

# The *Via Severiana* and the *Tabula Peutingeriana*: valuation of landmarks precision and town expansions

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## ABSTRACT

This work deals with the examination of the route of the *Via Severiana*, traced in the imperial period, from a historical and technical point of view. Then, results were compared with the *Tabula*, which we can consider one of the first synthetic representations of general viability. We have explored the development of the various sectors of the *via Severiana*, considering the various utility and presence on the territory, using the *Tabula Peutingeriana* as a benchmark. Subsequently, we have examined the precision of the positioning of the various landmarks along the *via Severiana* and the movement of the inhabited centers as a function of density. Modern technologies, including Geographic Information Systems and mathematical models, allow us to help the archaeologists to overlay ancient maps like the *Tabula Peutingeriana* onto contemporary maps, aiding in the identification of locations and understanding ancient landscapes.

Section: RESEARCH PAPER

Keywords: precision; measurements; landmarks; expansions

Citation: E. Petritoli, F. Leccese, The *Via Severiana* and the *Tabula Peutingeriana*: valuation of landmarks precision and town expansions, *Acta IMEKO*, vol. 13 (2024) no. 2, pp. 1-8. DOI: [10.21014/actaimeko.v13i2.1754](https://doi.org/10.21014/actaimeko.v13i2.1754)

Section Editor: Francesco Lamonaca, University of Calabria, Italy

Received January 20, 2024; In final form March 11, 2024; Published June 2024

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## 1. INTRODUCTION

Political, economic, and social life during the centuries of the Empire revolved around the *Urbe*. Rome was the seat of imperial authority and administration, the main place of commercial exchange between East and West as well as being by far the most populated city in the ancient world with around one million inhabitants. For this reason, thousands of people flocked to the capital every day by sea and by land, enriching it with artists and men of letters from all regions of the Empire.

The ancient Romans built long roads to connect the most distant provinces with the capital of the empire. Made as straight as possible to minimize distances, these infrastructures were essential for the growth of the empire, as they allowed the army to move quickly. Besides the army, they were also used for political, administrative, and commercial purposes. The Roman Road network constituted the most efficient and long-lasting road system of the ancient times, which made it possible to bring Roman civilization into contact with the most diverse peoples who populated the known world at the time. No other people in that historical era were able to match their ability to choose routes, construction techniques and the organization of assistance to travellers. This work focuses the *Via Severiana* and

the oldest cartographic reference system, the *Tabula Peutingeriana*, from a technical point of view.

As a first step, it is worth examining the *Via Severiana* from a historical and political point of view, its route, and the main segments. Subsequently, we should assess which of these landmarks coincide with those indicated on the "tabula" and how they have evolved over time. As a third step, we should examine the precision of the positioning of the various landmarks along the *via Severiana* and the movement of the inhabited centres as a function of density [1].

The *Tabula* is a valuable historical document that provides insights into Roman knowledge of geography and their extensive road network. While not being a precise map, according to modern standards, it offers a fascinating glimpse into how the Romans perceived and navigated their vast empire.

To interpret the *Tabula* in an effective way, it is crucial to consider its historical context and purpose. The map was likely used as a practical guide for travellers rather than as a comprehensive geographical record.

We have focused our attention on the exact measurement and comparison between the positions and the relative route of the Roman road through the centuries, appropriately evaluating the positional drift of the relative landmarks [2].

## 1.1. The Via Severiana

### Septimius Severus

Septimius Severus (*Lucius Septimius Severus Pertinax*) was born on April 11, 145 AD, in *Leptis Magna*, a city in Roman North Africa (modern-day Libya). He came from a relatively wealthy equestrian family and was the first Roman emperor from Africa. *Severus* had a distinguished military career, serving in various capacities and rising through the ranks: he held key military commands in the Danubian and Illyrian regions and earned a reputation as a capable and disciplined commander.

In 193 AD, the Roman Empire experienced a period of political instability known as the Year of the Five Emperors: after the assassination of Emperor *Pertinax*, *Severus* was proclaimed emperor by his troops in Pannonia (modern-day Hungary): then *Severus* marched on Rome to secure his position, defeating his rivals, including *Clodius Albinus* and *Pescennius Niger*, in a series of military campaigns. His victory marked the beginning of the Severan dynasty [3].

*Septimius Severus* ruled for nearly 18 years, making him one of the longest-reigning emperors of the Roman Empire. His reign was characterized by efforts to strengthen the Roman military, administrative reforms, and infrastructure projects, also undertook campaigns in Britain and in the East to secure and expand the empire's borders.

Severus is often remembered as a capable military leader who sought to maintain stability and secure the empire's borders during a period of challenges: he implemented significant military reforms in the Roman army, increasing the pay for soldiers and expanding the size of the legions. These reforms aimed to strengthen the loyalty of the army and enhanced the empire's defence capabilities.

