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# Application of the transposition method to uncertainty evaluation of uncorrelated measurements

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ABSTRACT
The application of the transposition method in the processing of indirect uncorrelated measurements with different numbers of observations of input quantities is considered. Expressions for evaluation the estimate of the measurand, as well as the combined standard and expanded uncertainties are obtained. An example of the implementation of the proposed method is considered.

**Section:** RESEARCH PAPER

**Keywords:** Transposition method; uncorrelated measurements; standard and expanded uncertainties

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## 1. INTRODUCTION

Increasing the reliability of measurement uncertainty estimates is an urgent task of metrology. To solve this problem, new data processing methods are being developed and already known methods are being improved [1], [2]. In the book [1], the application of the data enumeration (transposition) method for measurand estimate and uncertainty evaluation of indirect uncorrelated measurements was outlined. It is shown that the transposition method gives an unbiased estimate of the measurand and the most reliable estimate of its type A uncertainty without the need for assumptions about the probability distribution functions (pdf) of the measurement data of the input quantities and without using the Taylor expansion for nonlinear model equation [3], [4] as well as eliminates the "imaginary" correlation between the results of measurements of input quantities associated with a limited number of measurements.

When deriving the formula for the combined standard uncertainty of type A in [1], the authors limited themselves to the simplest case, when the number of observations of all input quantities is the same. In addition, the paper did not consider the issue of estimating the combined standard uncertainty of type B and calculating the expanded uncertainty considering it.

The purpose of this article is to extend the transposition method to indirect uncorrelated measurements with a different number of repeated observations of input quantities under repeatably conditions and with the evaluation of all types of uncertainties, including the expanded one.

## 2. THE ESSENCE OF THE TRANSPOSITION METHOD

The essence of the transposition method is to obtain all possible values of the measured quantity $y_{q,p,\dots,j}$ obtained by transposition of all values $x_{1q}, x_{2p}, \dots, x_{Nj}$ of the input quantities substituted into the measurement equation:

$$y_{q,p,\dots,j} = f\left(x_{1q}, x_{2p}, \dots, x_{Nj}\right), \tag{1}$$

where $q = 1, 2, \dots, n_1$; $p = 1, 2, \dots, n_2$; $j = 1, 2, \dots, n_N$.

For two input quantities with the number of observations $n_1$ and $n_2$ the process of obtaining an array can be represented in the form of Table 1.

Table 1. The transposition method for two input quantities.

| | $x_{21}$ | $x_{22}$ | … | $x_{2n_2}$ |
|---|---|---|---|---|
| $x_{11}$ | $f(x_{11}, x_{21})$ | $f(x_{11}, x_{22})$ | … | $f(x_{11}, x_{2n_2})$ |
| $x_{12}$ | $f(x_{12}, x_{21})$ | $f(x_{12}, x_{22})$ | … | $f(x_{12}, x_{2n_2})$ |
| … | … | … | … | … |
| $x_{1n_1}$ | $f(x_{1n_1}, x_{21})$ | $f(x_{1n_1}, x_{22})$ | … | $f(x_{1n_1}, x_{2n_2})$ |

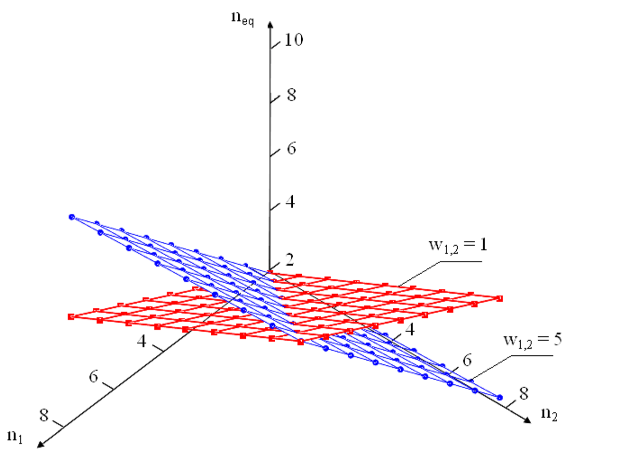


Figure 1. Dependence of the $n_{eq}$ on the ratio $w$ of contributions of input quantities and the number of their observations $n_1$, $n_2$.

