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Estimation of catapult size and bolt dimensions by comparison of design formulae for early torsion-based catapults

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ABSTRACT
A novel method for the analysis of ancient catapults by comparing two known ancient design formulae for early torsion-based catapults is presented. The hypothesis is that both formulae, one for a bolt shooting catapult and one for a stone thrower, give the optimum design regarding performance and that both express the same optimal design. It is hypothesized that a developed mathematical equation of 3rd order can be used to determine the length and diameter of catapult bolts from bolt point data, for optimally designed catapults, as no such catapult bolt parts so far are known to have been discovered. The method leads to an estimation of the catapult size/dimensions via scaling factors known from ancient sources. Published data from catapult points, known from excavations/literature, were used as the base of the analysis and the development of the mathematical model.

Section: RESEARCH PAPER

Keywords: Catapult analysis; projectile length; torsion catapult

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1. INTRODUCTION AND BACKGROUND

The background of this article is to describe the development of a method to determine the dimensions of catapult bolt projectiles for early Hellenistic catapults, constructed according to the known ancient design formulae. The method further enables the possibility of determining the key design parameters of a torsion-based catapult via the dimensions of the bolt point alone (under certain constraints). A series of recent publications address the mathematical analysis of the performance, energy storage capability etc. of torsion-based catapults, but, to the best of our knowledge, no attempt to estimate the full length of the bolt on basis of the bolt point dimensions is known to have been published by other authors. The method applied here is based on the kinetic projectile energy for two different ancient catapult types.

Section 2 gives a brief overview of the historical development of catapults and of early catapult research in general. The mathematical framework is developed in Section 3. The concept of virtual specific gravity of a bolt point will be introduced in Section 3 and elaborated in Section 4 and the possible solution to a resulting 3rd order equation is shown in Section 5. Section 6 describes the overall method/procedure to calculate the full bolt length L, and a sensitivity analysis of the involved parameters is presented in Section 7. Section 8 contains the conclusion of the work.

The use of catapults can, besides psychological impact (intimidation, deterrence), have physical effects by targeting specific targets such as city walls and soldiers. Smaller catapults, in general, were used for shooting bolts/spears, and larger catapults launched stones. The basic function of a catapult is to accelerate a given projectile of mass m to a velocity high enough to allow it to move a distance l via air to a given target. Adding such a high initial velocity v to said mass m during launching from the catapult enables the mass to hit enemy targets. The projectiles can have different shapes and forms, from bolts over irregularly formed objects such as natural stones to processed spherical stone balls of specific mass. The actual shooting distance l will, besides the stored energy, depend on the friction coefficient between the projectile and the air, but the aim here is to analyse the behaviour at the launch moment and, thus, not the travelled

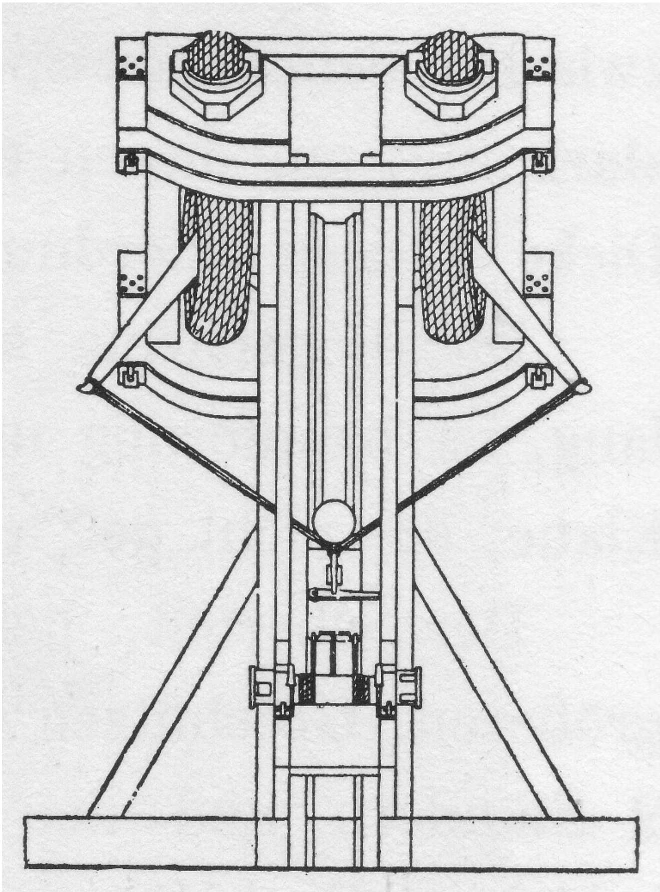

Figure 1. Drawing of a large stone throwing catapult. Note the rope-based torsion springs. Adapted from [3] with kind permission from the Saalburg Museum, Germany.

distance l. For information about research on actual shooting range, air resistance etc., see [1], [2].