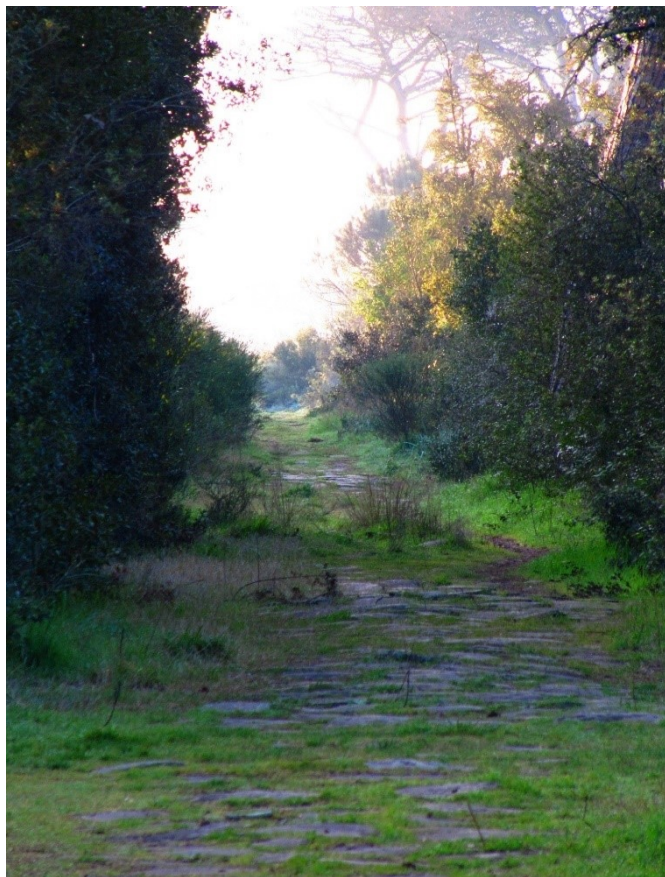


Figure 1. The *Via Severiana* near Castel Fusano (Rome – Italy). (Authors)

Septimius Severus' reign is notable for its military successes, administrative changes, and efforts to stabilize the Roman Empire during a time of political upheaval. His contributions had a lasting impact on the subsequent development of the Roman state and military institutions.

### The *Via Severiana*

The *Via Severiana* was an ancient Roman road that played a significant role in connecting the city of Rome to the regions of southern Italy. The *Via Severiana* was named after the Roman Emperor Septimius Severus, who ruled from 193 to 211 CE. It is believed that the road was constructed or renovated during his reign. The exact route of the *Via Severiana* can be somewhat challenging to trace precisely due to changes in landscape over time and modern development. Figure 1 shows a stretch of *Via Severiana* near Castel Fusano in Rome (Italy).

It is generally understood that the road ran southeast from Rome, passing through the Alban Hills and connecting with other major Roman roads. The *Via Severiana* had a particular significance in connecting Rome to the important port city of Ostia, which served as the gateway to the Mediterranean Sea. Ostia was a crucial harbour for the city of Rome, facilitating the transport of goods, people, and soldiers. As many Roman roads, the *Via Severiana* served both military and commercial purposes. It facilitated the movement of Roman legions and officials to and from various regions, ensuring effective governance and defence. The road also played a key role in trade and commerce, promoting economic activities between Rome and the southern parts of Italy [4].

### Archaeological Evidence

Archaeological evidence, including remnants of the road and associated structures, has been discovered along the suspected route of the *Via Severiana*.

These findings provide insights into the engineering and construction techniques employed by the Romans in the development of their extensive road network [5].

With the decline of the Roman Empire, many of the ancient roads fell into disrepair, and the once-thriving network diminished in importance. Factors such as invasions, changes in trade routes, and economic decline contributed to the abandonment of some roads, including the *Via Severiana*.

The *Via Severiana* is just one example of the extensive network of Roman roads that played a crucial role in the empire's infrastructure, facilitating communication, trade, and the movement of people and armies. While some parts of these roads have disappeared over time, others still exist or have been rediscovered through archaeological efforts.

Thomas Ashby said that it would have run along the shore only at first: just behind the line of villas which fronted the sea – which are now some 1 km inland – or even upon its edge (for an inscription records its being damaged by the waves). Farther southeast, the road would seem to have kept rather more distant from the shore, and it probably kept within the lagoons below the Circean promontory.

Today, only a few visible traces remain of the ancient road. Some fragments of the paving used to pave the road are visible inside the pine forest of Castel Fusano and in Ostia in the pine forest near the Canale dei Pescatori on the road that leads to Ostia Antica. Other remains are visible in the necropolis of Isola Sacra [6].

## 1.2. Tabula Peutingeriana

### Konrad Peutinger

Konrad Peutinger (1465–1547) was a German humanist, scholar, and antiquarian of the Renaissance period. He is particularly known for his association with the *Tabula Peutingeriana*, a medieval copy of a Roman road map [7], [8]. He was born on October 14, 1465, in Augsburg, a city in present-day Bavaria, Germany and received a humanist education, which was characterized by a focus on classical literature, philosophy, and the arts. Subsequently, he held various positions in public service, including serving as the city's legal advisor and holding the position of syndic (chief legal officer) in Augsburg and involved in diplomatic and political affairs, representing the interests of Augsburg in various capacities.

Peutinger was a prominent figure in the humanist movement, which sought to revive and emulate the classical learning and culture of ancient Greece and Rome and corresponded with other humanist scholars of his time, and his collection of manuscripts and antiquities reflected his interest in classical studies. It is important to note that Peutinger did not create the map but acquired it. The map is believed to be a copy of an earlier Roman map, possibly from the 2nd or 3rd century. After Peutinger's death, the map passed through various hands and probably ended up in the Austrian National Library in Vienna.

Konrad is remembered for his contributions to the humanist movement and for his role in preserving and disseminating classical knowledge. His extensive collection of manuscripts, including the *Tabula*, is a testament to his interest in antiquity and his efforts to preserve classical texts and artifacts.

While Peutinger's role in the creation of the *Tabula Peutingeriana* was that of an owner and preserver rather than a creator, his association with the map has left a lasting impact on our understanding of Roman geography and road networks and remains an important historical artifact and a testament to the interest in classical knowledge during the Renaissance [9].

### The Tabula Peutingeriana

The *Tabula Peutingeriana* (Figure 2), also known as the Peutinger Table or Peutinger Map, is a medieval copy of a Roman road map that provides a unique and valuable glimpse into the Roman road network and geographical knowledge of the time [10].