The number of values $y_{q,p}$ obtained in this way are defined as the product $n_1 n_2$.

Obviously, with $N$ input quantities, the process of obtaining an array of possible values $y_{q,p,...,j}$ of the measurand will look like a $N$-dimensional table with a total number of cells

$$M = \prod_{i=1}^{N} n_i \, . \tag{2}$$

Based on the resulting array of the measured value, you can get an unbiased estimate of the measurand:

$$\hat{y} = \bar{y} = \frac{1}{M} \cdot \left[ \sum_{j=1}^{n_N} \dots \sum_{q=1}^{n_1} y_{q,p,\dots,j} \right] \tag{3}$$

and the experimental variance of the individual observations $y_{q,p,...,j}$ is given by:

$$s^2\left(y_{q,p,\dots,j}\right) = \frac{1}{M-1} \cdot \left[ \sum_{j=1}^{n_N} \dots \sum_{q=1}^{n_1} \left(y_{q,p,\dots,j} - \bar{y}\right)^2 \right]. \tag{4}$$

Since the estimate of the measurand (3) and the variance estimate (4) were obtained from $M$ estimates of the measurand $y_{q,p,...,j}$, they should have less uncertainty than the estimates, obtained using the traditional method [5].

## 3.THE EQUIVALENT NUMBER OF OBSERVATIONS

To move from the variance of individual observations to the standard uncertainty of type A, it is necessary to determine the equivalent number of observations characterizing the uncertainty of the measurand. The use of the total number of enumeration options for this purpose is incompetent, since it does not characterize the number of actual measurement results, and its use will lead to an underestimation of the uncertainty.

To solve this problem in a general form for the input values, we expand the function (1) in a Taylor series in terms of the degrees of random errors of the input quantities and restrict ourselves to the first terms of the expansion:

$$\Delta_{q,p,\dots,j} = \frac{\partial f}{\partial x_1} \Delta_{1q} + \frac{\partial f}{\partial x_2} \Delta_{2p} + \dots + \frac{\partial f}{\partial x_N} \Delta_{Nj} \, , \tag{5}$$

where $\Delta_{1q}, \Delta_{2p}, \dots \Delta_{Nj}$ are the random errors in determining $y_{1q}, y_{2p}, \dots, y_{Nj}$; $\frac{\partial f}{\partial x_i} = c_i, i = 1, 2, \dots, N$ are the sensitivity coefficients.

Finding the variance of the right and left sides of this equality, we obtain the following equation:

$$s^2\left(y_{q,p,\dots,j}\right) = c_1^2 \, s^2\left(x_{1q}\right) + c_2^2 \, s^2\left(x_{2p}\right) + \dots + c_N^2 \, s^2\left(x_{Nj}\right) . \tag{6}$$

which can be rewritten, expressing for $s\left(y_{q,p,...,j}\right)$ and $s\left(x_{1q}\right)$, $s\left(x_{2p}\right), \dots, s\left(x_{Nj}\right)$ in terms of type A uncertainties of input $u_A(\bar{x}_i), i = 1, 2, \dots, N$ and output $u_A(\bar{y})$ quantities in the following form:

$$u_{\mathrm{A}}^2(\bar{y}) \cdot n_{\mathrm{eq}} = \sum_{i=1}^{N} \left[ c_i^2 \, u_{\mathrm{A}}^2\left(\bar{x}_i\right) \cdot n_i \right] , \tag{7}$$

where $n_i$ is the number of multiple observations that were made when measuring the $i$-th input quantity; $n_{eq}$ is the equivalent number of measurements that would need to be taken to measure the input quantities so that the type A standard uncertainty of the measurand would be the same as that determined by formula (7).

