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Measurement uncertainty evaluation of equivalent roughness in hydraulic pipes

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ABSTRACT
This paper addresses the quantification of the dispersion of equivalent roughness values obtained from the experimental study of hydraulic pipes used, for example, in water supply networks. This quantity is mainly used in the determination of the friction factor related to pipe fluid flow. In this context, non-linear and complex mathematical models, such as the Colebrook-White equation, are applied to characterize the equivalent roughness of hydraulic pipes composed of different types of materials. However, knowledge about the measurement uncertainty of the obtained estimates is still reduced, having a direct impact in the conformity assessment of this type of hydraulic component and in the technical comparison between different types of pipes (materials and manufacturers). The paper describes the application of a Monte Carlo method (MCM) in the measurement uncertainty evaluation of equivalent roughness. In addition to presenting the theoretical and experimental background, the paper describes the measurement uncertainty propagation, from the probabilistic formulation of the input quantities up to the output quantity. A numerical example, based on experimental data retrieved from field testing of hydraulic pipes integrated in a large-scale agricultural irrigation network, is shown in the paper, illustrating the suitability, advantages, and limitations of the proposed approach.

Section: RESEARCH PAPER

Keywords: hydraulics; equivalent roughness; measurement uncertainty; Monte Carlo method

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1. INTRODUCTION

Water supply networks have a critical role in our society in urban and rural areas. Water is a fundamental and essential resource for the subsistence and development of all countries and regions worldwide, justifying dedicated attention by United Nations (UN-Water) since the 1970s. These networks include several hydraulic elements, such as reservoirs, dams, wells, and pumping and treatment stations, and usually have a high extension. Pipes are essential to the water transportation between hydraulic elements, from an initial stage (collection) to the final stage (customer delivery).

From a design point of view, the friction of the water against the inner wall of a pipe is a crucial issue due to the need for pumping to overcome the corresponding pressure drop along the water supply network, directly impacting construction and operation costs. The friction factor of a pipe is directly related to the roughness of its inner surface. It is considered a complex problem in fluid mechanics, usually requiring an experimental approach under restricted conditions to obtain an accurate solution.

This paper describes the theoretical and experimental background for the determination of the equivalent roughness in hydraulic pipes (sections 2 and 3, respectively), being focused on the evaluation of the corresponding measurement uncertainty. Due to the non-linear and complex nature of this measurement problem, where the Colebrook-White equation [1] has a major contribution, a Monte Carlo Method (MCM) is proposed (in section 4) to achieve this goal. Knowledge about the equivalent roughness measurement uncertainty is still reduced but has a significant impact in the conformity assessment of hydraulic pipes, namely, for manufacturers. From a customer point of view, this knowledge is also important for the decision-making process, contributing for a rigorous technical comparison between different types of hydraulic pipes available for selection considering different suppliers and materials (steel, concrete, cast iron, among others).

In section 5, a numerical example is shown, illustrating the application of a MCM to the studied measurement problem, including the adopted probabilistic formulation of the input quantities and numerical simulation results obtained. Experimental data retrieved from field testing of hydraulic pipes integrated in a large-scale agricultural irrigation network was used for this purpose. A sensitivity analysis is also described, aiming at the identification of the major contributions for the obtained equivalent roughness measurement uncertainty.

Finaly, section 6 shows the main conclusions of this study, regarding the suitability, advantages, and limitations of the proposed measurement uncertainty evaluation approach.

2. THEORETICAL BACKGROUND

In the studied measurement problem, related to equivalent roughness, a straight and rigid hydraulic pipe is considered, having circular transverse cross-section of diameter D, subjected to a gravitational field characterised by an acceleration g. Considering an average velocity V of the fluid (water, for example) inside the pipe and, assuming a stationary Newtonian flow, the head loss, h, between two cross-sections separated by a mutual distance, L, is expressed by

$$h = f\,\frac{L}{D}\cdot\frac{V^2}{2g}, \tag{1}$$

where f is the friction factor [2], a dimensionless quantity, function of the pipe roughness and the Reynolds number, Re, which is defined as

$$Re = \frac{V\cdot D}{\nu}, \tag{2}$$

being ν the fluid kinematic viscosity (considered constant in the case of an isothermal flow) [2].

