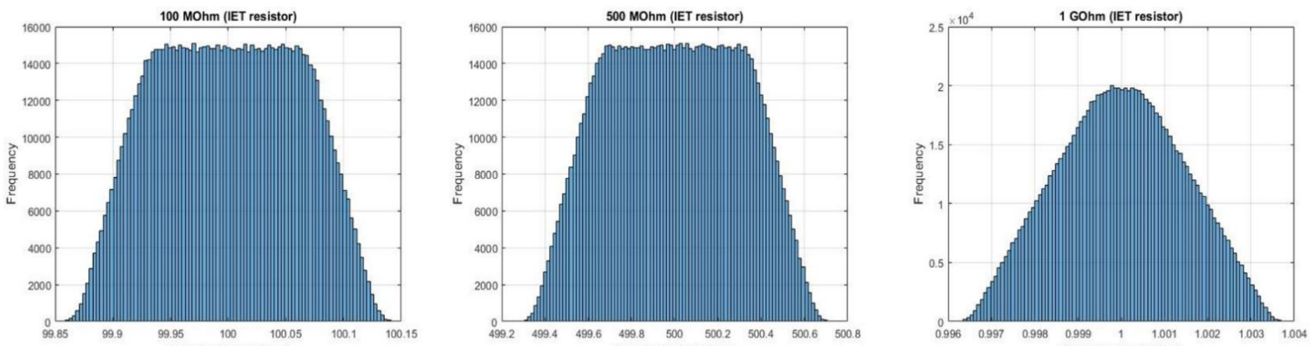


Figure 2. Distribution of values, attributed to the set-up resistance on RS No.1, according to the Monte Carlo simulation.

Table 3. Standard uncertainties attributed to the measured resistance with RS No.2.

Uncertainty component	Measurement point		
	100 MΩ	**500 MΩ**	**1 GΩ**
u_{A}	0.0028 MΩ	0.12 MΩ	0.002 GΩ
u_{res}	0.0000029 MΩ	0.000029 MΩ	0.000000029 GΩ
u_{acc}	0.058 MΩ	2.9 MΩ	0.0058 GΩ
u_{temp}	0.00058 MΩ	0.058 MΩ	0.000058 GΩ
u_{cal}	0.025 MΩ	1.1 MΩ	0.0025 GΩ

standard's specifications [31]. Regarding the practical reasons presented earlier, the evaluation is however conducted according to (16). The expanded uncertainty is calculated by multiplying the standard combined uncertainty with a coverage factor k = 1.96, by assuming normal distribution and a coverage interval of 95 %:

$$U_{95\,\%,\mathrm{par}} = U_{\mathrm{C,par}} = k \cdot u_{\mathrm{C,par}} = 1.96 \cdot u_{\mathrm{C,par}} \,. \tag{17}$$

If the Monte Carlo simulation [25] is implemented, it is expected that the resultant distribution will be more similar to Gaussian, in comparison to the case study of RS No.1 [29]. This is due to the fact that one of the influencing factors that dominantly shape the overall budget is evaluated by adopting a normal distribution. The verification of this assumption is illustrated in Figure 3, where the distributions of the measured resistances, with RS No.2 [31], are presented.

The implementation of the Monte Carlo simulation is conducted according to the setting introduced in subsection 3.1. The resultant distributions do not follow an exact pattern of a Gaussian distribution, therefore, a different outcome of the intra-laboratory comparison may be expected if different uncertainty evaluation methods are adopted. For the first two measurement points, 100 MΩ and 500 MΩ, the equivalent distribution may be approximated with a trapezoidal one. This is due to the highest contribution of u_{acc} in the overall budget, an influencing factor, mathematically evaluated by adopting a uniform distribution [37]. The distribution for 1 GΩ is closest to Gaussian, from all the contributions that have been presented so far. This is due to the fact that the Type A uncertainty is almost as significant as the calibration uncertainty, and it is evaluated by adopting a t-distribution, regarding the real-time recordings.

3.3. Case study – RS No.3

The third instrument in the intra-laboratory comparison is the 6 ½ digit multimeter, RS No.3 [32]. The analytical model for the representation of the influencing factors may be expressed as:

$$R_{\mathrm{par}} = R_{\mathrm{M}} + \delta_{\mathrm{res}} + \delta_{\mathrm{acc}} + \delta_{\mathrm{cal}} \,. \tag{18}$$

All the quantities in equation (18) have the same meanings as those already introduced in subsections 3.1 and 3.2. Once again, n = 10 single recordings were performed in a single measurement point. The statistical variations of the single recordings are introduced via the Type A uncertainty, calculated according to equation (3). The resolution-related uncertainty, u_{res}, is calculated according to equation (4). The uncertainty u_{acc} is evaluated according to equation (13) as the accuracy related parameters, $A_{\mathrm{meas,\%}}$ and $B_{\mathrm{ran,\%}}$, of RS No.3 [32] are given in the same form as the corresponding data for RS No.2 in [31]. The calibration uncertainty is calculated as:

$$u_{\mathrm{cal}} = \frac{U_{\mathrm{CAL,\%}}}{100} \cdot \frac{R_{\mathrm{M}}}{2} \,, \tag{19}$$

where $U_{\mathrm{CAL,\%}}$ is the expanded calibration uncertainty of the 6 ½ digit multimeter [32], presented in relative form, evaluated by assuming Gaussian distribution and approximately 95 % coverage interval.

