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Loss of accuracy compensation in determining a vessel's location with a mixed distribution of position line measurement errors

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ABSTRACT
The objective of the investigation is to improve the accuracy of a ship's position, based on navigational measuring equipment. The errors introduced into this process by key types of onboard equipment, including the GPS receiver, marine radar, and sextant, are analyzed. The errors of the sensors from which the measurement results are transmitted to the radar (gyrocompass, log, echo, etc.) are also considered. A multi-stage sequence in calculating the total positioning error from various navigational sources is also taken into account. The method for cumulative error compensation using redundant lines of position (LOPs) is proposed. The required measure of redundancy is determined based on derived analytic dependencies. The results of numerical experiments and natural observations are presented, confirming the appropriateness of using the proposed method. The proposed method and its results demonstrate its potential to improve positioning accuracy and ensure safe navigation.

Section: RESEARCH PAPER

Keywords: maritime vessel; ship; navigational measurement equipment; accuracy of position; redundant lines of position (LOPs); compensation of error; mixed probability distribution laws; numerical experiment; natural experiment; safe navigation; measurement

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1. INTRODUCTION

Navigation is the key element of safety at sea. The International Maritime Organization (IMO) has adopted the concept of electronic navigation — a digital revolution that aimed to obtain a maximum coverage of maritime shipping by 2018 [1]. The present-day bridge is a high-technology center equipped with Radar/ARPA, Automatic Identification System (AIS), Electronic Chart Display and Information Systems (ECDIS), Global Positioning System (GPS), satellite communication, etc. [2]. Change-over to electronic navigation definitely simplifies and makes data transmission more efficient. However, it also requires uninterrupted and high-quality digital communication between vessels and land facilities throughout the voyage. If any equipment malfunctions, failures also occur in maritime technologies, such as navigational sensors and bridge computer systems, as a result. Reliance on electronic navigation aids is not enough to achieve full appraisal of the situation, which can cause lack of situational awareness.

In navigation, we can determine the position of a vessel by various methods, such as cross-bearing obtained from two or more different beacons, beam and four-point bearing navigation method, horizontal sextant angle method, vertical sextant angle method, celestial bodies, dead reckoning, using modern electronic systems like GPS and maritime radar [3], [4]. Navigational officers can continuously monitor the position of the vessel in real-time on ECDIS, radar, GPS displays. The Integrated Navigation System (INS) and the Global Navigation Satellite System (GNSS) are widely used for vehicle positioning. In the context of GNSS, absolute positioning is broadly used not only in ship navigation, but also in many other fields, such as automotive, aerospace, and other types of navigation [5]. The combined use of different equipment facilitates better monitoring of the navigation. However, obstacles in the way of wireless signals often lead to GNSS malfunctions, which reduces positioning reliability. Post-processing kinematic GNSS requires an assessment of positioning accuracy [6].

Many deep learning methods have been proposed in combination with other technologies, because of their potential to improve the accuracy of determining the position of maritime vessels [7]. Though, they ignore the limitations imposed by limited computing resources and memory in real-time navigation systems [8].

One of the advanced products of automation technologies used for electronic navigation is the computer navigation system ECDIS (Electronic Chart Display and Information System). It displays information from electronic navigational vector charts (Electronic Navigational Chart, ENC) or raster charts (Raster Navigational Chart, RNC) and integrates it with Global Positioning System (GPS), Automatic Identification System (AIS), Radar data, Log, Echo-sounder, and other navigational equipment [9]. This integrated way results in the accumulation of errors from each measurement system and equipment type. Therefore, the development of a stochastic model, determining measurement error behavior, now requires further research and is part of this investigation. The aim of this article is to develop a method for the compensation of loss of accuracy in determining a vessel's position using excessive measurements when calculating the coordinates by the least squares' method.

In the rest of the article, a description of the solution to the problem under consideration is presented in the following order:

- Section 2 provides an overview of the technology to locate a vessel using Lines of Position (LOP).
- Section 3 describes the main types and models of navigational equipment used on ships for measurements, analyzing the level of errors of each type.
- Section 4 is dedicated to deriving analytical expressions, to ensure the necessary observation accuracy using redundant LOPs. A numerical experiment was conducted and described.
- Section 5 presents the results of natural experiments to determine the basic navigational parameters of a ship. Different durations and numbers of measurements were planned in order to identify the influence of the time factor on the measurement error distribution laws.
- Finally, Section 6 presents the conclusion of the article, which includes the main results of the analytical and natural experiments, the conclusions, and a description of the capabilities of the method for compensating for the error in determining the vessel's position proposed in the article.