The concept of energy states that a projectile at its launch moment must carry less kinetic energy E_{launch} than the energy E_{stored} initially stored in the energy storage medium of the catapult, such as bow limbs or torsion springs, see equation (1)

$$E_{\text{launch}} = \frac{1}{2}\, m\, v^2 < E_{\text{stored}}\,. \tag{1}$$

The maximum shooting distance l is not analysed in this investigation, as the main purpose is to establish a method to determine the catapult and bolt size based on excavated boltheads in general.

2. HISTORICAL DEVELOPMENT

The catapult is generally considered to have been invented approximately in the year 399 BC in Syracuse, at that time large bows mounted with a fixed release mechanism. At a later stage, around 350 BC, torsion-based energy storage was introduced, and the design of these late Hellenistic catapults was standardised around 270 BC (see [1] for a detailed historical description and [3], [4], [5] for further discussion). A drawing of a torsion-based stone thrower is shown in Figure 1 with the main catapult parts and frame structure shown in Figure 2.

The most important sources for information about ancient catapult design are Heron, Biton, Philon and Vitruvius, together spanning the period from 240 BC. to 25 AD [1], [6]. No design formulae for the early bow-based catapults are known to have existed, and only the artillery manuals of Heron of Alexandria, *Belopoeica* and *Cheiroballista*, contains information about the early non-torsion catapults as well as the design of the earliest torsion-based catapults, where energy is mainly stored in torsion springs fabricated out of rope. Heron, however, provides almost no measurements, and the emphasis is on the description of components. The work by Biton is difficult to interpret, mainly due to the absence of drawings, which presumably were lost over time. It is fragmentary and very hard to decode. The artillery manual by Philon of Alexandria, *Belopoeica*, is extremely important to understand the construction of a working catapult, as it provides useful lists of dimensions of components/parts. The Roman officer Vitruvius describes both the Greek and the Roman development from a historical perspective. Since none of the classical sources listed above contain all the necessary information for a reconstruction, it is necessary to combine information from several ancient sources to be able to build a working catapult.

The scientific investigation of the design and construction of Greek and Roman catapults began in the early 19th century and can overall be divided into a French and a German group. The French research was initiated by Emperor Napoleon III and was dominated by the works of the generals Dufour and Reffye [7]. They attempted to build reconstructions, based on the surviving ancient sources, and tried to develop a mathematical model for the shooting distance. Dufour and Reffye, however, seem to have misinterpreted several crucial design information from the Greek writers, and the reconstructions, therefore, are not considered correct, according to present knowledge [1]. The German group was dominated by the scholars Köchly and Rüstow, but their deep knowledge of Greek language was unfortunately not matched by their technical understanding, and they did not succeed in developing correctly functioning reconstructions or usable mathematical models [8]. The German tradition was later greatly influenced by artillery major Erwin Schramm, who in the period 1903-1920 experimented with the reconstruction of ancient catapults at the Saalburg Museum near Frankfurt in Germany [3]. The remains of a Greek bolt launching catapult was in 1914 discovered in Ampurias near Barcelona, Spain, dating from around 150 BC. Figure 3 shows the preserved frame, still with the 4 circular bronze washers for mounting of the torsion springs in their original positions. A reconstructed bolt launching catapult, based on a Ampurias catapult frame, with pre-tensioned rope springs inserted, shown in Figure 4. The design is similar to the version built by Erwin Schramm at the

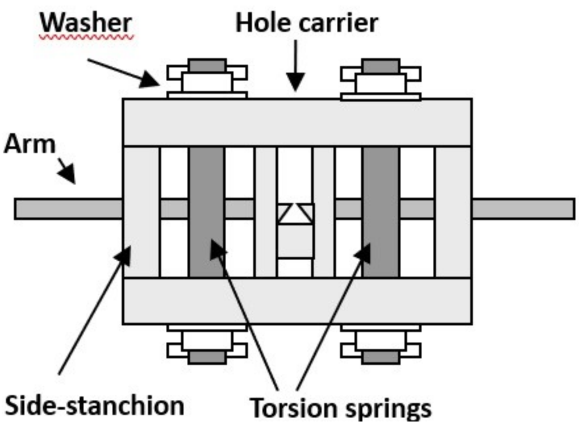

Figure 2. Drawing of main catapult parts and frame structure for a torsion catapult, front view.

Figure 3. Frame of the Ampurias catapult remains in Museu d'Arqueologia de Catalunya, Barcelona, Spain. The 4 circular bronze washers for the mounting of the torsion springs are visible. Photo by author K. M. Paasch.