In Table 4, the single uncertainty components are presented, regarding the actual measurements performed with RS No.3 [32]. For this instrument, the uncertainty propagation varies from one measurement point to another. As can be seen from Table 4, the accuracy-related component is by far the most dominant influencing factor in the measurement point of 100 MΩ. It equals approximately 10 times the value of the calibration uncertainty, which is the second largest component in the overall budget. As the accuracy-related uncertainty is mathematically evaluated by assuming a uniform distribution, it is expected that the equivalent distribution will not tend toward Gaussian, if the Monte Carlo distribution propagation method [25] is implemented. In the measurement point of 500 MΩ, u_{acc} and u_{cal} are almost equal, while the statistical variations of the real-time measurements contribute to an order of magnitude smaller value. In the last measurement point, the accuracy-related component is almost double the value of the calibration uncertainty. The finite resolution as an influencing factor may be neglected in all three measurement points.

The standard combined uncertainty, if the GUM-based [23] approach is adopted, is calculated as follows:

$$u_{\mathrm{C,par}} = \sqrt{u_{\mathrm{A}}^2 + u_{\mathrm{res}}^2 + u_{\mathrm{acc}}^2 + u_{\mathrm{cal}}^2} \,, \tag{20}$$

assuming the input influencing factors as mutually uncorrelated [38], according to the remarks presented in subsections 3.1 and 3.2. Even though both RS No.2 and RS No.3 are traceable to BIPM [12], via the NMI of the Republic of Serbia [33], no

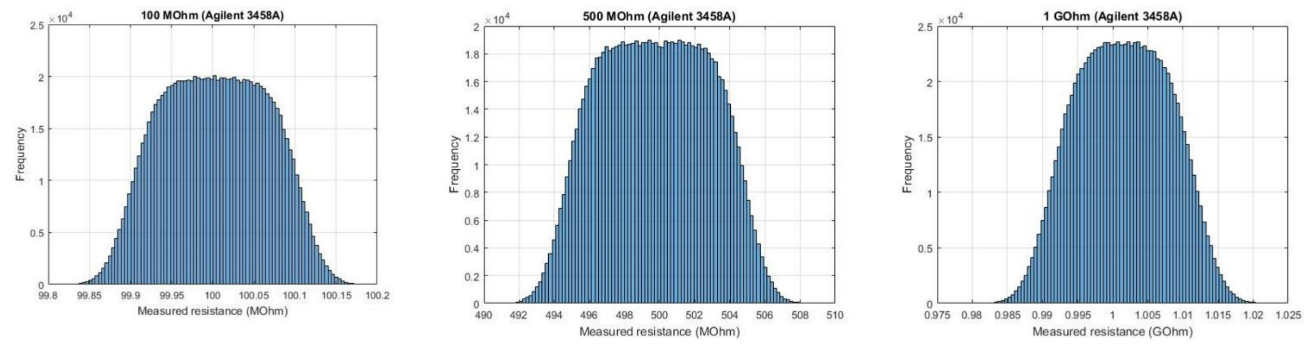


Figure 3. Distribution of values, attributed to the resistance measured with RS No.2 according to the Monte Carlo simulation.

Table 4. Standard uncertainties attributed to the measured resistance with RS No.3.

Uncertainty component	Measurement point		
	100 MΩ	500 MΩ	1 GΩ
u_A	0.0026 MΩ	0.12 MΩ	0.00015 GΩ
u_{res}	0.000029 MΩ	0.00029 MΩ	0.00000029 GΩ
u_{acc}	0.47 MΩ	5.8 MΩ	0.012 GΩ
u_{cal}	0.068 MΩ	3.4 MΩ	0.0068 GΩ

correlation between the accuracy of the 6 ½ digit multimeter [32] and its traceability via the 8 ½ digit multimeter [31] is supposed to be considered. This is due to the fact that u_{acc} in (20) emerges solely from the datasheet [32] of RS No.3, i.e. it is independent of all the influencing factors attributed to the measurements performed with RS No.2, which comprise the u_{cal} value in both equations (19) and (20). The expanded combined uncertainty is calculated according to equation (17), by adopting normal distribution and a 95 % coverage interval.

The results of the Monte Carlo simulation [25], realized with MATLAB®, are presented in Figure 4. The resultant distributions are obtained by performing $N = 1000000$ (one million) simulations in every measurement point, in order for the coverage interval of 95 % to be determined accordingly.

From the presented distributions, the assumptions made earlier, when the dominant influencing factors were analysed, may be validated. The distribution of the meter's recordings in the measurement point of 100 MΩ does not comply with Gaussian distribution. It tends more toward trapezoidal, or even uniform, due to the dominant accuracy-related influence. The equivalent distributions in the other two measurement points do follow the pattern of a normal distribution, as u_{cal} has a more significant contribution to the overall budget.