2. STATE OF THE ART

Line of position (LOP) measurement is a navigational term that has been used in marine practice for many decades [10]. LOP is a line established by observation or measurement on which a vessel can be expected to be located. A line of position is constructed based on the results of measurements, such as bearings, distances, angles, and other parameters. The determination of the vessel's position occurs by intersecting several LOPs obtained from different measurements.

2.1. Methods for obtaining LOP

The methods can be classified as follows:

1) Bearing: Measuring the compass or true bearing to a reference point and constructing an LOP as a straight line passing through the landmark at the bearing angle;
2) Distance: Measuring the distance to a reference point (e.g., using radar or on vertical angle) and constructing an LOP in the form of a circle, with the center at the landmark and a radius equal to the measured distance;
3) Combined methods: Using several methods simultaneously to obtain a more accurate determination of the ship's position, for example, bearing and distance to one reference point, or bearings to several reference points.

2.2. Determining the location and course of the vessel

The intersection points of several position lines (for example, two bearings and a distance) indicate the vessel's location.

Vessels must choose their course depending on the position of surrounding vessels (maintaining a safe speed) and taking into account the parameters of vessel movement in general, which are determined using navigational equipment. For example, bearing objects surrounding a maritime vessel (air, surface, and coastal), their sizes and movement characteristics, and the distance to them are measured using a marine radar.

When determining different characteristics of a vessel's movement, it is necessary to take into account and, if possible, compensate for the measurement errors introduced into the parameters of vessel movement by each type of device.

3. NAVIGATION MEASURING EQUIPMENT

It should be noted that the functionality and measurement accuracy of all navigational measuring equipment installed onboard is fully compliant with IMO and IEC [1]. However, the accumulated error of measurement from various equipment should be carefully investigated and taken into account during passage planning and ensuring the safety of navigation.

3.1. GPS receiver

The GPS receiver is a precise navigational equipment, which receives signals from the Navstar Global Positioning System. Users can navigate continuously with an accuracy of 5 meters 2 drms (distance root mean square) or better. The technology called Differential GPS (DGPS) allows users to get maximum accuracy from the GPS system. DGPS requires the use of two GPS receivers. One receiver, called a reference station, is placed at the point of the shot the coordinates of which are exactly known. The purpose of the DGPS is to use a reference station to measure GPS signal errors and calculate corrections for their elimination. Then these corrections are transmitted in real time to navigators, where they are combined with the signals received by the satellites, thus improving their navigation or positioning.

The maximum accuracy of positioning with the DGPS receiver can be improved to 1 meter or better [11] in 95 % of cases [12]. This is possible provided that the transmission of differential corrections (primarily pseudo range and range-rate corrections) is accessible, accurate, and controlled. Therefore, DGPS is designed for precise commercial navigation in coastal waters and in narrow corridors (water, air, etc.).

The equipment composition and Cross-Track Error (XTE) indication of KGP-920 are shown in Figure 1. For the KGP-920 model, the error of position is 10 m 2 drms for GPS, 5 m 2 drms for DGPS, 8 m 2 drms for Satellite Based Augmentation System (SBAS) — when using technology to increase the accuracy and reability of GPS [14] and [15].

The 2 drms (twice distance root mean square) value represents the radial position uncertainty at a 95 % confidence level [16]. This means that if the estimated position uncertainty is 2 m, there is a 95 % probability that the vessel's true location lies within a 2 m radius of the displayed coordinates. This is a