The *Tabula Peutingeriana* is believed to date from the 4th or 5th century AD, but it is a copy of an earlier Roman map, possibly from the 2nd or 3rd century; it is named after Konrad Peutinger, a 15th-century German humanist scholar who owned the map. The map is a scroll, measuring about 6.75 meters (22 feet) in length, but only a portion of it survives today. It is not a geographically accurate map but rather a schematic representation of the Roman road network, emphasizing the connections between various cities and regions.

The *Tabula* covers the Roman Empire from the British Isles in the northwest to the Indian subcontinent in the southeast. It portrays a stylized view of the Mediterranean, with important cities, rivers, and mountain ranges represented. The distances between locations are not to scale; instead, the emphasis is on depicting the road network and the connections between cities. One of the most significant features of the *Tabula Peutingeriana* is its detailed depiction of the Roman road system, including the names of many stopping points (mansions) along the roads. The map is divided into several segments, each highlighting a specific region of the Roman Empire. While the exact purpose of the

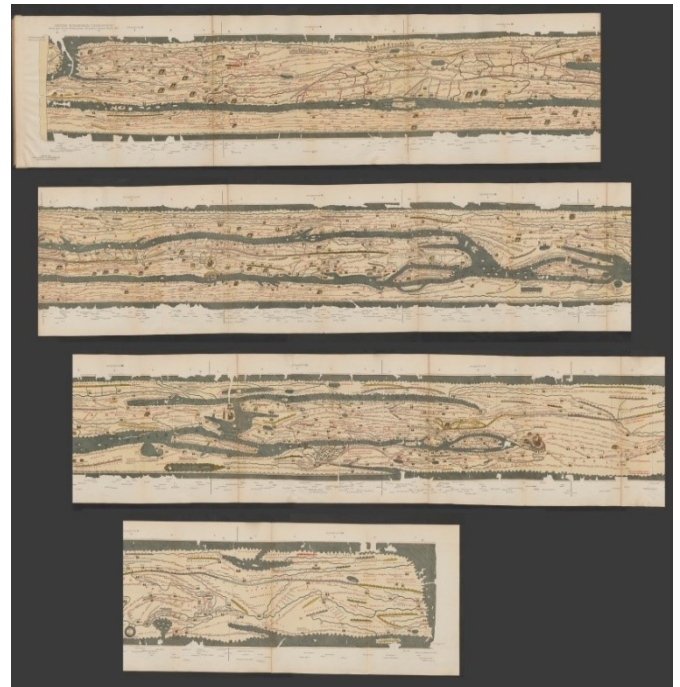


Figure 2. The Tabula Peutingeriana.

*Tabula Peutingeriana* is not entirely clear, it is believed to have served as a reference for travellers, military personnel, and administrators navigating the Roman road network, and it may have been used to calculate travel distances and estimate journey times between different locations.

The original *Tabula* is lost, but a medieval copy survived and is now housed in the Austrian National Library in Vienna. However, it suffered damage over the centuries and circa 25-30% of the original length remains to date.

### 1.3. Via Severiana and The Tabula

In the map, the *Via Severiana* is depicted as a line connecting Rome to the southeast, passing through the region that corresponds to the Alban Hills. The map provides information about the distances between various locations along the road but does not offer precise details about geographical features or the exact course of the road. The representation of the *Via Severiana* on the *Tabula Peutingeriana*, as well as other roads on the map, is more symbolic and intended to convey the connectivity of the Roman Empire rather than to provide a realistic geographical representation. The map emphasizes the road network's importance for travel, communication, and the movement of goods across the vast expanse of the Roman Empire.

### The Isola Sacra part

Inside a triangle of land, known in the past as *Isola Sacra*, stands the ancient Necropolis of Porto, a suggestive archaeological site a stone's throw from Fiumicino to Leonardo Da Vinci Airport. The island, which is bordered by the Tiber River to the southeast, the Fiumicino canal to the north and the Tyrrhenian Sea to the west, was called *Insula Portuensis* in ancient Roman times but in the 6th century AD, due to the strong Christian presence in the territory, acquired the nickname of "Sacred" [11], [12], [13], [14], [15].

The city of Porto, around the 1st century BC, achieved great development and, above all thanks to its proximity to the Port of



Figure 3. The villa of Quintus Hortensius Hortalus: panorama (Rome – Italy). (Authors)

Claudius, it gradually replaced the ancient city of Ostia as the main support for maritime activities.

The island was crossed by an important road, the Via Flavia Severiana, which connected the city of Porto with ancient Ostia. It was precisely the inhabitants of Porto who built the Necropolis on the side of the road. The floods of the Tiber and the consequent silting up of the entire area have allowed an excellent conservation of the tombs, with their characteristic pink colour [16].

### The villas part

Along the Via Severiana, after the Stagno di Ostia, there were a series of residential villas that followed one another along the coast, from the so-called “Palombara” area up to Torvaianica.

Most of these villas pre-existed the road as they mostly dated back to the late republican period. Back then the sea was further away than today, and the villas were immersed in the Mediterranean foliage that covered the area.

The villa of *Quintus Hortensius Hortalus* (Figure 3), a Roman orator who lived at the time of Cicero, is in the Palombara area. The first findings in the area date back to 1713, subsequent excavations carried out in the early 1800’s by the Chigi family, then owners of the area, and then in 1933-34, brought to light most of the remains of the villa, which are still visible today. These findings were for a long time associated with the villa of Gaius Plinius Caecilius Secundus (Pliny the Younger). Only recently, in 1984, the remains have been attributed to the villa of Hortensius, while the villa of Plinius is believed to be in the presidential estate of Castel Porziano, in the so-called Villa Magna in the locality of Grotte di Piastre. It should be noted that over the centuries the coastline has advanced by about 600 metres: this has led to a progressive burial of all the artefacts and the aggression by the colonizing plants of the dunes.