3.4. Case study – RS No.4

The last participant included in the intra-laboratory comparison is the multifunction calibrator [34] with its high resistance/low current measuring adapter [35], RS No.4. The high-resistance measurements with RS No. 4 are affected by the same influencing factors as are the recordings performed with RS No.3 [32]. Therefore, equation (18) is valid for the analytical representation of the measurement model. Once again, $n = 10$ recordings, per measurement point, are performed. The resolution uncertainty, u_{res}, is calculated according to equation (4). The accuracy related uncertainty, u_{acc}, is calculated according to equation (13), in which the parameters $A_{meas,\%}$ and $B_{ran,\%}$, obtained from the specifications [34]–[35] of the calibrator's measurement setup, are included. However, a slight modification of equation (13) is supposed to be made, considering that the accuracy specifications in [34]–[35] are given by assuming normal distribution and a coverage interval of approximately 95 %. This means that in equation (13), a division by factor of 2, instead by factor of √3, is supposed to be adopted [37]. The uncertainty related to the traceability of the measurement configuration is evaluated according to equation (19)), as the calibration uncertainty, $U_{CAL,\%}$, is available in relative form.

Table 5. Standard uncertainties attributed to the measured resistance with RS No.4.

Test voltage	Uncertainty component	Measurement point		
		100 MΩ	500 MΩ	1 GΩ
100 V	u_A	0.0018 MΩ	0.045 MΩ	0.000054 GΩ
	u_{res}	0.0029 MΩ	0.000029 MΩ	0.000029 GΩ
	u_{acc}	0.45 MΩ	3.3 MΩ	0.0045 GΩ
	u_{cal}	0.005 MΩ	0.025 MΩ	0.00005 GΩ
500 V	u_A	0.00047 MΩ	0.011 MΩ	0.000028 GΩ
	u_{res}	0.00029 MΩ	0.0029 MΩ	0.000029 GΩ
	u_{acc}	1.3 MΩ	2.3 MΩ	0.013 GΩ
	u_{cal}	0.005 MΩ	0.025 MΩ	0.00005 GΩ
1000 V	u_A	0.0015 MΩ	0.0053 MΩ	0.000031 GΩ
	u_{res}	0.0029 MΩ	0.00029 MΩ	0.000029 GΩ
	u_{acc}	0.45 MΩ	3.3 MΩ	0.0045 GΩ
	u_{cal}	0.005 MΩ	0.025 MΩ	0.00005 GΩ

The single uncertainty components' magnitudes are presented in Table 5, for the three measurement points and the three test voltages applied. In all the measurement points, no matter the magnitude of the applied test voltage, the dominant influencing factor is the accuracy of RS No.4 [34]–[35]. This uncertainty component is several orders of magnitude larger than the other influencing factors. The other three standard uncertainties have almost an equal order of magnitude value, except for the 500 MΩ measurement point, recorded with test voltage of 100 V, when u_{res} is completely negligible. Even though u_{acc} dominates the overall budget, it does not have a constant value for the same measured resistance if different test voltages are applied. Namely, with the alteration of the test voltage, measurement ranges change as well, and the component of u_{acc}, related to the measurement range, contributes differently to the uncertainty budget. As it is evaluated by assuming Gaussian distribution, the

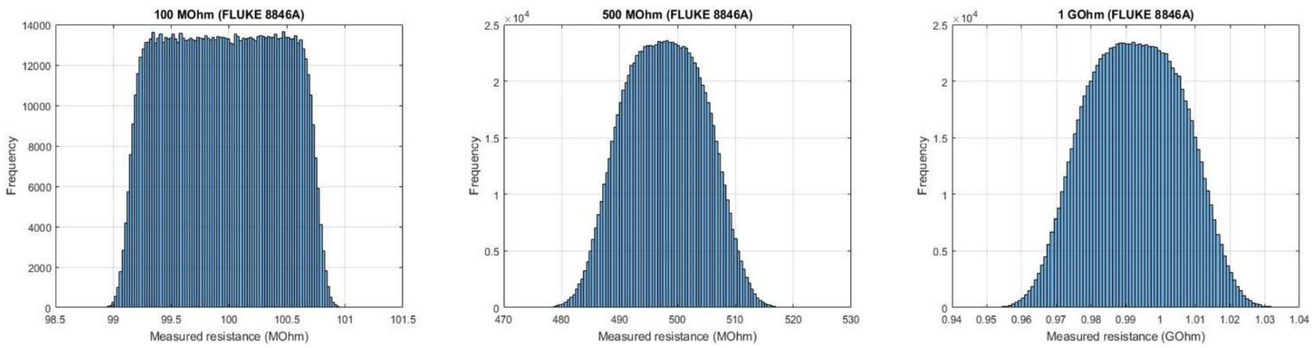


Figure 4. Distribution of values, attributed to the resistance measured with RS No.3, according to the Monte Carlo simulation.