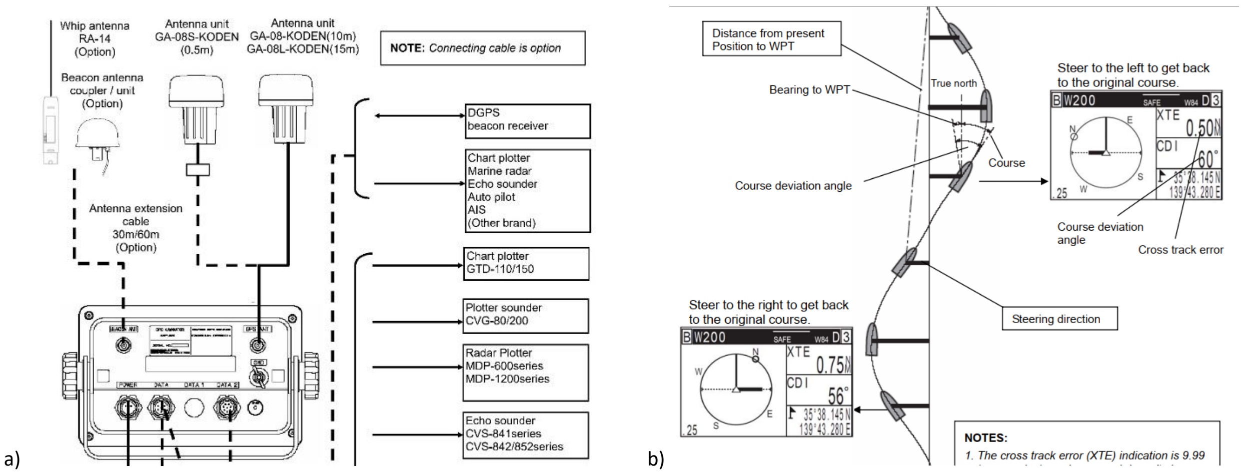

Figure 1. GPS navigator KGP-920: a) equipment composition; b) indicator of the Cross-Track Error (XTE) [13].

high value achieved under good reception conditions but can be significantly reduced by any interference [17].

It should be noted that the model KODEN KGP-920 is only 'SBAS ready' (that is, there is no specified module in the configuration, but it is possible to install it), so in order to increase accuracy, it is necessary to install additional blocks in the composition of the GPS (see Figure 1), or replace the GPS model with the KODEN KGP-925, for which the accuracy of converted positions can be off 1/4 mile [18]. But such an update is the most expensive, and impossible, during the voyage. Therefore, it is necessary to develop a mathematical apparatus for the compensation of the loss of accuracy in determining the position of the ship.

3.2. Sea radar

Marine radar is a navigation and detection system used on ships to detect and locate objects at sea by transmitting radio waves and analyzing the echoes. It helps with collision avoidance, as well as in navigation by showing land margins and other ships, and it is also used for applications such as oil spill detection and monitoring weather conditions. The range of the radar depends on the model, but it is not more than 40 nautical miles from the ship in issue [19].

For example, in the radar model VisionMaster FT [20], the radar workstation receives data from such sensors as gyrocompass, GPS, Log, Echo-sounder, AIS, etc. (Figure 2). Respectively, each of these sensors introduces a measurement uncertainty in the determination of bearing and the vessel`s course. According to the documentation, radar system bearing accuracy is better than ± 1°, [21]. However, it is only possible to judge whether such accuracy is sufficient based on the calculation of the minimum start time [22] and distance [23] required for a divergence maneuver to avoid a ship's collision.

In order to evaluate the accuracy with which the radar course and bearing from the gyrocompass (Gyro in Figure 2) are received, it is necessary to know what the design of the motors used for the rotating elements of the following gyrocompass systems is: incremental or synchronous. Depending on this, the accuracy of the rate will swing within 0.5° [21].

The collection of data about speed through the water is implemented via the pulse-log interface (Log in Figure 2). Accuracy of speed measurement in set mode is 5 % or 0.25 Kt (Knot), whichever is greater.

The radar system provides speed of target data based on echo signal (Ech in Figure 2).

The accuracy of the AIS system depends on the error of the GPS receiver installed on the ship (see Subsection 3.1). An accuracy of up to 10 m, with which AIS can transmit data, is sufficient for successful navigation, collision avoidance, and coordination of ship movements within the system [24].

3.3. Sextant

Nowadays, the sextant is rarely used, primarily as an auxiliary tool. However, in cases where other navigational aids, such as GPS, are unavailable or out of service, the sextant is an important tool [25].

The sextant is a navigational tool used to measure angles between celestial bodies and the horizon, as well as angles between terrestrial objects. These measurements allow the latitude and longitude of the ship to be determined.

Onboard, the accuracy of the sextant may be affected by the vibration of the vessel, caused by wind, engine operation, etc. Most of the errors are caused by horizontal acceleration, arising from the roll and pitch of the ship. If the roll were simple harmonic, errors could be partially eliminated by measuring with a sextant at the mid-point of the roll [26].

The marine sextant with digital display can work both day and night, in low light up to starlight. This sextant is less likely to make reading errors, simpler to operate, and more balanced than the traditional one, and it has an accuracy of 10 angular seconds instead of 30 angular seconds [27].

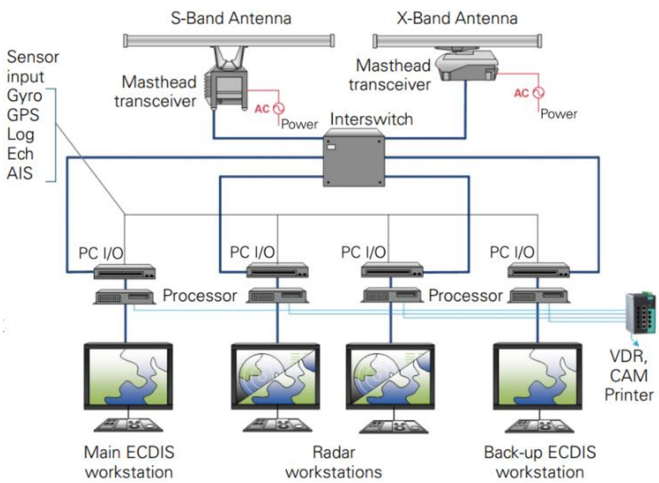

Figure 2. The block diagram of the radar VisionMaster FT [20].