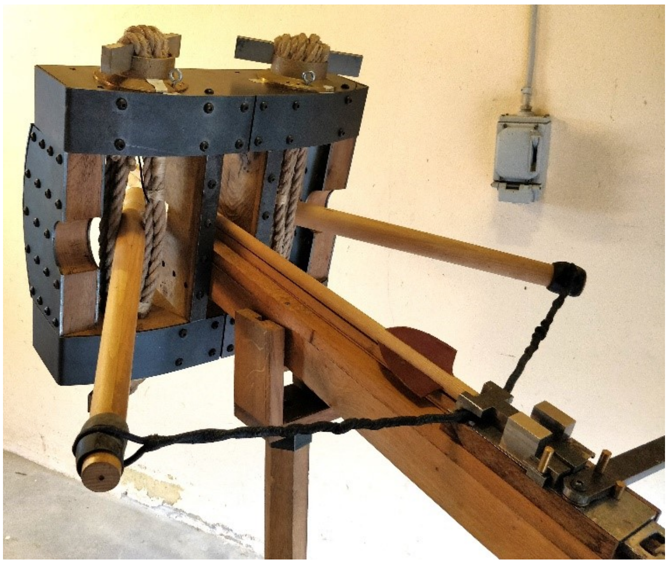

Figure 4. Torsion based Euthytonon type catapult, based on the Ampurias design. Construction by the authors.

Saalburg Museum in 1918 (see [3] and [9], [10] for details). The historical attempts to calculate catapult performance, based on torsion theory, were in general not successful and did not generate usable results. For a more contemporary approach, see [11].

The ancient sources are specific about two sets of standardised design rules. They cover bolts/spears/bolts and spherical stone balls, each with its own calibration formulae to be applied, see equation 2 and equation 3, compiled primarily by E. W. Marsden [1], [2]. The sources state that all component sizes for a given catapult can be constructed from one base measurement, which is then scaled according to the needed size of the catapult. This standardization of the designs apparently emerged around 270 BC, presumably in Alexandria.

The extremely important text *Belopoeica* by Philon of Alexandria, compiled around year 200 BC contains a section stating that: "*Now it is time to explain ….. the subject of artillery construction, called engine-construction by some people… the fundamental basis and unit of measure for the construction of engines was the diameter of the hole. This had to be obtained not by chance or at random, but by a standard method which could produce the correct proportions at all sizes (of a catapult). …… Later engineers looked exclusively for a standard factor with subsequent experiments as a guide……*" [2].

The ancient texts clearly state that the standard factor is given by the diameter of the torsion spring! This base measurement was derived by two different methods, one for the Euthytone (bolt thrower) (Figure 3) and another for the Palintone (stone thrower). The Euthytone was based on the length of the bolt and the Palintone was based on the weight of the stone. The two formulae will be briefly discussed below. The standardization introduced in 270 BC linked all vital geometries to the calculated diameter of the torsion spring (equations (2) and (3)), both for the bolt throwers and for the stone throwers. The design parameters given by the ancient authors Philon and Vitruvius are not complete and it is also necessary to combine them for building a complete catapult. A catapult of any size can be constructed by applying the appropriate scaling factors, as illustrated in Table 1. Philon stated that the ratios were discovered by experiment and experience. Similar tables exist for Palintones. For a full list, covering also winches and sliders, see [1], [2]. The factor in question is the diameter f of the torsion springs. The authors Heron, Philon and Vitruvius give the formulae, but only Philon and Vitruvius have also supplied the ratios of the individual catapult parts, without those, the calibration formulae would be worthless (see Table 1).

The measurements of all major components in a torsion-based catapult are expressed as a factor relative to a single design parameter, the diameter of the torsion spring f. This diameter is, according to Philon/Heron/Vitruvius, determined by two different methods, depending on the purpose of the catapult. The key factor is the diameter of the circular washer, holding the inserted torsion spring. For a bolt launcher (Euthytonon) the factor f_e is given by 1/9 of the actual full bolt length L (equation 2).

$$f_e = \frac{1}{9} L \,. \tag{2}$$

It is observed that equation (2) used the length L of a bolt, but the diameter d is unknown. No complete catapult bolt from before the 3rd century has been discovered, only the bolt points

Table 1. Euthytone multiplication factors for f_p. (x) indicates not provided, calculated from available information. [x] estimated value, difficult to calculate. Compiled from [1], [2]. *Vit.*=Vitruvius, *Ph.*=Philon.