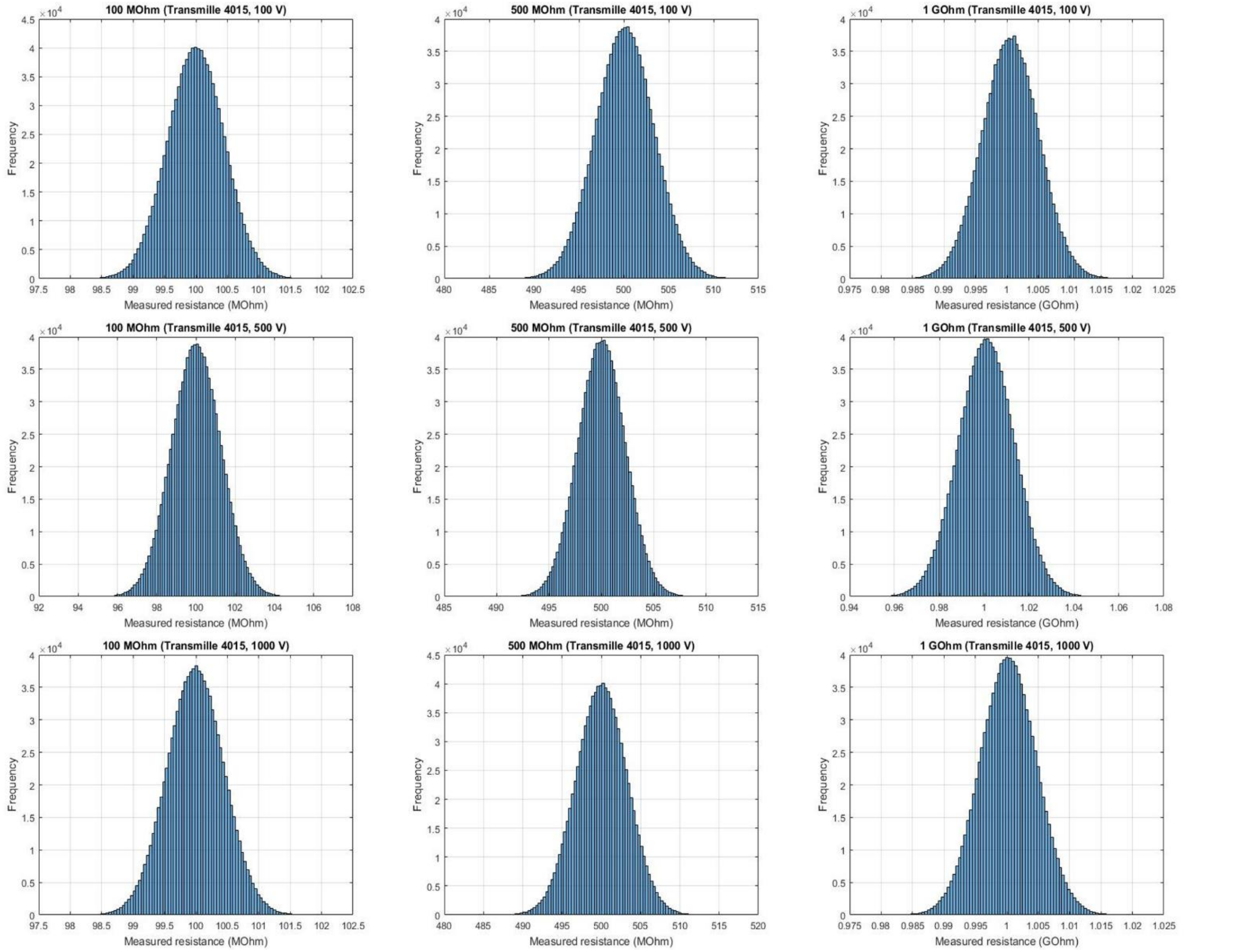


Figure 5. Distribution of values, attributed to the resistance measured with RS No.4, according to the Monte Carlo simulation.

resultant distribution, if all the influencing factors are regarded, is expected to be Gaussian as well, [23], [25], [37]. This implies that the measurement uncertainty, which corresponds to a coverage interval of 95 %, is expected to be more or less the same, no matter the adopted principle of evaluation.

If the GUM [23] approach is implemented, the standard combined uncertainty will be calculated according to equation (20). For the evaluation of the expanded uncertainty, a multiplication by the coverage factor $k = 1.96$ is supposed to be performed, as presented in equation (17). The Monte Carlo Simulation [25], conducted by using MATLAB® and $N = 1000000$ (one million) trials, provides justification for the point. The results, in the form of equivalent distributions, attributed to the resistance measured with RS No.4 [34]–[35], are illustrated in Figure 5. As can be seen from the figure, in every measurement point and for each value of the test voltage, the equivalent distribution is normal, with a different level of scattering of the single measurement results around the mean value.

4. INTRA-LABORATORY COMPARISON RESULTS AND DISCUSSION

In the following section, the results of the intra-laboratory comparison will be presented. The intra-laboratory comparison is performed by means of resistance recording, with RS No.2 [31], RS No.3 [32], and RS No.4 [34]–[35]. The resistance is set on the RS No.1 [29]. Two alternative approaches will be analysed, according to the background discussed in Section 3. Namely, the measurement uncertainty, corresponding to a 95 % coverage interval, will be calculated by using both the GUM-based concept [23] and the Monte Carlo simulation [25] results, obtained in the form of resultant distributions.