Table 1. Dependence of the coefficient k on the number of LOP n.

n	2	3	4	5	6	7	8	9	10
k	2.00	1.333	1.00	0.8	0.666	0.571	0.5	0.444	0.40

3.4. Complex use of equipment

Considering the functions and errors of the above-mentioned navigational equipment, it can be concluded that a radar station measures the distance and bearing to an immobile reference point, and the vessel`s latitude and longitude are identified by means of a GPS receiver. Given the multi-stage sequence of determining the total error of vessel parameters obtained by means of various navigational equipment (GPS, radar, connected sensors, AIS, etc.), a method, proposed in [17], which effectively combines marine radar data with camera images can be considered promising. Thereby the accuracy and reliability of object detection can be improved. However, the appropriateness of this method is disputed in difficult navigable conditions (fog, poor visibility, unstable image from cameras when rocking, etc.). Thus, analytical methods for reducing and/or compensating the errors of navigational equipment in determining the basic parameters of the movement of a ship require further research.

4. MATERIALS AND METHODS

4.1. Determination of the number of LOPs for the observation of the ship's location

To ensure the required observation accuracy, redundant LOPs are used, and the observed coordinates are calculated using the least squares method. To determine the variance of navigation measurement errors, a sample of their experimental values is formed by natural observations.

The navigation measurement errors in the sample have a mixed distribution type as a set of normally distributed errors with different variances, which depend on the measurement conditions.

When determining the vessel's position at a specific moment, the measurement conditions are considered constant. Consequently, the navigation measurement errors theoretically follow a normal distribution with a constant, albeit unknown, variance. Since the exact variance for the current conditions is not available, we use the average variance D_m derived from a large experimental sample as an estimator. Note that this sample, collected over time under varying conditions, exhibits a mixed probability distribution. If the variance of the observed position of the vessel, D_z, is specified, then, knowing the dependence of D_z on the number of LOPs, n, and the variance D_m, it is possible to determine the required number of LOPs that provide the variance D_m of the observed vessel's position (1):

$$D_z = F(n, D_m). \tag{1}$$

However, since the actual LOP errors follow a mixed distribution, rather than the normal distribution assumed, this leads to a loss of accuracy. Consequently, the actual variance of the observed position D_m will be greater than the specified target variance D_z.

To compensate for this loss of observation accuracy, it is necessary to increase the number of LOPs to ensure the required value of the variance D_z. If the least squares method is used to calculate the observed coordinates in the presence of redundant LOPs, then the efficiency of the coordinates is < 1. Therefore, the actual variance $D_n = F(n,\, D_m)$ taking into account the efficiency of the coordinates provides, in accordance with equation (2):

$$D_n = F(n, D_m) = D_z\, e, \tag{2}$$

where e is the statistical efficiency of the estimation of the observed coordinates.

From this equation, the required number n of LOPs for effective observation of the vessel's location is determined.

4.2. Choice of a probability distribution law to compensate for the loss of accuracy

The Gaussian normal distribution is usually used to analyze statistical data on the accuracy of determining the location of a moving object. However, if the sample histogram has 'heavy tails', determination of the latitude and longitude error via the Gaussian function is incorrect and requires an alternative approach. Mixed probability distribution laws of two types are also used to describe random errors in navigation measurements: the first type is the generalized Cauchy distribution; the second type is the Pearson distribution of type VII [28].

Paper [29] presents the dependence of the variance of the vectoral error module on the number n of LOPs and the LOPs' parameters, which for a normal distribution has the following form:

$$D_n = \frac{D_m\, n}{(\sum_{i=1}^{n} \cos^2 \alpha_i)(\sum_{i=1}^{n} \sin^2 \alpha_i) - (\sum_{i=1}^{n} \sin \alpha_i \cos \alpha_i)^2}\,, \tag{3}$$

where a_i is the angle between the transferring and the meridian line in the selected celestial coordinate system.