Cicero, in his letters and speeches, mentioned visiting the villa of Hortensius on multiple occasions. However, these references do not provide extensive details about the villa itself: the letters to his friend Atticus contain some mentions of interactions at the villa, but they are more focused on the people and events rather than describing the physical aspects of the property. While we have some information about Quintus Hortensius Hortalus and his prominence in Roman oratory, specific details about the architecture, location, or features of a villa owned by him are not well-documented in historical records. The study of Roman villas and their owners often relies on archaeological findings and historical texts that provide glimpses into the lifestyles of the Roman elite.

### The Swamps Part

The construction of the road also presented many difficulties for the skilled Roman engineers, due to the different watercourses which, descending from the inland mountains and

especially in the Pomptinus Ager (Pontine Marshes) area, often became marshy before flowing into the sea.

In ancient times, the Pontine Marshes were a notorious and unhealthy area due to the prevalence of malaria. The marshes were largely uninhabitable, and the surrounding hills served as a refuge for local populations. The Romans made attempts to drain the Marshes even during the Republican period. Efforts were made to channel water away from the region and reclaim the land for agricultural purposes.

The technique used to overcome this type of obstacle was that of driving wooden pillars into the ground and subsequently filling the space between the pillars with large stones which were then strongly compressed and further filled with earth to form raised embankments of one or two meters compared to the level of the swamp, on which the road was later built.

### The Circeo part

The road is illustrated by the Tabula Peutingeriana, where, as far as the Circeo is concerned, two important stations were indicated: “Circeios” (near Torre Paola) and Ad Turres (probably in Torre Vittoria). The series of stations that follow one another from Astura to Terracina can be summarized in Table 1.

Other studies place the ‘Ad Turres’ station near the current Torre Olevola instead of Torre Vittoria: from Astura to Clostra in the Chart they read nine miles, distance, if exact, determines the station of the Fogliano maquis, four miles distant from the walls of San Donato. This city which, in this case, would correspond to the following station called Ad Turres Albas, however correcting the number of III of the Chart in IV. Circejo is placed XIX miles distant from Turres Albas so that by measuring the space between San Donato and the ruins of Circejo on the mountain of the same name commonly called Santa Felicita, one counts at least XXIII miles, and therefore the number XIX in the map must be corrected. Four miles are placed on the Map, which roughly coincide with the Tower of Olevola or a little further. From this point to Terracina, the Map places XI miles but there are barely IX miles (Table 2).

The original route of the Severiana probably followed the current coastal road Borgo Grappa - Molella - San Felice. The byway, on the other hand, reached the port area of Torre Paola, Circeios flanked by various sepulchres, some of which are still visible in the locality of Selva Piana. The road continued, in the

Table 1. Ancient Stations.

Station	Distance	
Astura	VII miglia	10.400 km
Clostris	IX miglia	13.400 km
ad Turres albas	III miglia	4.500 km
Circeios	XII miglia	18.000 km
Ad Turres	IV miglia	6.000 km
Terracina	IX miglia	16.300 km

Table 2. Modern Stations.

Ancient station	Distance	New Position
Asturam	VII miglia	Torre Astura
Clostra	IX miglia	?
Ad Turres Albas	IV miglia	San Donato
Circeios	XXIII miglia	Torre Paola
Ad Turres	IV miglia	Torre Olevola
Terracinam	IX miglia	Terracina

main stretch, along the mountain up to Torre Vittoria (perhaps Torre Olevola) from where it continued towards Terracina.

#### 1.4. Map Precision

The intended purpose of the map influences its precision requirements. Navigational charts, for example, demand high precision to ensure safe navigation, while thematic maps may prioritize conveying specific information over fine details.

The precision of the Tabula, (i.e., whether the positions coincide with the real ones), is beyond question: the number of travellers who have used it over the centuries is so high and the “returns from the field” have been so high that the distances can be considered reliable. This does not mean that the centuries have in any case taken their toll on the document and, despite him, the inaccuracies may be attributed to various factors: in all cases, we can consider the absolute invariance of the Severiana, i.e., its layout has not changed at all over the centuries. A short list of reasons for inaccuracy follows [16].

#### The “Floating Stations”

The stations must lie next to the Severiana, but human and historical events (wars, famines, etc.) and meteorological factors (floods, landslides etc.) may have moved the precise position of the building or complex that was designed to welcome travellers. This “fluctuating” movement may have been carried out several times over time, making the current geolocation the station difficult.

#### The Ghost Station

For the same reasons discussed in the previous point, many stations may have simply been abandoned. Additional causes could be the swamping of the area and the consequent unhealthy air, or the abandonment of the countryside or adjacent inhabited centres, which made the supply extremely difficult or impossible.

#### Urbanization of the Station

Conversely, simple post stations become rather populous towns: the service to the passenger therefore becomes much more varied, widespread, and multiplied across multiple suppliers. It is not uncommon for inns and horse exchange stations to multiply in these locations and, if very populous, we also have the appearance of various entertainment activities.

#### The Fall from Grace

An additional important aspect to consider is the variation in importance of the station. Over the centuries, with the movement of people and the variation of the surrounding countryside, localities that were very important in the first centuries declined for social or political reasons, being reduced to simple agglomerations of houses whose economy depended on the passage of travellers.