Part	Height		Diameter / Length		Width	
Author	*Vit.*	*Ph.*	*Vit.*	*Ph.*	*Vit.*	*Ph.*
Spring hole			1	1		
Hole carrier	1	1	6 1/2	(6)	1 1/2	1 1/2
Washer	[3/4]	[3/4]	1 1/4	1 1/4		
Sides	3 1/2	4			1 1/2	1 1/2
Tenons	1/2					
Case	1			16	1	

of iron. The socket diameter of a bolthead is likely to be equal to the diameter of the wooden shaft, but the length of the inserted wooden shaft is unknown.

The washer diameter f_p of a stone-throwing catapult (palintonon) is calculated as a function of the weight w of the stone (equation 3) [1], [2]. The expression is shown in SI-unit (kg for the mass m and meter for the diameter f_p).

$$f_p = 0.130 \sqrt[3]{w} \, . \tag{3}$$

The original design formula used the units of Attic-Euboic minae, where 1 mina = 436.6 grammes [1]. The torsion springs were installed under extreme tension to ensure sufficient energy storage and individually adjusted by turning the washers. For details regarding stretching/pre-tensioning of rope see [1], [9], [10]. The actual amount of energy stored in a torsion spring/washer design, as shown in Figure 1 and Figure 2, is traditionally calculated based on a solid cylinder approximation [11]. However, more recent research has shown that the missing material in the torsion spring below the crossbar should be considered [10], [12].

3. COMPARISON OF DESIGN FORMULAE

The physical principle behind the function of the catapults is, as stated in section 1, that both types of catapults accelerate a given mass m to a given launch velocity v with a kinetic launch energy E_{launch}, as shown in equation (1). The main text by Philon can be interpreted as if the designs for both types of catapults are optimal. In case both design formulae express a maximum performance, it is to be considered if both equations express the same function of projectile weight m. In that case it is hypothesised that it is possible to establish the actual length L and shaft diameter d of catapult bolts, where only the points have survived [9], [10], as well as determining the diameter of the torsion spring and thus the size of catapult parts found without corresponding complete bolts.

The "standardisation" in design of catapults is, as stated, considered to have taken place around 270 BC. Both formulae relate to the weight of the projectile, the palintonon formula with direct use of the mass m of the stone and the euthytonon indirectly by the length L of the bolt. The mass of a bolt as a function of its length, therefore, can be determined based on the physical dimensions and material properties, such as the specific gravity of iron and wood. Under the initial hypothesis that both calibration formulae (equations (2) and (3)) express the same physical performance, we have

$$f_e = f_p \, , \tag{4}$$

giving

$$\frac{1}{9} L = 0.130 \sqrt[3]{m} \, . \tag{5}$$

Under the hypothesis that both calibration formulae are optimized regarding the resulting mass of the projectile (stone/bolt/spear), both formulae for a given projectile mass m will give the optimal torsion spring diameter f. Combining the calibration formulae and solving for the mass m (in kg) gives

$$m = \frac{L^3}{(0.130 \cdot 9)^3} \, . \tag{6}$$

The projectile mass m of a bolt is expressed as a function of the bolt length L and its other physical dimensions, shape and material compositions. The full-length L can further be divided into the shaft length L_{shaft} and the point length L_{point}, as illustrated in Figure 5

$$L = L_{\mathrm{shaft}} + L_{\mathrm{Point}} \, . \tag{7}$$

L_{shaft} can be expressed as the ratio α between the shaft length L_{shaft} and the full-length L, for $\alpha < 1$.

$$L_{\mathrm{shaft}} = \alpha \, L \, . \tag{8}$$

The mass m of a bolt can be calculated as the sum of the mass of the visible shaft (m_{shaft}) and the mass of the point section (m_{point})

$$m = m_{\mathrm{shaft}} + m_{\mathrm{point}} \, . \tag{9}$$

The mass of the shaft part m_{shaft} is calculated via its volume and the specific gravity of the wooden shaft material (δ_{shaft}) and the length L_{shaft}. The mass of the fletching is considered much lower than the other components and is not included. The real specific gravity of the virtual point volume will depend on the weight m as well as the shape of the actual bolt point, which might vary from type to type. As the bolt point can have a multitude of shapes, a virtual specific gravity $\delta_{\mathrm{virtual}}$ is introduced into the model. This expresses the specific gravity as if the bolt point section was an enclosing cylinder of a pseudo-material with a virtual density $\delta_{\mathrm{virtual}}$, resulting in the same mass m as the bolt point section as illustrated in Figure 5. The mass of the bolt point m_{point} is thus calculated via the enclosing cylinder volume and the virtual specific gravity ($\delta_{\mathrm{virtual}}$).