Taking into account the given equation (3), equation (2) can be written in the following form:

$$k\, D_z = F(n, D_m), \tag{4}$$

where

$$k = \frac{n}{(\sum_{i=1}^{n} \cos^2 \alpha_i)(\sum_{i=1}^{n} \sin^2 \alpha_i) - (\sum_{i=1}^{n} \sin \alpha_i \cos \alpha_i)^2} \tag{5}$$

depends on the number of LOPs n.

By substituting the actual variance equation (2) into the relationship $D_n = k\, D_m$, we obtain the condition for the coefficient k:

$$k = \frac{D_z}{D_m} e\,. \tag{6}$$

Based on the obtained value k, the required number n of LOPs is determined (Table 1):

$$n = \sum_{i \neq j}^{n} \sin^2(\alpha_i - \alpha_j)\,. \tag{7}$$

Table 1 shows the dependence of the values of the coefficient k on the number n of LOPs. The values in Table 1 are calculated assuming an optimal (uniform) angular distribution of the LOPs. This configuration minimizes the geometric factor and implies that the angle Δa_i between adjacent position lines is constant $\Delta a_i = 360°/n$. For example, for $n = 3$, the angles are 0°, 120°, 240° relative to the reference.

For this case, the valid expression is $D_n = k\, D_m$.

As noted earlier, the errors of navigation measurements in the sample have a mixed type of distribution, which obeys a mixed law of the 1st or 2nd type [30].

4.3. A numerical experiment

Let us assume that the LOP error obeys a mixed distribution law of the 1st type, the density of which is expressed as follows [30]:

$$f(\xi) = \frac{A_m}{(\xi^2/2 + \lambda)^{m+1}}, \tag{8}$$

where A_m the normalizing factor is, and

$$A_m = \frac{2^{2\,m}(m!)^2}{\sqrt{2}\,\pi\,(2\,m)!}\lambda^{m+1/2},$$

where m is an essential parameter that takes integer values, and λ is the scale parameter.

In the work [31], it is shown that the efficiency e of the estimation of observed coordinates calculated by the least squares method, in the case when random errors of navigation measurements are distributed according to a mixed law of the 1st type, is < 1. The condition $e < 1$ indicates a loss of statistical efficiency. According to [29], this means that the variance of the coordinates obtained by the least squares method is larger than the minimum possible variance achievable by an optimal estimator for this distribution type.

The values of e depending on the essential parameter m are calculated by the formula:

$$e = 1 - \frac{3}{2\,m^2 + 3\,m + 3}. \tag{9}$$

The efficiency e, calculated using equation (9) as a function of the parameter m, is given in Table 2.

As follows from the given Table 2, with the growth of m, the mixed distribution tends to normal, and the efficiency e tends to 1. For the case of the distribution of errors of the LOP according to the mixed law of the 1st type, we will consider an example that demonstrates the application of the proposed method of compensation for the loss of accuracy in determining the vessel's location.

Let us assume that the given dispersion of the observed location of the vessel must be equal to the dispersion of the LOP, i.e., $D_z = D_m$. For this, as shown in Table 1, without taking into account the loss of accuracy, 4 LOPs are needed. Let us also assume that the errors of the LOP sample are distributed according to a mixed law of the 1st type with a significant parameter $m = 1$. As follows from Table 2, in this case, the efficiency $e = 0.5$, i.e., the accuracy deteriorates by half.

To compensate for the loss of accuracy, the variance of the location determination must be reduced by 2 times, which in Table 1 corresponds to the coefficient $k \approx 0.5$. This is achieved at 8 LOPs, as follows from the same Table 1.

Thus, to compensate for the loss of observation accuracy, it is necessary to use additional (redundant) LOPs. Thus, at $m = 1$, it is necessary to use 8 LOPs instead of 4 LOPs. Based on them, it is possible to calculate the coordinates using the least squares method. In this case, $D_n = D_m$. If $m = 3$, then $e = 0.893$ (according to Table 2). Analysis of Table 1 shows that in this case, 5 LOPs are needed instead of 4 LOPs.

Now let us consider the case when the LOP error obeys a mixed distribution law of the 2nd type, the density of which, as shown in [30], has the following form:

$$f(\xi) = \frac{A_m}{(\xi^2/2 + \lambda)^{m+\frac{3}{2}}}, \tag{10}$$

where A_m the normalizing factor is, and

$$A_m = \frac{1 \times 3 \times 5(2\,m+1)\,\lambda^{m+1}}{\sqrt{2} \times 2^{m+1}\,m!}.$$

As shown in [31], the efficiency e of the estimation of the observed coordinates, which are calculated by the least squares method for the case of distribution of the random error of navigation measurements according to the mixed law of the 2nd type, is also < 1. Depending on the essential parameter m, it is calculated using the expression:

$$e = 1 - \frac{3}{2\,m^2 + 5\,m + 3}. \tag{11}$$

Moreover, the efficiency e, calculated using equation (11) as a function of the parameter m, is given in Table 3.