Hence, we have:

$$n_{\mathrm{eq}} = \frac{1}{u_{\mathrm{A}}^2(\bar{y})} \sum_{i=1}^{N} \left[ n_i \, c_i^2 \, u_{\mathrm{A}}^2\left(\bar{x}_i\right) \right] \tag{8}$$

and since from the law of propagation of uncertainty [5]

$$u_{\mathrm{A}}^2(\bar{y}) = \sum_{i=1}^{N} \left[ c_i^2 u_{\mathrm{A}}^2\left(\bar{x}_i\right) \right] \tag{9}$$

then the final expression for the equivalent number of degrees of freedom when using the transposition method will take the form:

$$n_{\mathrm{eq}} = \frac{\sum_{i=1}^{N} \left[ n_i \, c_i^2 \, u_{\mathrm{A}}^2\left(\bar{x}_i\right) \right]}{\sum_{i=1}^{N} \left[ c_i^2 \, u_{\mathrm{A}}^2\left(\bar{x}_i\right) \right]} . \tag{10}$$

For two input quantities, this formula will look like:

$$n_{\mathrm{eq}} = \frac{n_1 \, c_1^2 \, u_A^2(\bar{x}_1) + n_2 \, c_2^2 \, u_A^2(\bar{x}_2)}{c_1^2 \, u_A^2(\bar{x}_1) + c_2^2 \, u_A^2(\bar{x}_2)} = \frac{n_1 \, w + n_2}{w + 1} , \tag{11}$$

where $w$ is the ratio of the contributions of the uncertainty of the first and second input quantities to the uncertainty of the measurand.

Dependence $n_{eq}$ on $w$, $n_1$ and $n_2$ is shown in Figure 1, from which it can be seen that for any $w$, the value $n_{eq}$ is between $n_1$ and $n_2$, and takes on the value $n_{\mathrm{eq}} = n$ at $n_1 = n_2 = n$.

Knowing $n_{\mathrm{eq}}$, one can find the combined standard uncertainty of type A by the formula:

$$u_{\mathrm{A}}(\bar{y}) = \frac{s\left(y_{q,p,\dots,j}\right)}{\sqrt{n_{\mathrm{eq}}}} = \\ = \sqrt{\frac{1}{n_{\mathrm{eq}} \cdot (M-1)} \left[ \sum_{j=1}^{n_N} \dots \sum_{q=1}^{n_1} \left(y_{q,p,\dots,j} - \bar{y}\right)^2 \right]} . \tag{12}$$

For an equal number of observations of all input quantities we have $n_{\mathrm{eq}} = n$ and expression (12) turns into the expression

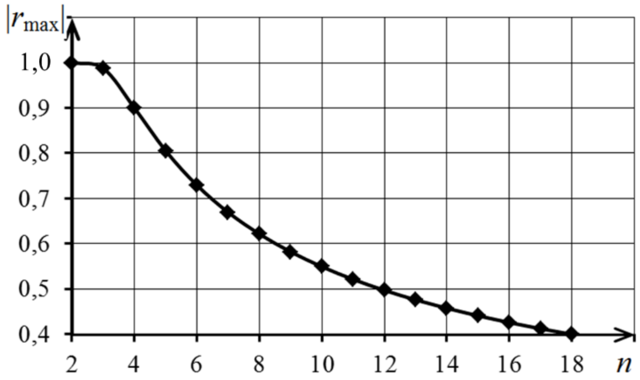


Figure 2. Dependence $|r_{\max}|$ on the number of measurements *n*.

given in [1]

$$u_{\mathrm{A}}(y) = \frac{s\left(y_{q,p,\ldots,j}\right)}{\sqrt{n}}. \tag{13}$$

If necessary, the expanded uncertainty of type A can be determined by the formula:

$$U_{\mathrm{A}}(y) = t_{0.95;(n_{\mathrm{eq}}-1)} \cdot u_{\mathrm{A}}(\bar{y}), \tag{14}$$

where is $t_{0.95;(n_{\mathrm{eq}}-1)}$ the Student's coefficient for the probability of 0.95 and the number of degrees of freedom $(n_{\mathrm{eq}} - 1)$.

## 4. CORRELATION OF DATA OBTAINED BY TRANSPOSITION

When processing data using the traditional method [5], the need to consider the correlation coefficient is determined by the Student's inequality [1]:

$$\frac{|r|}{\sqrt{1-r^2}}\sqrt{n-2} \geq t_{p;(n-2)}\ , \tag{15}$$

where $t_{p;(n-2)}$ is the Student's coefficient for a given confidence level $p$ and the number of degrees of freedom $(n-2)$.