In this study, the equivalent roughness, ε_s, is assumed homogenous and uniform along the pipe, expressing the dimensional irregularities of its inner surface and considering an equal sand grain diameter (in the first roughness studies in pipes, their inner surface was coated with standard sand with a known grain dimension). In this context, the quantity relative roughness is defined as the quotient between the equivalent roughness and the pipe inner diameter, ε_s/D.

The Colebrook-White equation [1] is an implicit function which allows determining (using interpolation tables, graphical diagrams, analytical or numerical approaches) the friction factor based on the relative roughness and the Reynolds number, i.e.

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\frac{\varepsilon_s}{D}}{I} + \frac{M}{Re\cdot\sqrt{f}}\right), \tag{3}$$

where $I = 10^{0.87}/2$ and $M = 10^{0.4}$. If the pipe friction factor is known, the Colebrook-White equation can be used to express the equivalent roughness explicitly:

$$\varepsilon_s = D\cdot I\left(10^{-\frac{1}{2\sqrt{f}}} - \frac{M}{Re\cdot\sqrt{f}}\right). \tag{4}$$

By introducing the concept of equivalent hydrostatic pressure [2] in expression (1), the friction factor can be obtained from

$$f = \frac{2\cdot\Delta p\cdot D}{\rho\cdot L\cdot V^2}, \tag{5}$$

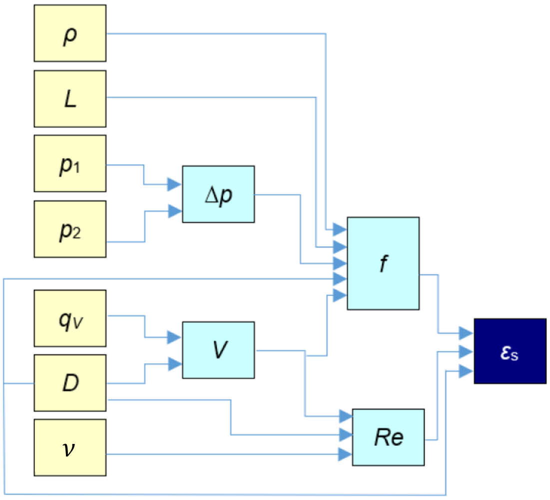

Figure 1. Functional diagram of the equivalent roughness measurement approach.

where ϱ is the fluid density (for a given temperature and pressure inside the pipe), and Δp is the pressure drop between two cross-sections separated by a distance of L.

In the studied equivalent roughness indirect measurement approach, input quantities can be grouped in four categories:

(i) the physical properties of the fluid – density and kinematic viscosity – both dependent on pressure and temperature, with values known from literature [3];

(ii) dimensional properties of the pipe – transverse cross-section diameter, and distance between two cross-sections (where pressure measurements can be performed);

(iii) the average velocity of the flow inside the pipe, which can be known indirectly, based on the knowledge of the pipe cross-section area, A, and in the volumetric flow measurement, q_v, i.e.

$$V = \frac{q_v}{A} = \frac{q_v}{\pi\,\frac{D^2}{4}}, \tag{6}$$

and

(iv) the pressure drop, defined as

$$\Delta p = p_1 - p_2, \tag{7}$$

obtained from pressure measurements p_1 and p_2 in two transverse cross-sections of the pipe separated by a known distance.

The functional diagram for the equivalent roughness measurement is shown in Figure 1.

3. EXPERIMENTAL BACKGROUND

The experimental determination of the equivalent roughness implies the availability of a hydraulic infrastructure composed by a pipeline, being capable of establishing a pressurized flow.

Taking into account the usual measurement resolution of pressure instrumentation (between 0.1 mbar and 1 mbar), a large-scale infrastructure is required to observe a readable pressure drop between two transverse cross-sections of the pipe. Between these two cross-sections, the presence of other hydraulic elements, such as valves and fittings, is not recommended since they will contribute to the increase of the pressure drop, therefore, originating a systematic measurement error in the equivalent roughness.

The hydraulic infrastructure must also allow the volumetric flow measurement between the two cross-sections, where the pressure drop will be measured. In each cross-section, service plugs (in air valves, for instance) can be used to perform pressure measurements and to retrieve fluid samples for temperature measurement in the beginning and end of the test.

In this experimental context, the following measurement instruments (with suitable measurement intervals and resolutions, traceable to the corresponding SI metrological domains) are recommended:

(i) two pressure transducers;
(ii) one ultrasonic flowmeter;
(iii) one thermometer.