As in the previous case, with an increase in the essential parameter m, the efficiency e tends to 1, and the mixed distribution approaches normal distribution (Gauss distribution).

Let us also consider an example for the case when the LOP errors are distributed according to the mixed law of the 2nd type, demonstrating the application of the proposed method for compensating for the loss of accuracy in determining the vessel's position. As in the previous case, let us assume that the given variance of the observed vessel's position should be equal to the LOP variance, i.e., $D_z = D_m$. For this, as follows from Table 1, 4 LOPs are required without taking into account the loss of accuracy. Let us assume that the LOP sample errors are distributed according to the mixed law of the 2nd type with an essential parameter $m = 1$. As follows from Table 3, the efficiency value $e = 0.7$, i.e., the accuracy deteriorates by 30 %. To compensate for the loss of accuracy, the variance of position determination should be reduced by the value e, which, in Table 1, corresponds to the coefficient $k = 0.7$. This is achieved with 6 LOPs, as follows from Table 3.

Thus, to compensate for the loss of observation accuracy, it is necessary to use additional (redundant) LOPs. Thus, with $m = 1$, instead of 4 LOPs, it is necessary to use 6 LOPs, according to which the coordinates are calculated using the least squares method; in this case, $D_n = D_m$.

5. RESULTS

To check the possibility of using mixed laws of both types to describe the distribution of errors in measuring navigation parameters in real operating conditions, natural observations were carried out. To form the initial samples of errors in measuring navigation parameters, a series of navigation parameter measurements was made, in the amount of more than 100 measurements in each sample. Measurements of navigation parameters were made on a moving ship (the status 'Underway'

Table 2. Efficiency of 1st type mixed distribution.

m	1	2	3	4	5	6
e	0.625	0.824	0.900	0.936	0.956	0.968

Table 3. Efficiency of 2nd type mixed distribution.

m	1	2	3	4	5
e	0.700	0.857	0.917	0.945	0.962

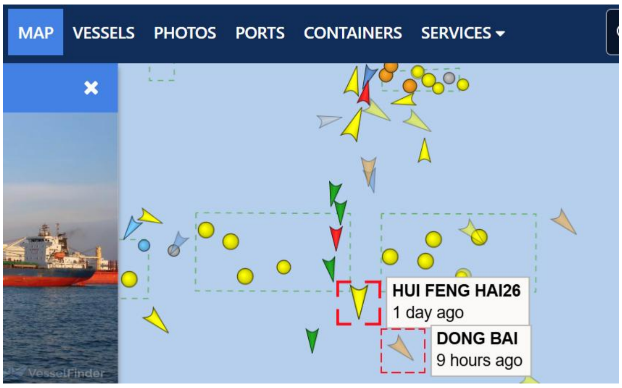

Figure 3. The current positions of HUI FENG HAI26 and DONG BAI ships at South China Sea by AIS [32].

was displayed on the NAV radar). The measurements of the distance and bearing to the target were made with the radar. The GPS receiver was recording the latitude and longitude of the ships.

The measurements were carried out for three ships (HUI FENG HAI26, FENGYUNHE, and DONG BAI), which were within the visibility of the ship from which the measurements were made. The current positions of the ships in the South China Sea by AIS [32] are shown in Figure 3.

These ships are displayed on the ECDIS of the ship making the measurements (Figure 4).

The time interval for all measurements, in accordance with the vessel traffic (see Figure 3), was distributed as follows: for the ship FENGYUNHE, it was the shortest (about 2 hours, every minute; a total of 120 measurements). For the ship DONG BAI, measurements were taken for 9 hours (every 5 minutes; a total of 108 measurements), and for the ship HUI FENG HAI26, during one day (every 5 minutes; a total of 244 measurements). Different durations and numbers of measurements were planned to identify the influence of the time factor on the distribution law of measurement error.

Figure 5 shows an example of one measurement result from a series of observations of the bulk carrier HUI FENG HAI26. At the time of the measurements, HUI FENG HAI26 was displayed on our ship's navigational instruments as Target541 (the target number is assigned by our ship's radar when a target appears on the radar).

Thus, in each port, for each of the three ships, 4 series of measurements of navigation parameters (bearing, distance, latitude, and longitude) were obtained, with a total of 12 series of measurements during the experiment. For each series of measurements, the average value of the navigation parameter was determined, and the values of the measurement errors were calculated, which together represented a sample of errors. The dispersion and standard deviation of the error of each sample were calculated using standard statistical formulas for sample variance and standard deviation (Table 4).