In Table 6, the measurement results, alongside with the expanded combined uncertainties evaluated according to GUM [23] are presented. In the measurement point of 100 MΩ, the best overall result, in terms of both the mean recorded value and the lowest combined uncertainty, is obtained with RS No.2 [31]. The expanded uncertainty attributed to this measurement is equal to the uncertainty prescribed to the sourced resistance. The result confirms that both RS No.1 [29] and RS No.2 [31] act as the laboratory's primary RSs in the concrete measurement point. An even better result, in terms of the mean recorded value, is obtained by using the multifunction calibrator measurement setup, RS No.4 [34]–[35]. On the other hand, the expanded uncertainty is almost an order of magnitude larger than the one attributed to the decade resistor's performance. For the other two measurement points, 500 MΩ and 1 GΩ, RS No.4 provides better results than the 8 ½ digit multimeter [31], from the perspective of lower deviation between the measured value and

Table 6. Measurement results and GUM-evaluated uncertainties of the four RSs participating in the intra-laboratory comparison.

Reference standard	Measurement point	R_{ref}/R_{par}	$U_{95\,\%,ref}/U_{95\,\%,par}$
RS No.1	100 MΩ	100 MΩ	± 0.12 MΩ
	500 MΩ	500 MΩ	± 0.59 MΩ
	1 GΩ	1 GΩ	± 0.0028 GΩ
RS No.2	100 MΩ	100.0034 MΩ	± 0.12 MΩ
	500 MΩ	499.7681 MΩ	± 6.1 MΩ
	1 GΩ	1.0015016 GΩ	± 0.013 GΩ
RS No.3	100 MΩ	99.949 MΩ	± 0.93 MΩ
	500 MΩ	497.762 MΩ	± 13.2 MΩ
	1 GΩ	0.992633 GΩ	± 0.026 GΩ
RS No.4 (100 V)	100 MΩ	100.001 MΩ	± 0.88 MΩ
	500 MΩ	500.1276 MΩ	± 6.4 MΩ
	1 GΩ	1.0006 GΩ	± 0.0088 GΩ
RS No.4 (500 V)	100 MΩ	100.012 MΩ	± 2.5 MΩ
	500 MΩ	500.04 MΩ	± 4.4 MΩ
	1 GΩ	1.001 GΩ	± 0.025 GΩ
RS No.4 (1000 V)	100 MΩ	99.99 MΩ	± 0.88 MΩ
	500 MΩ	500.043 MΩ	± 6.4 MΩ
	1 GΩ	1.0003 GΩ	± 0.0088 GΩ

the sourced reference resistance. The presented uncertainties of the measurements with the two instruments are, however, comparable and equal almost ten times the value of the expanded combined uncertainty, attributed to the sourced resistance. As mentioned earlier, the expanded uncertainty of the measured resistance with the calibrator's measurement setup is strongly correlated to the measurement ranges, which are dependent on the selected test voltage. For test voltages of 100 V and 1000 V, the same measurement ranges are used, hence, the corresponding uncertainties are equal. The measurement ranges, when testing with 500 V signals, are different and, consequently, smaller or larger uncertainties are obtained. The RS No.3 [32] provides measurement results that deviate the most in relation to the sourced resistance value. Additionally, the largest expanded uncertainties are attributed to these measurements. The variations in both the mean recorded value and the expanded combined uncertainty are not that significant when 100 MΩ are being measured. For the other two measurement points, significant differences are detected, with respect to the other RSs. The expanded measurement uncertainty, when resistance of 500 MΩ is being measured with the 6 ½ digit multimeter [32], is more than double the value, attributed to the result obtained with the calibrator [34]–[35]. When 1 GΩ resistance is being recorded, the uncertainty of RS No.3 [32] is comparable to the one evaluated for RS No.4 [34]–[35] measurement with 500 V test voltage. If the other two test voltages of RS No.4 are applied, the multimeter's uncertainty is, once again, more than twice the value attributed to the calibrator's performance.

Table 7. Simulated results and Monte Carlo-evaluated uncertainties of the four RSs participating in the intra-laboratory comparison.

Reference standard	Measurement point	R_{ref}/R_{par}	$U_{95\,\%,ref}/U_{95\,\%,par}$
RS No.1	100 MΩ	99.99994 MΩ	± 0.11 MΩ
	500 MΩ	500.0003 MΩ	± 0.53 MΩ
	1 GΩ	0.9999988 GΩ	± 0.0027 GΩ
RS No.2	100 MΩ	100.00344 MΩ	± 0.11 MΩ
	500 MΩ	499.7631 MΩ	± 5.5 MΩ
	1 GΩ	1.0014989 GΩ	± 0.012 GΩ
RS No.3	100 MΩ	99.9496 MΩ	± 0.79 MΩ
	500 MΩ	497.752 MΩ	± 12.4 MΩ
	1 GΩ	0.992615 GΩ	± 0.025 GΩ
RS No.4 (100 V)	100 MΩ	100.002 MΩ	± 0.88 MΩ
	500 MΩ	500.1174 MΩ	± 6.4 MΩ
	1 GΩ	1.0006 GΩ	± 0.0088 GΩ
RS No.4 (500 V)	100 MΩ	100.014 MΩ	± 2.5 MΩ
	500 MΩ	500.04 MΩ	± 4.4 MΩ
	1 GΩ	1.001 GΩ	± 0.025 GΩ
RS No.4 (1000 V)	100 MΩ	99.99 MΩ	± 0.88 MΩ
	500 MΩ	500.039 MΩ	± 6.4 MΩ
	1 GΩ	1.0003 GΩ	± 0.0089 GΩ