Synchronization between flow and pressure measurements is a key issue in this type of experiment and must be guaranteed. Hydraulic stability of the fluid flow is also very important, requiring testing steps with a sufficient time interval to minimize transient phenomena and assure stationary conditions. In this case, special attention must be given to the pumping control system of the hydraulic infrastructure.

4. MEASUREMENT UNCERTAINTY EVALUATION

A MCM is proposed for the measurement uncertainty propagation from the input quantities to the equivalent roughness (output quantity, shown in Figure 1). The selection of this method is motivated by the non-linearity and complexity of the applied mathematical models, namely, the Colebrook-White equation [1].

In this context, the main guidelines of the GUM – Guide to the expression of Uncertainty in Measurement / Supplement 1 [4] were followed. A total of 10^6 runs were performed to achieve a convergent solution for the dispersion of values related to the equivalent roughness (output quantity), with a computational accuracy level below 0.001 mm.

A dedicated numerical simulation routine was developed for this purpose in a MATLAB environment, using the Mersenne-Twister pseudo-random number generator [5]. The same routine was used to perform a sensitivity analysis of the input quantities, aiming the identification of the main contributions for the measurement uncertainty of the equivalent roughness. In this case, an individual increase of 25 % in the magnitude of each input measurement uncertainty was considered, and the corresponding increase of the output quantity measurement uncertainty was normalized.

5. EXAMPLE

This section exemplifies the application of the proposed MCM approach to the studied measurement problem, based on experimental data retrieved from field testing of hydraulic concrete pipes (with a 1.2-meter inner diameter) integrated in a large-scale agricultural irrigation network located in the South region of Portugal. The studied segment connects two water reservoirs at different altitudes. Water pressurization is assured by a pumping station located near the lowest reservoir.

Three measurement points were defined in this pipe (as shown in Figure 2): (i) the flow measurement near the highest reservoir (shown in Figure 3); (ii) the pressure measurement in two air valves (see example in Figure 4) installed in different cross-sections of the conduct, 804 meters away from each other, without any significant hydraulic elements between them.

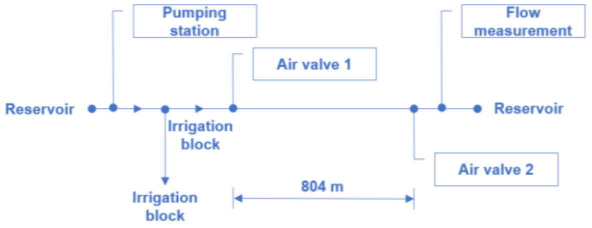

Figure 2. Schematic representation of the hydraulic infrastructure.

Figure 3. Flowmeter ultrasonic sensors installed in the pipe.

Figure 4. Pressure transducer connected to one of the pipe's air valve plug.

Automatic data acquisition of flow, pressure and temperature measurements was defined, considering an acquisition period of five seconds during 10 minutes records.

5.1. Measurement estimates

In the performed field test, a constant water temperature was observed (20.1 ºC) between the experimental campaign's beginning and end. Based on this information and in the static pressure measurements performed in each air valve (shown in Table 1), average water density (998.30 kg/m^3) and kinematic viscosity ($1.0008 \cdot 10^{-6}$ m^2/s) values were obtained from literature [3].

Table 1. Static pressure measurement results (average values and sample experimental standard deviations).

Static pressure	Air valve 1 / bar	Air valve 2 / bar
Test beginning	5.088 3 ± 0.008 6	1.769 6 ± 0.002 1
Test end	5.088 6 ± 0.001 5	1.769 4 ± 0.001 1

Table 2. Dynamic pressure measurement results for each flow testing step.

Volumetric flow / (m^3/h)	Dynamic pressure in air valve 1 / bar	Dynamic pressure in air valve 2 / bar
576 ± 34	5.096 3 ± 0.001 0	1.772 6 ± 0.000 6
765 ± 65	5.100 1 ± 0.000 9	1.773 7 ± 0.001 1
828 ± 71	5.100 5 ± 0.000 8	1.773 4 ± 0.001 0
1020 ± 100	5.106 5 ± 0.000 7	1.775 8 ± 0.000 9
1402 ± 139	5.122 1 ± 0.001 2	1.783 5 ± 0.000 6
1676 ± 134	5.140 7 ± 0.003 8	1.794 8 ± 0.001 5
1721 ± 117	5.139 3 ± 0.003 9	1.792 9 ± 0.001 4

Table 2 presents the average values and sample experimental standard deviations related to the flow and dynamic pressure measurements.