From this expression, one can get the dependence of the maximum absolute value of the "imaginary" (insignificant) correlation coefficient on the number - $n$ of paired observations, for two physically uncorrelated quantities:

$$|r_{\max}| \geq \frac{1}{\sqrt{\dfrac{n-2}{t^2_{p;(n-2)}}+1}}. \tag{16}$$

This dependence is shown in Figure 2 (given $p$ = 0.95), from which it can be seen that even at $n$ = 15, insignificant, pseudo-correlation value can reach 0.45.

When processing data by transposition, the question arises whether the "imaginary" correlation that occurs with a limited number of observations between two sets of data will not affect the estimate of measurement uncertainty.

When data is rearranged in any of the sets, the imaginary correlation may remain (only the correlation coefficient will change and even cross the maximal limit mentioned above), this can significantly affect the assessment of measurement uncertainty when implementing the transposition method.

To prove the lack of correlation between the data taken for the transposition method, we take independently obtained results of measuring two quantities: $x_{11}, x_{12}, \ldots, x_{1n}$; $x_{21}, x_{22}, \ldots, x_{2n}$. Then the covariance for all data used in the implementation of the transposition method will be determined by the formula:

$$\begin{aligned}K = \frac{1}{n^2-1}&[(x_{11}-\bar{x}_1)(x_{21}-\bar{x}_2)+(x_{12}-\bar{x}_1)(x_{21}-\bar{x}_2)+\\ &+\ldots+(x_{1n}-\bar{x}_1)(x_{21}-\bar{x}_2)+(x_{11}-\bar{x}_1)(x_{22}-\bar{x}_2)+\\ &+(x_{12}-\bar{x}_1)(x_{22}-\bar{x}_2)+\ldots+(x_{1n}-\bar{x}_1)(x_{22}-\bar{x}_2)+\\ &+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+\\ &+(x_{11}-\bar{x}_1)(x_{2n}-\bar{x}_2)+(x_{12}-\bar{x}_1)(x_{2n}-\bar{x}_2)+\ldots+\\ &+(x_{1n}-\bar{x}_1)(x_{2n}-\bar{x}_2)].\end{aligned}$$

It is easy to see that this expression can be transformed into the following:

$$\begin{aligned}K = \frac{1}{n^2-1}&\{(x_{21}-\bar{x}_2)[(x_{11}-\bar{x}_1)+(x_{12}-\bar{x}_1)+\ldots+\\ &+(x_{1n}-\bar{x}_1)]+(x_{22}-\bar{x}_2)[(x_{11}-\bar{x}_1)+(x_{12}-\bar{x}_1)+\ldots+\\ &+(x_{1n}-\bar{x}_1)+\ldots\ldots\ldots\ldots\ldots\ldots+\\ &+(x_{2n}-\bar{x}_2)[(x_{11}-\bar{x}_1)+(x_{12}-\bar{x}_1)+\ldots+(x_{1n}-\bar{x}_1)],\end{aligned}$$

from where one can finally get:

$$\begin{aligned}K = \frac{1}{n^2-1}&[(x_{21}-\bar{x}_2)+(x_{22}-\bar{x}_2)+\ldots+(x_{2n}-\bar{x}_2)]\times\\ &\times[(x_{11}-\bar{x}_1)+(x_{12}-\bar{x}_1)+\ldots+(x_{1n}-\bar{x}_1)].\end{aligned}$$

As can be seen from the last expression, both the first and second factors of its right side are the sum of the deviations of the observational results of each input quantity from their average values. Therefore, each of them is equal to zero, i.e. the correlation moment (and the correlation coefficient) for the data taken to implement the transposition method will also be equal to zero.

## 5. EXPANDED UNCERTAINTY EVALUATION

Expanded uncertainty evaluation for data obtained by transposition, can be done using several approaches.

The first approach corresponds to the basic uncertainty estimation algorithm [5]. To find the total standard uncertainty of type B, one should use the law of propagation of uncertainty:

$$u^2_{\mathrm{B}}(y) = \sum_{i=1}^{N} c_i^2\, u^2_{\mathrm{B}}(x_i)\ , \tag{17}$$

where $c_i = \frac{\partial f}{\partial x_i}$ are the sensitivity coefficients, and $u_{\mathrm{B}}(x_i)$ are the B-type standard uncertainties of the input quantities.