## 2. MATERIALS AND METHODS

### 2.1. The Via Severiana

It is worth noting that the Tabula Peutingeriana is not a geographical map in the modern sense: it is a stylized, schematic representation designed for practical use, probably serving as a guide for travellers. It provides valuable insight into the Roman road network and major landmarks, the emphasis is placed on illustrating the connections (roads) between highlights rather than providing precise geographical detail. Accordingly, it is necessary to use the map in conjunction with other historical sources and modern mapping technologies such as mathematical models, to evaluate its precision.

#### Landmarks precision

The precision of landmarks on the Tabula is limited: the map is a stylized and schematic representation that prioritizes illustrating the connectivity of roads rather than providing accurate geographic details. While major cities and some notable landmarks are depicted, the representation may lack the precision found in modern maps.

#### Landmarks Motion

Each landmark evolves over time and hardly remains still in a single place even if forced by the conditions of the landscape. Accordingly, it is possible to attribute them a speed of movement (albeit very low) which intrinsically distorts the distance between the previous and the following landmarks, respectively. The final aim is to determine how the precision of the measurement evolves over time (months, years, centuries) while the total measurement remains constant (first and last terrestrial reference). The speed or displacement in our case is moved for clarity on two cardinal axes, north-south and east-west.

The town or landmark expands with two direction components (or speed although slow), the first is  $\vec{s}_{\text{random}}$ , is a pure casual expand/direction combination [17]:

$$\vec{s}_{\text{random}}(t) = \begin{bmatrix} \text{random}_{\text{SN}}(t) \\ \text{random}_{\text{SE}}(t) \end{bmatrix} \quad (1)$$

Every axis is composed by two components:  $\text{random}_{\text{S}}(t - \Delta t)$  is the speed that consider the “history” of the landmark,  $\mathcal{R}(\Delta t)$  is a random zero mean vector, and  $\delta(\Delta t)$  is an algebraic scalar that modifies the direction.

Accordingly, the full expression becomes:

$$\vec{s}_{\text{random}}(t) = \begin{bmatrix} \text{random}_{\text{SN}}(t - \Delta t) + \mathcal{R}_{\text{N}}(\Delta t) \cdot \delta_{\text{N}}(\Delta t) \\ \text{random}_{\text{SE}}(t - \Delta t) + \mathcal{R}_{\text{E}}(\Delta t) \cdot \delta_{\text{E}}(\Delta t) \end{bmatrix}. \quad (2)$$

We impose that the landmark moves constantly in a sort of bi-dimensional “moving plane” so due to the negligible (impossible) movement on the vertical plan. In mathematical terms, we have:

$$\delta_z(t) \approx 0 \quad (3)$$

The movement of the single landmark is influenced by the position of the other landmarks: we define  $\vec{s}_{\text{diff}}(t)$  as the second component of speed that “diffuse” the landmarks in the full area as:

$$\vec{s}_{\text{diff}}(t) = \begin{cases} \sum_{i=1}^n \frac{|r - r_i|^3}{|r - r_i|} \\ |r - r_i| \leq d_{\text{min}} \end{cases} \quad (4)$$

where  $n$  is the full number of landmarks,  $r$  is the distance from the reference (Rome) of the considered landmark,  $r_i$  is the distance of the  $i^{\text{th}}$  landmark, and  $d_{\min}$  is the *reference* distance between two landmarks. The “*minimum reference distance*” indicates the minimum distance assumed by two landmarks.

Thus, the total movement speed of the single landmark is:

$$\vec{s}_{\text{TOT}}(t) = \vec{s}_{\text{random}}(t) + \vec{s}_{\text{diff}}(t) \quad (5)$$

So, the complete expression of  $\vec{s}_{\text{TOT}}(t)$  is:

$$\left\{ \begin{array}{l} \text{random } S_N(t - \Delta t) + \mathcal{R}_N(\Delta t) \cdot \delta_N(\Delta t) \\ \text{random } S_E(t - \Delta t) + \mathcal{R}_E(\Delta t) \cdot \delta_E(\Delta t) \\ |r - r_i| \leq d_{\min} \end{array} \right\} + \left\{ \begin{array}{l} \sum_{i=1}^n \frac{|r_N - r_{iN}|^3}{|r_N - r_{iN}|} \\ \sum_{i=1}^n \frac{|r_E - r_{iE}|^3}{|r_E - r_{iE}|} \\ 0 \end{array} \right\} \quad (6)$$

Figure 4 shows the evolution of the  $\vec{s}_{\text{diff}}(t)$  as a function of time. The time axis is normalized to 1 to compare trends while the speeds are also normalized to 1, indicating the maximum speed of the single landmark. It is noted that as the number of landmark members' increases, the total speeds are always higher but are reached later.

## 2.2. Urban expansion

Urban expansion is the physical growth and expansion of urban areas, which includes the outward expansion of city boundaries, increased population density, and the development of new infrastructure and land uses. It is a complex phenomenon influenced by various social, economic, political, and environmental factors. These phenomena are often driven by population growth as people migrate from rural to urban areas in search of economic opportunities, better living standards, and improved amenities. Economic growth and development contribute to urban expansion. Cities often expand as economic activities increase, attracting businesses, industries, and a greater manpower.

The expansion of cities is closely linked to the development of infrastructure, including transportation networks (roads, highways, public transit), utilities (water supply, sewage systems),

and communication networks: this involves changes in land use patterns, with agricultural or natural land being converted into residential, commercial, or industrial areas.

Zoning regulations and urban planning policies influence the direction and extent of urban expansion. These policies guide the allocation of land for different purposes and aim to manage a sustainable growth.

Real estate development is a key driver of urban expansion. Developers collect land to build housing, commercial buildings, and other structures to meet the demands of a growing population.