Based on the results shown in Table 1 and Table 2, the corresponding differential pressures and pressure drops were calculated (see Table 3) and used to determine the intermediate (average flow velocity, Reynolds number and friction factor) and output (equivalent roughness) quantities. The results are shown in Table 4 and Figures 5 and 6.

Table 3. Differential pressure and pressure drop estimates.

Volumetric flow / (m^3/h)	Differential pressure in air valve 1 / bar	Differential pressure in air valve 2 / bar	Pressure drop / bar
576	0.007 9	0.003 1	0.004 8
765	0.011 7	0.004 2	0.007 5
828	0.012 1	0.003 9	0.008 2
1020	0.018 0	0.006 3	0.011 8
1402	0.033 6	0.014 0	0.019 7
1676	0.052 2	0.025 3	0.027 0
1721	0.050 9	0.023 4	0.027 5

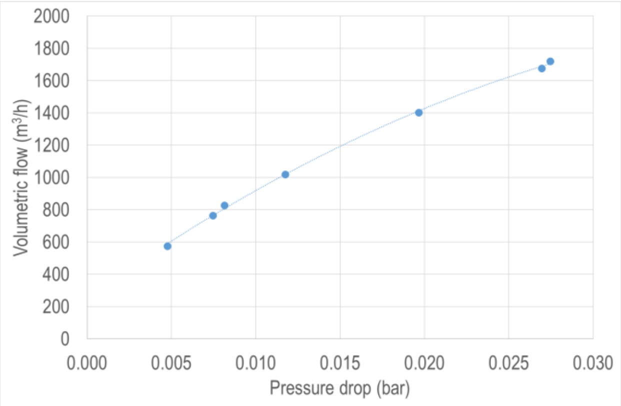

Figure 5. Relation between pressure drop and flow values.

Table 4. Estimates for the intermediate and output quantities.

Flow velocity / (m/s)	Reynolds number	Friction factor	Equivalent roughness / mm
0.141	$1.7{\cdot}10^5$	0.072	0.060
0.188	$2.3{\cdot}10^5$	0.064	0.046
0.203	$2.4{\cdot}10^5$	0.059	0.039
0.251	$3.0{\cdot}10^5$	0.056	0.034
0.344	$4.1{\cdot}10^5$	0.049	0.025
0.412	$4.9{\cdot}10^5$	0.047	0.022
0.423	$5.1{\cdot}10^5$	0.046	0.021

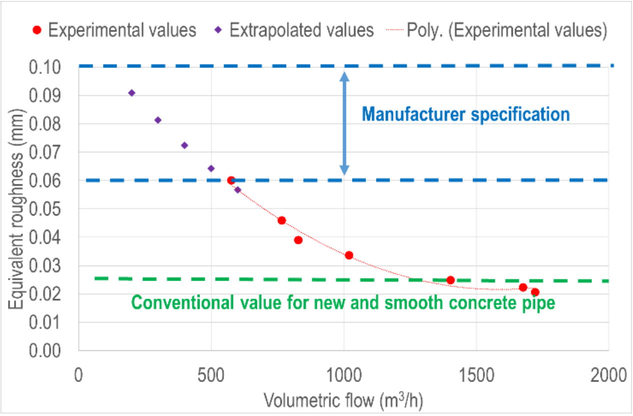

Figure 6. Relation between equivalent roughness and flow values.

In the studied case, the pressure drop measurement for a low magnitude volumetric flow (lower than 500 m^3/h) was not possible due to the high instrumental sensitivity required for the pressure measurement and the proximity relative to hydraulic stability in the pipe (near one mbar). Volumetric flow measurements above 1750 m^3/h were not performed due to an instrumental limitation of the used flowmeter related to the dimension of the test pipe (inner diameter equal to 1.2 m). In addition, the flow measurement surrounding region (near a standard tee) also contributed for turbulence effects in a high average flow velocity.

Figure 6 shows that a second-degree polynomial curve can represent the functional relation between the volumetric flow and the equivalent roughness, due to its proximity to the experimental values. It should be noticed that the extrapolation of this curve, for reduced volumetric flow values (below 500 m^3/h), originates equivalent roughness values comprised in the interval specified for the type of studied pipe (between 0.06 mm and 0.1 mm).