In Table 7, the simulated mean resistances, as well as the measurement uncertainties, evaluated for a coverage interval of 95 %, are presented, following the implementation of the Monte Carlo distribution propagation method [25]. From the results of the simulation, presented in Section 3, a symmetry of all the distributions around the mean value may be detected. Considering this symmetry, the uncertainty, corresponding to a coverage interval of 95 %, is calculated from the distribution specific points, $R_{2.5\,\%}$ and $R_{97.5\,\%}$, which refer to a cumulative probability of 2.5 % and 97.5 %, respectively [25]:

$$U_{95\,\%} = \frac{R_{97.5\,\%} - R_{2.5\,\%}}{2}. \quad (21)$$

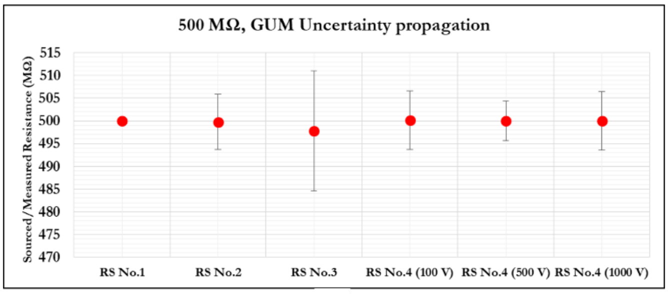
As may be seen from Table 7, the results from the Monte Carlo simulation [25] are similar to the mean measurement results, as the statistical scatterings of the instruments' readings are introduced in the distribution propagation modelling. Small deviations, regarding the presented uncertainties, may be detected, especially when non-normal distributions are obtained. That is the case when the uncertainty of the sourced resistance is regarded, as well as with the uncertainties prescribed to the measurements performed with both digital multimeters [31]–[32]. In both cases, the distribution propagation method [25] provides smaller total uncertainties. This leads to a conclusion that the uncertainty propagation, according to GUM [23], may be regarded as a safe side approach. When the performance of the calibrator's setup [34]–[35] is analysed, the presented uncertainties are equal, no matter which concept for evaluation

Table 8. Results from the intra-laboratory comparison in the form of E_n criterion values.

Reference standard	Measurement point	GUM		Monte Carlo	
		E_n	Status	E_n	Status
RS No.2	100 MΩ	0.02	Pass	0.023	Pass
	500 MΩ	-0.038	Pass	-0.043	Pass
	1 GΩ	0.11	Pass	0.12	Pass
RS No.3	100 MΩ	-0.055	Pass	-0.063	Pass
	500 MΩ	-0.17	Pass	-0.18	Pass
	1 GΩ	-0.28	Pass	-0.3	Pass
RS No.4 (100 V)	100 MΩ	0.0011	Pass	0.0023	Pass
	500 MΩ	0.02	Pass	0.018	Pass
	1 GΩ	0.065	Pass	0.065	Pass
RS No.4 (500 V)	100 MΩ	0.005	Pass	0.0057	Pass
	500 MΩ	0.0079	Pass	0.0089	Pass
	1 GΩ	0.04	Pass	0.041	Pass
RS No.4 (1000 V)	100 MΩ	-0.0079	Pass	-0.011	Pass
	500 MΩ	0.0066	Pass	0.0061	Pass
	1 GΩ	0.028	Pass	0.033	Pass

is adopted. However, the outcome of the intra-laboratory comparison may still be different, due to the simulated output and uncertainty magnitude of the reference resistor [29].

The results of the intra-laboratory comparison, following the two different approaches for uncertainty evaluation, are shown in Table 8. According to the presented E_n criterion values, all the participating instruments successfully completed the intra-laboratory comparison. The E_n criterion value is well within the -1 and +1 limits in every measurement point. So, the conducted procedure provides an extended level of confidence and quality assurance of the data presented by the laboratory to its clients.



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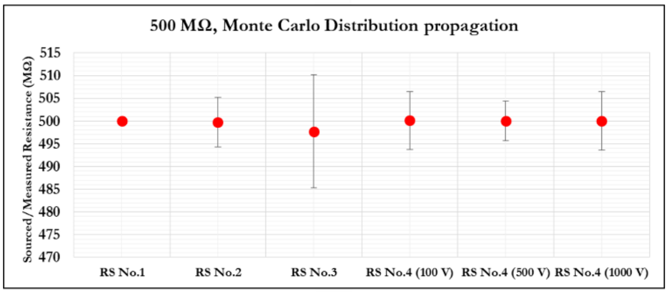

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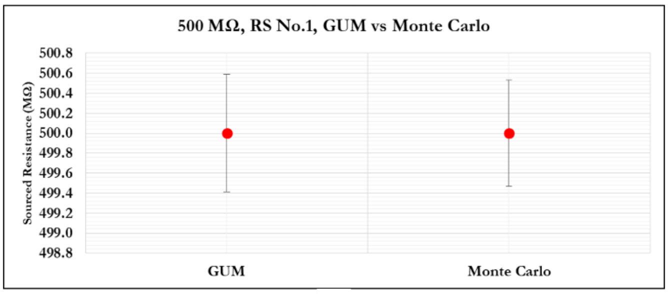