Urban sprawl is a form of uncontrolled or low-density urban expansion characterized by the spread of development into surrounding rural areas. It often leads to increased automobile dependence and environmental concerns. In some cases, urban expansion may outpace the development of necessary infrastructure, leading to challenges such as traffic congestion, inadequate public services, and environmental degradation.

The expansion can have significant environmental implications, including habitat loss, increased pollution, and changes in the natural landscape. Sustainable urban planning aims to minimize these impacts. This can exacerbate social and economic inequities, with certain areas experiencing more significant growth and development while others face neglect and lack of infrastructure.

Some cities adopt smart growth principles, emphasizing compact and efficient development to minimize the environmental footprint and enhance the quality of life of the residents.

Effective land-use planning strategies, such as mixed-use development and transit-oriented development, can help manage urban expansion in a way that supports sustainability and community well-being. This fact may involve the redevelopment of brownfield sites, repurposing former industrial or contaminated areas for new uses, contributing to sustainable development.

Urban expansion is a global phenomenon, with many regions experiencing rapid urbanization. Understanding these trends is crucial for addressing the challenges associated with urban growth.

## Population Density Evolution

Population density is a key demographic indicator that provides insights into how concentrated or dispersed a population is within a specific geographic area.

The expansion of cities is closely linked to the development of infrastructure, including transportation networks (roads, highways, public transit), utilities (water supply, sewage systems), and communication networks: this involves changes in land use patterns, with agricultural or natural land being converted into residential, commercial, or industrial areas.

Now we can describe our system using the Fokker-Planck equation. It describes the temporal evolution of the population density function and can be generalized to other observable entities [18], [19], [20], [21], [22], [23], [24], [25].

All this is expressed with the time evolution of the probability density:

$$\frac{\partial \chi(\mathbb{R}_{NE}, t)}{\partial t} = -\nabla [f_1(\mathbb{R}_{NE}, t) \cdot \chi(\mathbb{R}_{NE}, t)] + \frac{1}{2} |d_{\min}| \cdot \nabla^2 [(f_2(\mathbb{R}_{NE}, t))^2 \cdot \chi(\mathbb{R}_{NE}, t)] \quad (7)$$

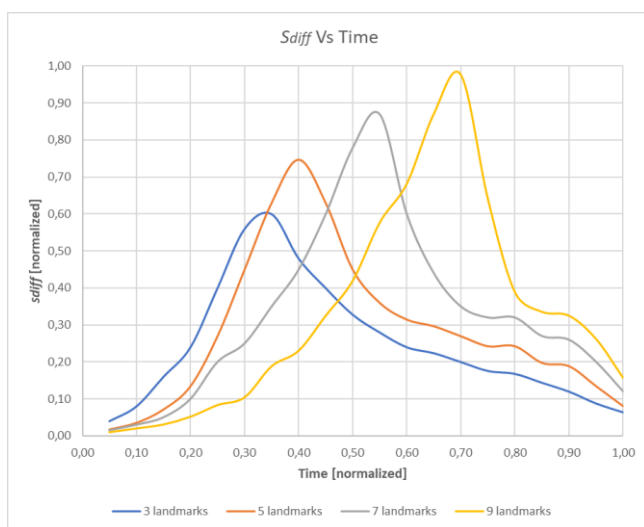


Figure 4. Speed of the landmark vs time: parametric curves with 3, 5, 7 and 9 bases (time and speed are normalized).

where  $\mathbb{R}_{NE}$  is the position of population in the horizontal plane (*northbound-eastbound plane*),  $\chi$  is the population density at time  $t$ ,  $\nabla\chi(\mathbb{R}_{NE}, t)d\mathbb{R}_E d\mathbb{R}_N$  is the probability to find a population with position  $\mathbb{R}_{NE}$  in the area defined by  $d\mathbb{R}_E$  and  $d\mathbb{R}_N$  at  $t$  time,  $f_1(\mathbb{R}_{NE}, t)$  is a direction and describes the deterministic motion, and  $f_2(\mathbb{R}_{NE}, t)$  describes the random component of the motion.

For  $f_1(\mathbb{R}_{NE}, t)$  we consider the definition:

$$f_1(\mathbb{R}_{NE}, t) = \sqrt{1 - \mu(\chi)} \quad (8)$$

where:

$$\mu(\chi) = \frac{\min(\chi, \varphi_r)}{\varphi_r}, \quad (9)$$

in which:  $\varphi_r$  is the maximum expected density in the area defined by  $\mathbb{R}_E$  and  $\mathbb{R}_N$ .

For  $f_2(\mathbb{R}_{NE}, t)$  we consider the definition:

$$f_2(\mathbb{R}_{NE}, t) = \varphi_{a1} + \varphi_{a2} \cdot \mu(\chi)^{\varphi_{a3}} \quad (10)$$

where  $\varphi_{a1}$  is the random factor,  $\varphi_{a2}$  and  $\varphi_{a3}$  are the factors due to the virtual proximity of other people.