In this rather simple model, the small cone of wood inside the socket is not included as, for example, an inner diameter of 16 mm and length of 70 mm inside will add around 3-4 grams to the weight, only a few percent of the weight of the iron point itself. The mass of the parts is approximated as follows:

$$m_{\mathrm{shaft}} = \delta_{\mathrm{shaft}} \, \pi \, \frac{d^2}{4} \, L_{\mathrm{shaft}} \tag{10}$$

$$m_{\mathrm{point}} = \delta_{\mathrm{virtual}} \, \pi \, \frac{d^2}{4} \, L_{\mathrm{point}} \, . \tag{11}$$

Inserting equation (10) and equation (11) into equation (9) gives

$$m = \frac{\pi}{4} d^2 \cdot (\delta_{\mathrm{shaft}} \, L_{\mathrm{shaft}} + \delta_{\mathrm{virtual}} \, L_{\mathrm{point}}) \, , \tag{12}$$

resulting in

$$k_1 \, L^3 = k_2 \, d^2 \cdot (\delta_{\mathrm{shaft}} \, L_{\mathrm{shaft}} + \delta_{\mathrm{virtual}} \, L_{\mathrm{point}}) \, , \tag{13}$$

where the constants k_1 and k_2 below contain the numerical values

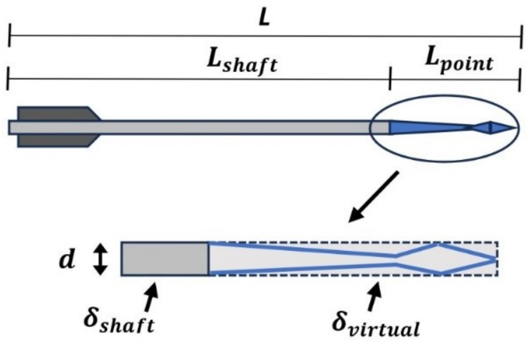

Figure 5. Bolt composition and parameters.

$$k_1 = \frac{1}{(0.130 \cdot 9)^3}$$
and
$$k_2 = \frac{\pi}{4}.$$

By applying the ratio α from equation (8) and using equation (13), it can be shown that the diameter d of the bolt shaft can be calculated by solving the following equation. The steps in the solving process are shown in some detail to illustrate the process.

$$d^2 = \frac{k_1}{k_2} \cdot \frac{L^3}{\delta_{\text{shaft}}\, \alpha\, L + \delta_{\text{virtual}}\, (L - \alpha\, L)}. \quad (14)$$

Solving for the shaft diameter d, by implementing k_1 and k_2 and relabeling δ_{virtual} as δ_{point} gives

$$d = \sqrt{\frac{4}{\pi \cdot (0.130 \cdot 9)^3}} \cdot \frac{L}{\sqrt{\delta_{\text{shaft}}\, \alpha + \delta_{\text{point}}\, (1 - \alpha)}}. \quad (15)$$

Rearranging for full bolt length L gives

$$L = \frac{d}{2} \cdot \sqrt{\pi \cdot (0.130 \cdot 9)^3\, (\delta_{\text{shaft}}\, \alpha + \delta_{\text{point}}\, (1 - \alpha))}. \quad (16)$$

The total length L is now expressed as the diameter d times a factor containing the other physical properties. The parameters L_{point} and d are measured values from the given bolt point under investigation, as shown in Figure 5. By introducing the variable β as the length of the bolt point L_{point} divided by the diameter d, the full bolt length L can be determined as shown below.

$$\beta = \frac{L_{\text{point}}}{d} \rightarrow L_{\text{point}} = \beta\, d. \quad (17)$$

Inserting equation (17) into equation (14) and rearranging terms and reduction, the parameter d can be isolated

$$d^2 = \frac{k_1}{k_2} \cdot \frac{L^3}{\delta_{\text{shaft}}\, L_{\text{shaft}} + \delta_{\text{point}}\, (L - L_{\text{shaft}})}$$

$$d = \sqrt{\frac{4}{\pi \cdot (0.130 \cdot 9)^3}} \cdot \frac{L}{\sqrt{\delta_{\text{shaft}}\, \alpha + \delta_{\text{point}}\, (1 - \alpha)}}. \quad (18)$$

It can be observed that the only unknown in equation (18) is the full bolt length L. Rewriting and rearranging for powers of L gives

$$\beta\, d^3 \left(\delta_{\text{point}} - \delta_{\text{shaft}}\right) = \frac{k_1}{k_2} L^3 - d^2\, \delta_{\text{shaft}}\, L$$

$$\frac{k_1}{k_2} L^3 - d^2\, \delta_{\text{shaft}}\, L - \beta\, d^3 \left(\delta_{\text{point}} - \delta_{\text{shaft}}\right) = 0$$

$$L^3 - \frac{k_2}{k_1} d^2\, \delta_{\text{shaft}}\, L - \frac{k_2}{k_1} \beta\, d^3 \left(\delta_{\text{point}} - \delta_{\text{shaft}}\right) = 0. \quad (19)$$

According to equation (19), the length of the projectile can be calculated via 4 parameters:

- the length of the bolt point L_{point}
- the socket diameter d
- the virtual specific gravity of the point δ_{point}
- the specific gravity of the wooden shaft δ_{shaft}.