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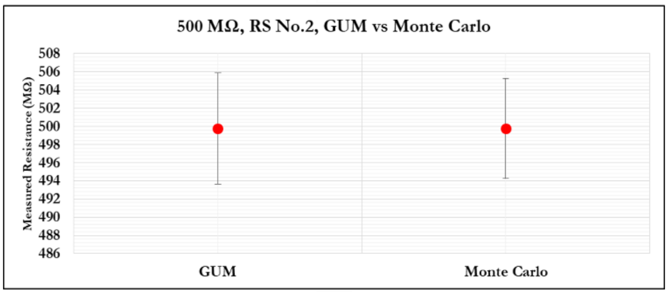

d)

Figure 6. Intra-laboratory comparison results in measurement point of 500 MΩ, regarding: a) all the participating RSs and GUM uncertainty propagation method b) all the participating RSs and Monte Carlo distribution propagation method c) RS No.1 set output and both methods for uncertainty evaluation, d) RS No.2 mean measured resistance and both methods for uncertainty evaluation.

The results provide assurance of the performance consistency of the reference standards, as well as proven reliability regarding their implementation for the maintenance of an unbroken traceability chain in the domain of high resistance in LEM.

An example illustration of the intra-laboratory comparison results is given in Figure 6, for the measurement point of 500 MΩ. These results are selected for graphical presentation, as according to Table 6 and Table 7 the most significant differences between the GUM [23] and the Monte Carlo [25] uncertainties are detected in the concrete measurement point. In Figure 6a) the set/measured values with the four RSs are presented, together with the measurement uncertainties, calculated according to the GUM [23] uncertainty propagation method. Figure 6b) shows the set and the measured mean resistances, together with their attributed uncertainties if the Monte Carlo distribution propagation method [25] is adopted. Figure 6c) and Figure 6d), illustrate a comparison between the results when traditional GUM [23] and Monte Carlo [25] evaluation methods are implemented, regarding the sourced resistance with RS No.1 [29] and the mean measured resistance with RS No.2 [31], respectively. From Figure 6 it may be concluded that the measurement uncertainties cover the differences between the standards' set output and mean recordings, thereby resulting in a favourable outcome of the intra-laboratory comparison.

5. SENSITIVITY ANALYSIS

In the final part of the paper, the possibility of different intra-laboratory comparison outcomes will be analysed from a perspective of underestimation or variation of the different input influencing factors. Four scenarios will be presented regarding alterations in the uncertainty budgets of the single reference standards. In each scenario, only one uncertainty component will be addressed. The change in the uncertainty budget, attributed to the reproduced resistance, may potentially result in a different intra-laboratory comparison outcome, regarding all the participating instruments, according to equation (1). The underestimation or alteration of a single standard uncertainty, attributed to the participating instrument's recordings, will have a potential impact only on that specific participant. The results, in the form of E_n criterion values, are given in Table 9.

For the realization of this part of the analysis, several constraints are included, regarding the feasible options of underestimation or alteration of the individual influencing factors. In Section 3, it was highlighted that the accuracy-related uncertainty is dominant in every measurement point, for every RS that participate in the intra-laboratory comparison. The accuracy-related uncertainty cannot be just removed from the budget, as it contains the primary information about the standard's performance and its position in the traceability chain. This component is calculated according to a unique mathematical expression, as presented in the manufacturers' datasheets, so the possibilities for eventual reductions are very limited. On the other side of the spectrum, the underestimation of the smallest influencing factors (e.g., resolution) will not be covered in the further evaluation either, as it is expected that their neglect will have almost no impact on the intra-laboratory comparison outcome.

The case scenario 1 (CS1) resembles a situation where the long-term stability of the reference resistor [29] is not considered in the overall budget. This is a realistic option, as mentioned earlier, the routine periodic check-ups showed stable resistance output in a several-year period. The measurement point of

Table 9. E_n criterion values, obtained by underestimation or alteration of single uncertainty components.

Scenario	Meas. Point	Reference standard	GUM		Monte Carlo	
			E_n	Status	E_n	Status
CS1	500 MΩ	RS No.2	-0.038	Pass	-0.043	Pass
		RS No.3	-0.17	Pass	-0.18	Pass
		RS No.4 (100 V)	0.02	Pass	0.018	Pass
		RS No.4 (500 V)	0.0079	Pass	0.009	Pass
		RS No.4 (1000 V)	0.0066	Pass	0.0061	Pass
CS2	1 GΩ	RS No.2	0.12	Pass	0.12	Pass
		RS No.3	-0.28	Pass	-0.3	Pass
		RS No.4 (100 V)	0.067	Pass	0.068	Pass
		RS No.4 (500 V)	0.04	Pass	0.041	Pass
		RS No.4 (1000 V)	0.029	Pass	0.034	Pass
CS3	1 GΩ	RS No.2	-0.24	Pass	-0.26	Pass
CS4	1 GΩ	RS No.2	1.5	Fail	1.6	Fail

500 MΩ is chosen for depicting the practical implications. The underestimation of u_{st} will result in a slight total uncertainty decrease. The GUM-based [23] uncertainty, attributed to the resistor's set-up value, then equals ± 0.57 MΩ. The Monte Carlo-evaluated [25] uncertainty equals ± 0.48 MΩ. As can be seen from Table 9, no practical implications are detected on the intra-laboratory comparison results. The E_{n} criterion derives the same values as the ones obtained in the primary study.