Subsequently, statistical hypotheses were tested for each sample. During the testing process, the degree of agreement between the statistical material of the sample and the selected laws of probability distribution of errors was determined. The normal distribution law (Gauss's law) and mixed laws of the first and second types were chosen as alternative laws.

The alternative laws were chosen. For each sample, the values of the Pearson goodness-of-fit criterion χ^2 with the specified error probability distribution laws were calculated. The distribution law chosen was the one for which the Pearson goodness-of-fit criterion χ^2 had a minimum value. The final results of the field observations are given in Table 5.

Figure 4. Displaying the ships-targets on the ECDIS of the measuring ship.

Based on the statistical analysis, the following conclusions can be made about the errors in determining the location. The errors in measuring the navigation parameters (bearing and distance) obtained over a limited time interval (2 hours) are subject to the normal probability distribution law (Gauss's law).

The errors in determining the latitude and longitude over the same time interval are distributed according to mixed laws of both types.

The errors in measuring the bearing and distance on a larger time interval (9 hours or more) are subject to mixed distribution laws. The degree of their difference from the normal law is proportional to the time intervals of measuring the series of values of the navigation parameter.

Furthermore, the calculated values of Pearson's chi-squared statistic (χ^2), presented in Table 5, demonstrate a high goodness-of-fit between the experimental data and the proposed mixed distribution models. The low values of χ^2 confirm that the selected mixed laws (Type 1 and Type 2) provide a statistically valid description of the vessel's positioning errors for both latitude and longitude, validating their application in the proposed accuracy compensation method.

6. CONCLUSIONS

In modern electronic navigation (E-navigation), various types of equipment are used to determine the location and course of a vessel. The accumulated measurement errors from various types of devices should be carefully studied and taken into account, and attempts should be made to compensate for plotting the vessel's course and ensuring the safety of navigation.

It is shown that the errors in measuring the navigation parameters of bearing and distance, with a sample of more than 100 measurements, obey the normal probability distribution law (Gauss's law) only if they are obtained over a limited time interval (several hours). The errors in determining latitude and longitude over the same time interval have a mixed type of distribution of type 1 or type 2.