The mentioned equation converges, in the highest volumetric flow region (around 1750 m^3/h), for the conventional equivalent roughness value indicated in literature [6], for circular pipes composed of new and smooth concrete (0.025 mm). The geometrical shape of the obtained curve is comparable to those shown in Moody's diagram [7].

5.2. Probabilistic formulation of input quantities

The probabilistic formulation of the input quantities related to the equivalent roughness comprises both type A (experimental data) and type B (available knowledge), as shown in Table 5.

In the case of the pipe diameter and distance quantities, the corresponding standard uncertainties were quantified based on

Table 5. Example of the probabilistic formulation of the input quantities.

Uncertainty component	Uncertainty source	Type	Probability distribution	Standard uncertainty
$u(D)$	Pipe diameter	B	Gaussian	2.5 mm
$u(L)$	Distance	B	Gaussian	50 mm
$u(\rho)$	Water density	B	Gaussian	0.03 kg/m^3
$u(\nu)$	Water kinematic viscosity	B	Gaussian	$2.9{\cdot}10^{-9}$ m^2/s
$u(q_\mathrm{v})$	Volumetric flow	A	Gaussian	34 m^3/h (minimum) 117 m^3/h (maximum)
$u(p_1)$	Differential pressure (valve 1)	A	Gaussian	0.7 mbar (minimum) 3.9 mbar (maximum)
$u(p_2)$	Differential pressure (valve 2)	A	Gaussian	0.6 mbar (minimum) 1.4 mbar (maximum)

Table 6. Measurement estimates and uncertainties of the pipe's equivalent roughness.

Average / mm	Mode / mm	2.5 % percentile / mm	97.5 % percentile / mm	95 % expanded uncertainty / mm
0.063	0.054	0.019	0.129	0.055
0.056	0.039	0.011	0.123	0.056
0.043	0.033	0.011	0.103	0.046
0.038	0.026	0.011	0.092	0.041
0.028	0.022	0.008	0.071	0.031
0.025	0.016	0.006	0.063	0.028
0.023	0.017	0.006	0.053	0.024

dimensional tolerances related to the pipe manufacturing and hydraulic infrastructure construction. With respect to the water physical properties (density and kinematic viscosity), the mentioned standard uncertainties are related to the measurement samples of pressure and temperature, obtained in the beginning and in the end of the test. Finally, the standard uncertainties shown in Table 5 for the volumetric flow and differential pressure quantities correspond to the observed sample experimental standard deviations.

5.3. Numerical simulations and sensitivity analysis results

Table 6 shows the results obtained from the numerical simulations performed by a MCM, namely, average values, modes, 2.5 % and 97.5 % percentiles and the expanded measurement uncertainty (95 % confidence interval) of the equivalent roughness dispersion of values, for each volumetric flow testing step in the studied pipe.

The obtained 95 % expanded measurement uncertainties varied between 0.056 and 0.024 mm, having a magnitude which is close to the average and mode equivalent roughness values.

Figure 7 to Figure 9 show the output probability density distributions obtained from the MCM simulations, related to the maximum, average and minimum values obtained for the equivalent roughness quantity.

As shown in Figure 7 to Figure 9, the output probability density distributions related to equivalent roughness, show a non-symmetrical geometrical shape. This is justified by the proximity of the obtained values relative to the physical limit of zero roughness imposed by the Colebrook-White equation. This fact also justifies the differences found between the estimates obtained from average and mode values of the numerical sequences obtained from the simulations.

The performed sensitivity analysis – results shown in Figure 10 – revealed that the main contributions for the dispersion of values related to the equivalent roughness are mainly related to the measurement of the volumetric flow (near 53 %) and pressure drop (approximately, 44 %).

These contributions are directly related to the observed hydraulic stability in the studied irrigation infrastructure. Lower measurement uncertainty values are expected when performing laboratorial experiments.

The remaining input quantities have individual contributions equal or lower than 1 %.

6. CONCLUSIONS

The performed study allowed concluding that a MCM is suitable numerical approach for the measurement uncertainty evaluation of equivalent roughness in hydraulic pipes. It is now possible to dispose of accurate and rigorous measurement uncertainties values when performing experiments for the determination of equivalent roughness. The obtained knowledge can improve technical comparisons between pipes of different manufacturers and materials, and better assist conformity assessment processes performed by suppliers and consumers.