The second case scenario (CS2) considers the measurement of the 1 GΩ resistance on the smallest resistor decade [29]. The resistance values of 1 GΩ, 10 GΩ, and 100 GΩ on RS No.1 can be sourced from two decades. The decade with a maximal value of up to 10 GΩ is the primarily selected source of 1 GΩ value. If the lower decade with a maximal value of 1 GΩ is chosen instead, lower accuracy and stability boundaries are given in the resistor's specifications [29]. According to equations (6) and (7), the selection of this setting will affect the standard uncertainties, as well. The GUM-evaluated [23] expanded uncertainty then equals ± 0.0012 GΩ, while the uncertainty obtained according to the Monte Carlo simulation [25] equals ± 0.001 GΩ. Even though more than double reduction of the 95 % coverage interval uncertainty is obtained, it does not significantly affect the calculated E_n criterion value, as may be seen form Table 9.

The third and the fourth case scenarios (CS3 and CS4) are related to the underestimation of the statistical variations in the recordings performed with the 8 ½ digit multimeter [31]. The measurement point of 1 GΩ is selected, as according to Table 3, the Type A uncertainty has a value that is comparable to the magnitude of the accuracy-related component. For performing this part of the sensitivity analysis, it is assumed that only one reading, per measurement point, is recorded. The two most extreme values are then selected. In CS3, the minimal reading of the initial 10 measurements is adopted as the measured value, R_{M} = 0.9969802 GΩ. The uncertainties, evaluated according to the GUM [23] and the Monte Carlo [25] concepts, equal ± 0.012 GΩ and ± 0.011 GΩ, respectively. In CS4, only the maximal reading of the initial 10 measurements is regarded, i.e. R_{M} = 1.0189182 GΩ. The 95 % coverage interval uncertainties will equal ± 0.013 GΩ if the GUM [23] approach is adopted, or ± 0.011 GΩ if the Monte Carlo [25] simulation is performed. The uncertainties, in both CS3 and CS4, do not differ significantly, in relation to the values obtained from the primary experiment. The calculated E_{n} criterion, on the other hand, alters significantly, as presented in Table 9. This is mostly due to the differences between these extreme single recordings and the mean value. In CS3, this difference results in the alteration of the sign of the E_n value, although it is still within the acceptance limits. On the other hand, when CS4 is regarded, the adoption of the maximal recording only results in the E_{n} criterion value beyond the ± 1 limits, i.e. the intra-comparison result is Fail.

6. CONCLUSIONS

In the paper, an intra-laboratory comparison for reference standard consistency assessment and calibration data quality assurance is presented. The proposed concept is experimentally verified in an accredited calibration laboratory, by introducing different types of high-accuracy class reference standards for high-resistance reproduction and measurement. For the numerical evaluation of the results, the E_{n} acceptance criterion, according to ISO/IEC 17043, is adopted. Two methodologies for measurement uncertainty evaluation are included: the traditional uncertainty propagation concept, by adopting the pathway presented in GUM, and the distribution propagation concept, based on the Monte Carlo simulation.

The measurement uncertainty evaluation stage showed that the 95 % coverage interval uncertainty does not have a unique value if different concepts for calculation are adopted, even if the same influencing factors are regarded. The distribution of measured/sourced values, obtained as a result of the Monte Carlo simulation, does not always follow the pattern of a normal distribution, as adopted according to the traditional GUM concept. This is especially relevant when the dominant influencing factors are evaluated by assuming some of the geometrical distributions, i.e. uniform in this particular case. In such a case, the measurement uncertainty, evaluated from the resultant distribution, for 95 % coverage interval, is smaller than the value obtained according to the GUM-based concept. The adoption of the GUM uncertainty propagation framework, which is the simpler of the two to implement, in such a scenario, may be regarded as a safe side approach.

From the perspective of the primary analysis, it may be concluded that all the participating reference standards successfully passed the introduced intra-laboratory comparison. The E_{n} criterion values are well within the acceptance limits, between -1 and 1, in every measurement point, a conclusion valid for both uncertainty evaluation concepts adopted. The results provide assurance of the consistency and the top-level performance of the introduced reference standards, which lead to an enhanced confidence in the laboratory's calibration results. As these reference standards are based on different measuring principles, the applied protocol provides additional quality assurance, regarding the consistent use of different methods for high-resistance unbroken traceability chain maintenance.

The primary experiment is supplemented with a sensitivity analysis, in which the E_{n} criterion variations, if some of the influencing factors are underestimated or altered, are regarded. From this study it may be concluded that if the random variations of the measurement quantity are not considered, i.e. only one recording, per measurement point, is performed, a non-favourable outcome of the intra-comparison may be expected.

Further research on the topic may include the implementation of the intra-laboratory comparison concept on reference standards for other electrical quantities, which are reproduced or measured with the equipment available at the laboratory's disposal. For the realization of the proposed protocols, both approaches for uncertainty evaluation may be adopted. The analysis may be conducted by regarding electrical quantities, which are covered within the laboratory's accreditation scope, or it may go even further, in the domain of electrical signals for which no documented traceability is available, such as non-sinusoidal, harmonically distorted voltages and currents.

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