The virtual specific gravity parameter of each individual bolt point must be estimated.

4. VIRTUAL SPECIFIC GRAVITY

The wood used for the shaft of roman spears/throwing weapons such as a pilum is expected to be a type of hardwood, typically ash, hazelnut or similar [13]. Due to lack of knowledge, similar types of wood are expected to have been used also for bolts. The specific gravity of these types of hardwood are typically around 670-740 kg/m³ [14]-[16]. A value of 700 kg/m³ is used in this analysis. The value will also depend on the humidity content. To the best of our knowledge, the so far only known complete catapult bolts discovered (Dura-Euporos, Qasr Ibrim) are from later Roman periods and are thus not considered representative for this analysis [17], [18].

The virtual specific gravity δ_{point} of the point will depend on the detailed shape and the wood inside the socked structure. A detailed analysis has been performed by the authors on data from a find of a Roman Republican weapon hoard from Grad near Šmihel under Mt. Nanos in Slovenia [19], covering many socketed catapult points of various sizes, as illustrated by examples in Figure 6. The analysis determined the virtual specific gravity of the catapult points together with an estimation of their mass, for a structure illustrated in Figure 5. The virtual specific gravity of catapult points can easily be calculated in case there is only little corrosion present.

Thirteen values of virtual specific gravity were calculated based on the drawings from [19], plate 14. The values span the range of 1372-3136 kg/m³ with an average value is 2330 kg/m³. The calculated corresponding β-values are illustrated in Figure 7. A linear fit is estimated, showing an R^2-value in the range of 0.7. This indicates that the points have comparable (but not identical) shapes and are not scaled version of the same shape, as the β-values express the point length versus socket diameter. Identical shapes would, regardless of length, show a linear relationship. For heavily corroded points it is recommended that the dimensions are approximated by the geometries of the points and not by insertion in a liquid, due to the loss of mass, however, swelling caused by corrosion can make the estimate difficult. This unfortunately was the case for the catapult points found together with the catapult from Ampurias described in Section 2. Their very corroded state unfortunately made them not suitable for inclusion in this analysis.

The difference between the weights of the points recorded in [19] and the calculated weight vary from -29 % to 19 %, as shown in Table 2, indicating the importance of correct modelling.

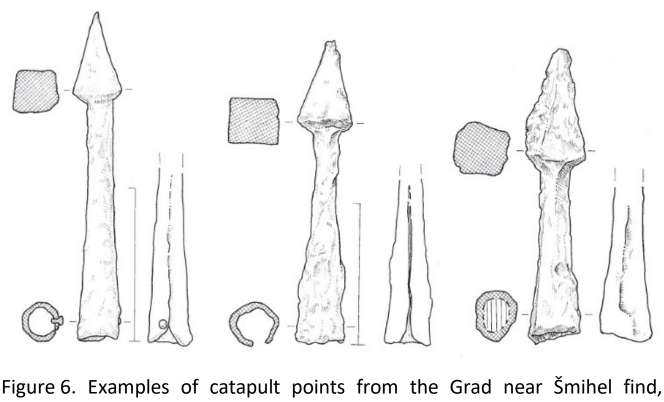

Figure 6. Examples of catapult points from the Grad near Šmihel find, Slovenia [19], plate 14. Adapted from [19] with kind permission from Arheološki vestnik.

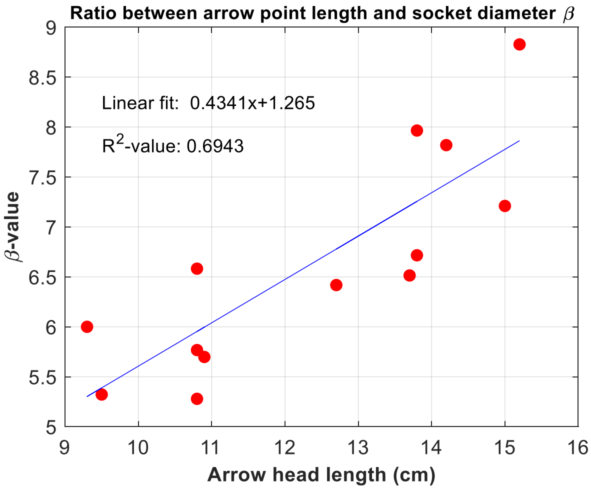

Figure 7. Calculated point β-values as function of diameter size of the 13 catapult points investigated from [19], plate 14.