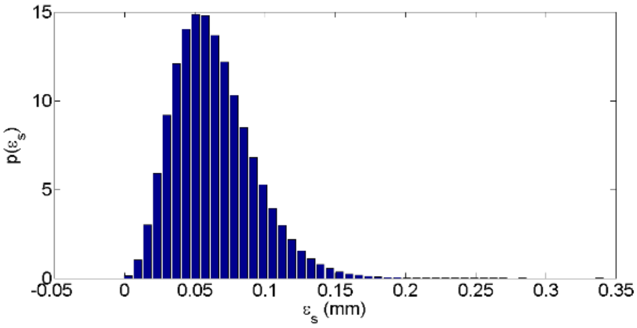

Figure 7. Output probabilistic distribution (0.063 mm average roughness).

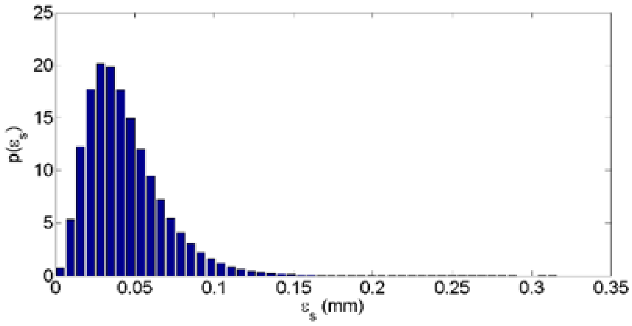

Figure 8. Output probabilistic distribution (0.043 mm average roughness).

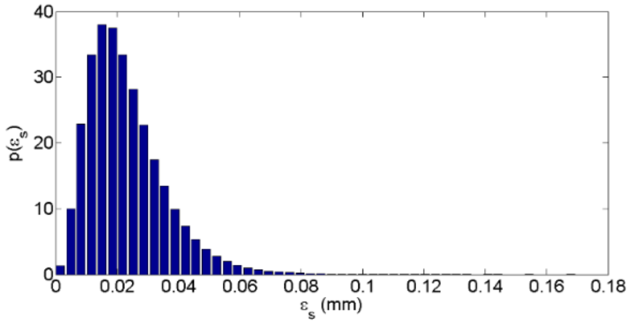

Figure 9. Output probabilistic distribution (0.023 mm average roughness).

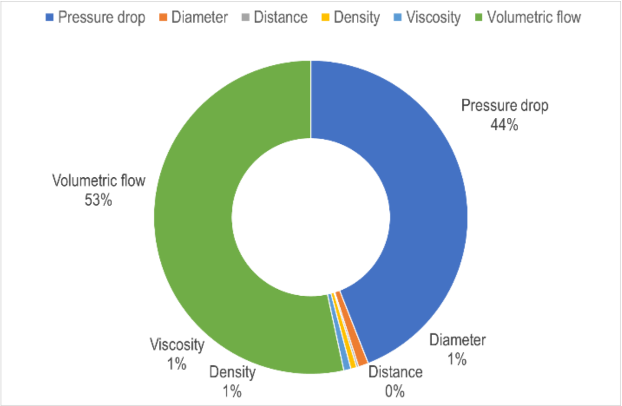

Figure 10. Contributions of the input quantities for the equivalent roughness measurement uncertainty.

In addition to the quantification of the measurement uncertainty, the performed evaluation showed differences between measurement estimates (average and mode values), related to the non-symmetrical shape of the output probability distribution that results, in the studied experimental case, from the proximity to the physical limit of zero roughness. In the case of a rough hydraulic pipe, with higher equivalent roughness, a more Gaussian and, therefore, symmetrical probability distribution is expected, considering the Central Limit Theorem and the presented probabilistic formulation of the input quantities.

In addition, knowledge about the probability distributions and measurement uncertainties of intermediate quantities such as fluid velocity, friction factor and the Reynolds number are also available in the proposed approach.

Special attention must be given to MCM simulations related to smooth pipes, where near-zero equivalent roughness values are expected which can cause numerical instability or unrealistic physical values such as negative or complex values. In these cases, the use of a Bayesian approach [8] is considered suitable to overcome this MCM limitation.

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