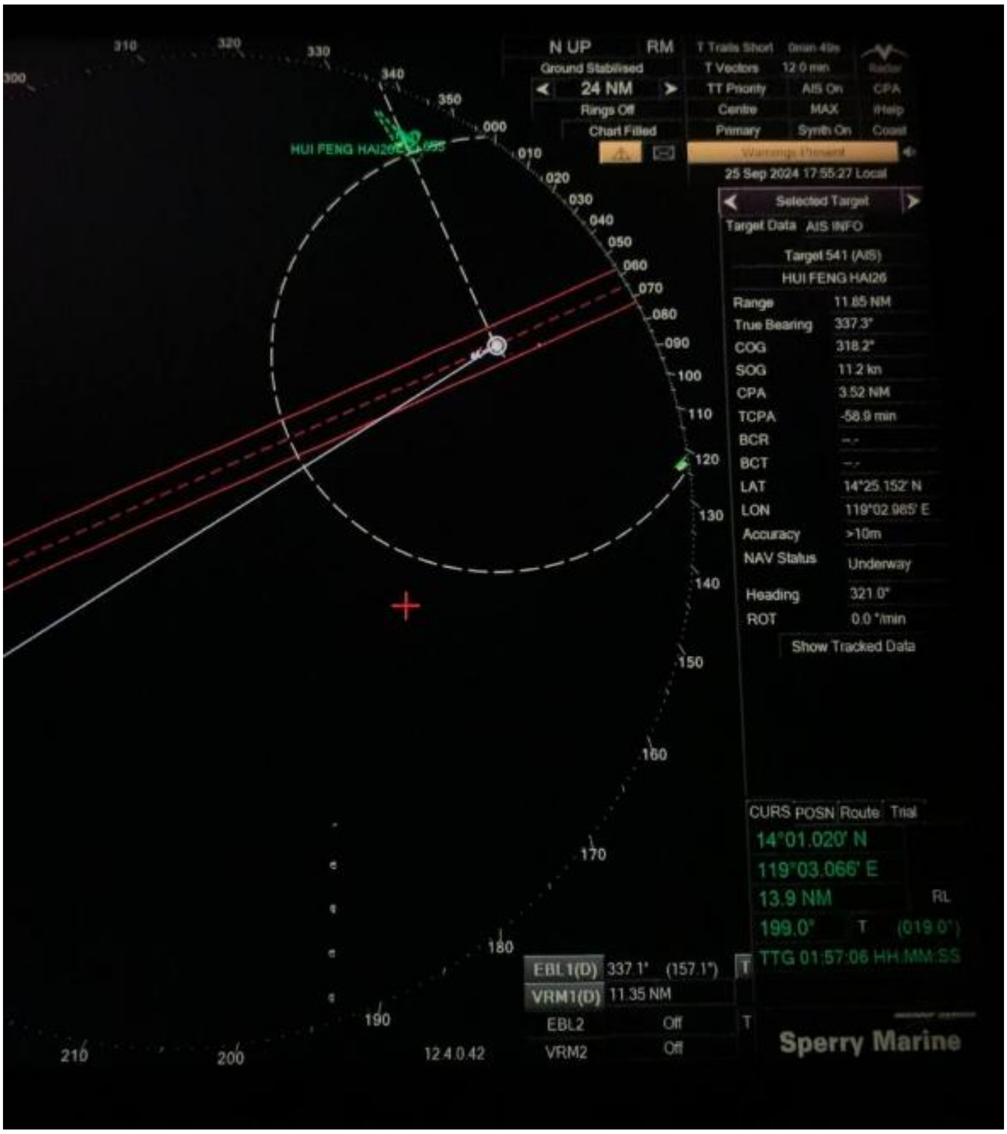

Figure 5. The navigation parameters of ship-target HUI FENG HAI26 on the radar of the measuring ship.

Table 4. Characteristics of the samples.

Ship-target	Sample No.	Navigation parameter	Number of measurements	Average value	Variance D	Standard deviations σ
HUI FENG HAI26	1	latitude	244	14°24'.874 N	31.42 m^2	5.61 m
HUI FENG HAI26	2	longitude	244	119°02'.985 E	40.24 m^2	6.34 m
HUI FENG HAI26	3	distance	244	11.51 NM	6.01 m^2	2.45 m
HUI FENG HAI26	4	bearing	244	336.8°	0.184 deg^2	0.429°= 25.7'
DONG BAI	5	latitude	108	13°89'.146 N	46.09 m^2	6.79 m
DONG BAI	6	longitude	108	119°03'.718 E	43.41 m^2	6.59 m
DONG BAI	7	distance	108	4.05 NM	18.20 m^2	4.27 m
DONG BAI	8	bearing	108	142.8°	0.1822 deg^2	0.427°= 25.6'
FENGYUNHE	9	latitude	120	14°1'.112 N	37.81 m^2	6.15 m
FENGYUNHE	10	longitude	120	119°03'.787 E	38.44 m^2	6.2 m
FENGYUNHE	11	distance	120	8.45 NM	5.16 m^2	2.27 m
FENGYUNHE	12	bearing	120	30.2°	0.214 deg^2	0.463°= 27.8'

A method is proposed for compensating for the loss of accuracy in determining a vessel's location using redundant measurements when calculating coordinates using the least squares method.

Analytical expressions and data for the implementation of the proposed method are given. As a result of the numerical experiment, it is shown that with the growth of the essential parameter, the efficiency of the estimation of the observed coordinates tends to 1, and the mixed distribution approaches the normal one (Gaussian law of normal distribution).

The results of the natural observations are presented, confirming the reliability of the proposed method.

Table 5. Final results of natural observations.

Sample No.	Navigation parameter	Number of sample members	Distribution law	χ^2	Observation period
1	bearing	244	2nd type at n = 3	0.091	1 day
2	distance	244	1st type at n = 4	0.091	1 day
3	latitude	244	1st type at n = 1	0.010	1 day
4	longitude	244	2nd type at n = 1	0.009	1 day
5	bearing	108	1st type at n = 2	0.011	9 hours
6	distance	108	2nd type at n = 2	0.010	9 hours
7	latitude	108	1st type at n = 6	0.011	9 hours
8	longitude	108	1st type at n = 1	0.013	9 hours
9	bearing	120	Gaussian distribution	0.007	2 hours
10	distance	120	Gaussian distribution	0.006	2 hours
11	latitude	120	1st type at n = 3	0.013	2 hours
12	longitude	120	2nd type at n = 1	0.015	2 hours

The study showed that errors in determining the bearing, course, and coordinates of a vessel during long-term observation do not occur randomly but are subject to mixed probability distribution laws of 1st and 2nd types. Thus, it is shown that the Gaussian distribution is not optimal for modeling errors in navigation positioning systems.

It is possible to compensate for the measurement errors, which are introduced into the parameters of vessel movement by each type of device, using combined measurements, and due to the calculated redundant number of LOPs, using the proposed method.

AUTHORS' CONTRIBUTION

Conceptualization, methodology, resources, supervision: Igor O. Burmaka;

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