5. SOLUTION OF THE 3rd DEGREE EQUATION SYSTEM

A cubic equation such as equation (19) can be written in the general form

$$L^3 + pL + q = 0\,, \tag{20}$$

where in this case

$$p = -\frac{k_2}{k_1}\, d^2\, \delta_{\text{shaft}} \tag{21}$$

$$q = -\frac{k_2}{k_1}\, \beta\, d^3 \left(\delta_{\text{point}} - \delta_{\text{shaft}}\right). \tag{22}$$

Cubic equations, in general, cannot easily be solved analytically. Except in cases, where one root is obvious and the equation is reduced to a quadratic equation, cubic equations are, in practice, mostly solved by an approximate or numerical method. The method used here is based on Cardano's method [20] and a set of assumptions related to its use.

In case p and q are real numbers

$$\Delta = \frac{q^2}{4} + \frac{p^3}{27} > 0\,, \tag{23}$$

then the real root (in this case L) is given by equation (24).

$$L = \sqrt[3]{u_1} + \sqrt[3]{u_2}\,, \tag{24}$$

where

$$u_1 = -\frac{q}{2} + \sqrt{\Delta} \tag{25}$$

$$u_2 = -\frac{q}{2} - \sqrt{\Delta}\,, \tag{26}$$

for details see [20]. Equations (21)-(26) are easily implemented in software, calculating the expected full-length L of the catapult bolt of which only the bolt point has been found. Figure 8 shows the calculated expected full bolt length L as a function of the point length L_{point}, for selected values of the socket/shaft diameter (15-26 mm). A selected specific gravity of the wooden shaft of 700 kg/m³ and a selected virtual specific gravity of 2400 kg/m³ of the bolt point section was used.

Table 2. Calculated full bolt length L and torsion spring diameter f_p.

Figure in [19], plate 14	Length L_{point} (mm)	Socked d (mm)	Weight (gram) [19]	Specific grav. (kg/m³)	Weight difference (%)	L (cm)	f_p (cm)
No. 1	152	17.2	93	2670	-2	65.6	7.3
No. 2	150	20.8	92	1870	7	71.7	8.0
No. 5	142	18.2	98	2753	-14	68.5	7.6
No. 6	138	17.3	84	2650	4	64.8	7.2
No. 7	127	19.8	98	2580	-6	71.4	7.9
No. 10	138	20.5	112	2510	10	74.1	8.2
No. 11	137	21.0	136	2894	-29	77.7	8.6
No. 12	108	20.5	62	1770	-5	67.8	7.5
No. 13	108	16.4	52	2331	-25	58.2	6.5
No. 15	108	18.7	63	1370	7	60.1	6.7
No. 16	109	19.1	50	1635	-12	62.9	7.0
No. 17	93	15.5	54	3137	-13	57.5	6.4
No. 18	95	17.9	32	1372	19	57.2	6.4

6. PROCEDURE AND EXAMPLES

The procedure to establish the length of a bolt, where only the dimensions of the bolt point is known, is as follows:

- Determine if the period of the bolt point is in the Hellenistic/Republican period.
- Measure the overall length L_{point} and the socket diameter d of the point.
- Estimate the volume of the bolt point, either by geometry or by immersion in a liquid [21].
- Calculate the virtual density δ_{point} of the bolt point.
- Estimate the length L by applying equation (20)-(26) or by applying the graphical solution chart shown in Figure 10.

The 13 data sets used from [19], plate 14, are used to calculate the expected corresponding full-length L of the bolt, based on equations (21)-(26) and the diameter of the torsion spring f_p via equation 1. The results are shown in Table 2. For weight differences below 10 %, the estimated bolt lengths are in the range of 60-74 cm and the catapult torsion spring diameters are in the range of 6.7-8.2 cm. Results for differences larger than 10 % are consider too uncertain to be included in the analysis.

7. SENSITIVITY ANALYSIS

The theoretical framework developed in the previous sections showed a method to estimate the full bolt length, but the analysis of available data in [19] also showed an uncertainty in estimating the require input values, especially the correct geometry of the bolt points and the derived virtual specific densities. To illustrate the influence of the main parameters has a sensitivity analysis been made. The results of the sensitivity analysis around the selected parameter set of shaft diameter d = 20 mm, point length L_{point} = 180 mm, wooden shaft δ_{shaft} = 700, iron 7800 kg/m³ and the virtual specific density δ_{point} = 2400 kg/m³ are shown is Table 3.