Now let us consider a random rectangular area of the surface, of the type:

$$D = [E_a, E_b] \times [N_a, N_b] \quad (11)$$

Of this area, we are going to consider the probability density at time  $t$  by integrating eq. (8)

$$\mathbb{R}_{NE} = \iint_D -\nabla[f_1(\mathbb{R}_{NE}, t) \cdot \chi(\mathbb{R}_{NE}, t)] + \frac{|d_{\min}|}{2} \cdot \nabla^2[(f_2(\mathbb{R}_{NE}, t))^2 \cdot \chi(\mathbb{R}_{NE}, t)] d_N d_E \quad (12)$$

Posing

$$t = t_\gamma, \quad (13)$$

we then have:

$$\mathbb{R}_{NE} = \iint_D -\nabla[f_1(\mathbb{R}_{NE}, t_\gamma) \cdot \chi(\mathbb{R}_{NE}, t_\gamma)] + \frac{|d_{\min}|}{2} \cdot \nabla^2[(f_2(\mathbb{R}_{NE}, t_\gamma))^2 \cdot \chi(\mathbb{R}_{NE}, t_\gamma)] d_N d_E, \quad (14)$$

and expanding:

$$\mathbb{R}_{NE} = \int_{N_a}^{N_b} \left( \int_{E_a}^{E_b} -\nabla[f_1(\mathbb{R}_{NE}, t_\gamma) \cdot \chi(\mathbb{R}_{NE}, t_\gamma)] + \frac{|d_{\min}|}{2} \cdot \nabla^2[(f_2(\mathbb{R}_{NE}, t_\gamma))^2 \cdot \chi(\mathbb{R}_{NE}, t_\gamma)] d_N \right) d_E. \quad (15)$$

The equation (15) thus explained provides us with two things: the position of the people on the horizontal plane (density) but also its evolution over time or, if we look at the expressions of the function as temporal slides, we have the historical development of the city or settlement.

### The "Splashdown" Effect

Town expansion refers to the planned and controlled growth of urban areas, including the increase in physical size, population, and infrastructure. It is a complex process influenced by various

Table 3. Parameters of the simulation.

Parameter	Value
$\varphi_r$	0.0066
$\varphi_{a1}$	0.715
$\varphi_{a2}$	5
$\varphi_{a3}$	0.099

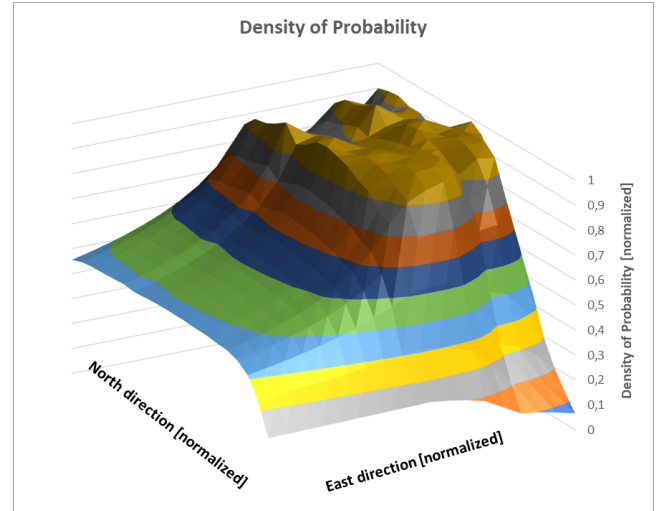


Figure 5. Density of probability vs surface directions.

factors, including population growth, economic development, land-use policies, and urban planning.

Expansion is ideally guided by comprehensive urban planning that considers factors such as land use, transportation, infrastructure, and environmental sustainability. Zoning regulations and master plans help shape the growth in an organized manner, so by further examining the parameters of function (15), to carry out a simulation, we can set the values shown in Table 3.

The simulation results can be seen in Figure 5. It highlights a sort of "splashdown", meaning that diffusion occurs as if it were liquid, i.e. with an ever-increasing density, from the centre towards the edge. This is also an imaginable behaviour since urban agglomerations, unless forced by the environment, rarely tend to separate, or divide into distant islands unless there is a desire to move away. In a nutshell, the density is constant in the centre or the historical centre or first aggregation of the city and then thins out towards the edges.

### 3. CONCLUSIONS

We have examined the route of the Via Severiana, traced in the imperial period, from a historical and technical point of view. Later it has been compared with the Tabula, which can be considered one of the first synthetic representations of general viability. The comparison between the study of the route with the Tabula showed as the difference between them is very small and this demonstrates as the Tabula can be applied even today.

Subsequently, we have examined the precision of the positioning of the various landmarks along the via Severiana and the movement of the inhabited centres as a function of density.

While the Tabula Peutingeriana provides valuable insights into Roman road networks, its limitations in terms of precision and detailed representations of town expansions are inherent to its nature as a medieval copy of an ancient document. We have

used a multidisciplinary approach, combining various sources and technologies, to enhance our understanding of Roman geography and urban development.

Modern technologies, including Geographic Information Systems and mathematical models, allow us to help the archaeologists to overlay ancient maps like the Tabula Peutingeriana onto contemporary maps, aiding in the identification of locations and understanding ancient landscapes.

## REFERENCES

- [1] E. Petritoli, F. Leccese, The *Via Severiana* and its Representation in the *Tabula Peutingeriana*, IMEKO TC4 Int. Conf. on Metrology for Archaeology and Cultural Heritage (MetroArchaeo 2023), Roma, Italy, 19-21 October 2023. DOI: [10.21014/tc4-ARC-2023.177](https://doi.org/10.21014/tc4-ARC-2023.177)
- [2] P. Gautier-Dalché “La trasmissione medievale e rinascimentale della Tabula Peutingeriana,” in *Tabula Peutingeriana. Le Antiche Vie Del Mondo*, a cura di Francesco Prontera. Florence: Leo S. Olschki 2003, pp. 43-52. [In Italian]
- [3] R. J. A. Talbert, R. Unger, *Cartography in Antiquity and the Middle Ages: Fresh Perspectives, New Methods*. BRILL. p. 119. 2008. ISBN 978-90-474-4319-3.
- [4] *Fragmenta Tabulae antiquae in quis aliquot per Rom. Provincias Itinera ex Peutingerorum bibliotheca*. Online [Accessed 26 February 2021] [In Latin] [www.europeana.eu](http://www.europeana.eu)
- [5] E. Albu, *Imperial Geography and the Medieval Peutinger Map*, *Imago Mundi* 57:2005, pp. 136–148.
- [6] E. Petritoli, F. Leccese, M. Botticelli, S. Pizzuti, F. Pieroni, A RAMS analysis for a precision scale-up configuration of the ‘Smart Street’ pilot site: An industry 4.0 case study, *Acta IMEKO*, 8 (2) 2019, pp. 3-11. DOI: [10.21014/acta\\_imeko.v8i2.614](https://doi.org/10.21014/acta_imeko.v8i2.614)
- [7] M. Rathmann, The Tabula Peutingeriana and Antique Cartography, In: *Brill’s Companion to Ancient Geography: The Inhabited World in Greek and Roman Tradition*. Edited by S. Bianchetti, M. R. Cataudella, and H.-J. Gehrke. Leiden, The Netherlands, and Boston, pp. 337–362.
- [8] A. Nibby, *Analisi della carta dei Dintorni di Roma*, Rome 1849, III, p. 610. [In Italian]
- [9] H. Westphal, *Agri Romani Tabula*, Berlin 1829. [In Latin]
- [10] J. Desjardin, *La table de Peutinger*, Paris 1873, p. 241. [In French]
- [11] Th. Ashby, in “*Mélanges École Française de Rome*”, 1905, p. 166. [In French]
- [12] R. Lanciani, *Le antichità del territorio Laurentino nella Reale tenuta di Castel Porziano*, in *Monumenti antichi dei Lincei XIII*, 1903, pp. 133-198. [In Italian]
- [13] P. Bellotti, L. Davoli, C. Tarragoni, L’evoluzione del litorale tiberino negli ultimi 3000 anni sotto le forzanti naturali e antropiche, *Studi costieri*, 22, 2014, pp. 33-43. [In Italian]
- [14] E. Petritoli, F. Leccese, The Geodesic Base of Piombino: The Measurement of Italy under an Historical Point of View, IMEKO TC4 Int. Conf. on Metrology for Archaeology and Cultural Heritage, Consenza, Italy, 19-21 September 2022, pp. 421-425. DOI: [10.21014/tc4-ARC-2022.080](https://doi.org/10.21014/tc4-ARC-2022.080)
- [15] P. Bellotti, L. Davoli, L. Sadori, Landscape diachronic Reconstruction in the Tiber Delta during historical Time: a holistic Approach, *Geogr. Fis. Dinam. Quat.*, 41, 2018, pp. 3-21.
- [16] M. Marcelli, La via Severiana e la c.d. villa di Plinio a Castel Fusano: nuove prospettive di ricerca e valorizzazione, *Geologia dell’Ambiente*, suppl 3/2019 “Alle foci del Tevere: territorio, storia, attualità”, 2019, ISSN 1591-5352. [In Italian]
- [17] L. Brotz, W. W. L. Cheung, K. Kleisner, E. Pakhomov, D. Pauly, Increasing jellyfish populations: trends in large marine ecosystems. *Hydrobiologia*, 690(1) 2012, pp. 3–20.
- [18] F. Bartoli, L. D’Amato, A. Nucera, G. Albani Rocchetti, G. Caneva, Understanding the Lost: Reconstruction of the Garden Design of Villa Peretti Montalto (Rome, Italy) for Urban Valorization. *Plants*, 13(1) 2023, 77. DOI: [10.3390/plants13010077](https://doi.org/10.3390/plants13010077)
- [19] E. Petritoli, F. Leccese, High accuracy attitude and navigation system for an autonomous underwater vehicle (AUV), *Acta IMEKO*, 7 (2) 2018, pp. 3-9. DOI: [10.21014/acta\\_imeko.v7i2.535](https://doi.org/10.21014/acta_imeko.v7i2.535)
- [20] S. Luke, C. Gioffi-Revilla, L. Panait, K. Sullivan. Mason: A new multi-agent simulation toolkit, *Proc. of the 2004 SwarmFest Workshop*, volume 8, 2004.
- [21] M. Vigelius, B. Meyer, G. Pascoe, Multiscale modelling and analysis of collective decision making in swarm robotics, *PloS one*, 9(11):e111542, 2014. DOI: [10.1371/journal.pone.0111542](https://doi.org/10.1371/journal.pone.0111542)
- [22] G. Albani Rocchetti, F. Bartoli, E. Cicinelli, F. Lucchese, G. Caneva, Linking Man and Nature: Relictual Forest Coenosis with *Laurus nobilis* L. and *Celtis australis* L. in Antica Lavinium, Italy. *Sustainability*, 14(1) 2021, 56. DOI: [10.3390/su14010056](https://doi.org/10.3390/su14010056)
- [23] E. Petritoli, M. Cagnetti, F. Leccese, Simulation of autonomous underwater vehicles (AUVs) swarm diffusion (2020) *Sensors (Switzerland)*, 20 (17), art. no. 4950, pp. 1 - 16. DOI: [10.3390/s20174950](https://doi.org/10.3390/s20174950)
- [24] E. Petritoli, F. Leccese, Precise Takagi-Sugeno fuzzy logic system for UAV longitudinal stability: An Industry 4.0 case study for aerospace, *Acta IMEKO*, 9 (4) 2020, pp. 106-113. DOI: [10.21014/acta\\_imeko.v9i4.723](https://doi.org/10.21014/acta_imeko.v9i4.723)
- [25] O. Samoilenko, S. Tsiporenko, Traceability ensuring by organizing the Metrological Measurements Network, *Acta IMEKO*, Vol. 12 No. 4 (2023), pp. . DOI: [10.21014/actaimeko.v12i4.1586](https://doi.org/10.21014/actaimeko.v12i4.1586)