Table 3. Result of sensitivity analysis.

Parameter	Point of analysis	Variation in L
d	20 mm	3.2 mm/mm
L_{point}	180 mm	0.6 mm/mm
δ_{point}	2400 kg/m³	7 mm / (100 kg/m³)
δ_{shaft}	670-740 kg/m³	20 mm

The sensitivity analysis showed that especially the estimation of the socked diameter d is important, as a variation of 1 mm can generate a deviation of 3.2 mm in the estimation of the full bolt length L. The socket diameter can be very difficult to estimate precisely due to corrosion. The variation in L from the point length L_{point} is 0.6 mm/mm. The virtual specific gravity δ_{point} of the point will vary with 7 mm for every 100kg/m^3 and is as such a critical parameter. Based on the wood density variations estimated in [14], [15], [16], the length of the bolt could vary up to 2 cm.

8. CONCLUSION

The main purpose of this paper was to develop a theoretical basis for the calculation of the bolt size and the diameter of the torsion spring for early torsion based euthytonon and palintonon catapults, by measurement of bolt point characteristics. This was achieved by combining the two known formulae for the optimal diameter of the torsion springs for those catapults, listed in ancient Greek/Roman sources. The hypothesis is based on the authors assumption that both formulae represent the same basic principle, to accelerate a given mass to an optimized velocity. Archaeological finds of ancient catapult bolts in general consist only of metal parts, as all wooden elements have deteriorated over time and this imposed uncertainties in the estimation of type of wood and actual dimensions (due to corrosion). The combination of both formulae and the introduction of a virtual specific gravity for the point section of the bolt has led to a polynomial of 3rd degree, solvable under certain assumptions. As examples, the published geometrical forms of 13 catapult points from the Grad near Šmihel find in Slovenia, have been analysed and used as input data for the analysis. The points show corrosion so a deviation in the results were to be expected. The application of the developed theory showed that the full length of the bolts matching those bolt points were estimated to be in the range of 60-74 cm, matching a catapult with a torsion spring diameter range of 6.7-8.2 cm. A 10 % difference in weight from the calculated weight and the measured weight of the points was accepted. A sensitivity analysis showed that a critical parameter in the process is the estimated diameter d of the bolt shaft, identified as the outer diameter of the socket part of the bolt point. A 1 mm variation in d will result in a 3.2 mm variation of the full bolt length. The variation of the specific gravity of the wood used as shaft material can give up to 20 mm in difference. A procedure for the calculation has been described and a graphical tool for the estimation of the bolt length has been presented.

In the best of our knowledge, it is the first time that it has been demonstrated that a combination of the apparently different calibration formulae for standardized bolt-throwing and stone throwing torsion-based catapults can be used to approximate the full length of a bolt made for an early Hellenistic euthytonon bolt shooting catapult. The standardized design was developed around the year 270 BC in Greece. To the authors best knowledge, no full-length catapult bolts from the Hellenistic period have been found or identified as such so far. Therefore, it has not been possible to test the developed theory against archaeological finds.

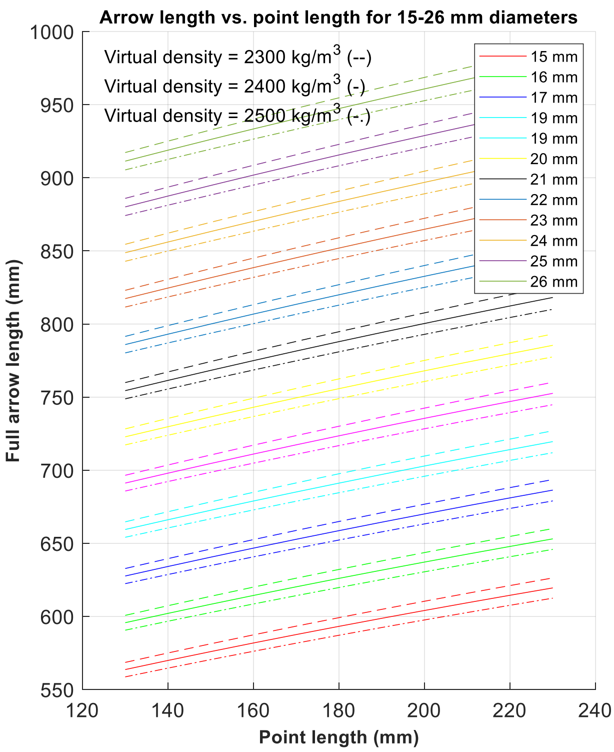

Figure 8. Estimated full bolt length L as function of the point length L_{point} for socket diameters 15-26 mm and point virtual densities of 2300-2400-2500 kg/m^3).

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