Then the combined standard uncertainty of indirect uncorrelated measurements will be equal to:

$$u_{\mathrm{c}}(y) = \sqrt{u^2_{\mathrm{A}}(\bar{y}) + u^2_{\mathrm{B}}(y)}, \tag{18}$$

and the expanded uncertainty is determined by the expression:

$$U(y) = t_{0.95;\nu_{\mathrm{eff}}}\, u_{\mathrm{c}}(y), \tag{19}$$

where $t_{0.95;\nu_{\mathrm{eff}}}$ is the Student's coefficient for the probability of 0.95 and the effective number of degrees of freedom, determined by the Welch–Satterthwaite equation, which for this case will look like:

$$\nu_{\mathrm{eff}} = \frac{u^4_{\mathrm{c}}(y)}{[u^4_{\mathrm{A}}(\bar{y})/(n_{\mathrm{eq}}-1)] + [u^4_{\mathrm{B}}(y)/\nu_{\mathrm{B}}]}, \tag{20}$$

where $\nu_B$ is number degrees of freedom assigned to $u_{\mathrm{B}}(y)$.

Table 2. Student's coefficient values for fractional degrees of freedom.

| $\nu$ | $t_{0.95}(\nu)$ | $\nu$ | $t_{0.95}(\nu)$ | $\nu$ | $t_{0.95}(\nu)$ |
|---|---|---|---|---|---|
| **1.0** | **12.706** | 2.9 | 3.245 | 5.1 | 2.556 |
| 1.1 | 10.277 | **3.0** | **3.182** | 5.2 | 2.541 |
| 1.2 | 8.649 | 3.1 | 3.125 | 5.3 | 2.527 |
| 1.3 | 7.501 | 3.2 | 3.073 | 5.4 | 2.514 |
| 1.4 | 6.657 | 3.3 | 3.025 | 5.5 | 2.502 |
| 1.5 | 6.017 | 3.4 | 2.981 | 5.6 | 2.490 |
| 1.6 | 5.517 | 3.5 | 2.940 | 5.7 | 2.478 |
| 1.7 | 5.119 | 3.6 | 2.902 | 5.8 | 2.468 |
| 1.8 | 4.795 | 3.7 | 2.868 | 5.9 | 2.457 |
| 1.9 | 4.527 | 3.8 | 2.835 | **6.0** | **2.447** |
| **2.0** | **4.303** | 3.9 | 2.805 | 6.1 | 2.437 |
| 2.1 | 4.112 | **4.0** | **2.776** | 6.2 | 2.428 |
| 2.2 | 3.949 | 4.4 | 2.750 | 6.3 | 2.419 |
| 2.3 | 3.807 | 4.5 | 2.725 | 6.4 | 2.410 |
| 2.4 | 3.684 | 4.6 | 2.702 | 6.5 | 2.402 |
| 2.5 | 3.575 | 4.7 | 2.621 | 6.6 | 2.394 |
| 2.6 | 3.478 | 4.8 | 2.603 | 6.7 | 2.386 |
| 2.7 | 3.392 | 4.9 | 2.586 | 6.8 | 2.379 |
| 2.8 | 3.315 | 5.0 | 2.571 | 6.9 | 2.372 |

Since $\nu_{\text{eff}}$, as well as $n_{\text{eq}}$, can be fractional, it is recommended to determine the values of the Student's coefficient according to Table 2 obtained in the article [6].

The second approach is to use the expanded uncertainty propagation law described in [7]:

$$U_{\text{c}}(y) = \sqrt{U_{\text{A}}^2(y) + U_{\text{B}}^2(y)}, \tag{21}$$

where the expanded uncertainty of type A, $U_A(y)$, is found by the formula (14) and the expanded uncertainty of type B, $U_B(y)$, is found by the formula:

$$U_{\text{B}}(y) = k_{0.95} \cdot u_{\text{B}}(y) \tag{22}$$

where the coverage factor $k_{0.95}$ is convenient to find using the kurtosis method [8].

The expediency of applying this or that approach should be determined for the specific conditions of their application (model equation, the number of results of observations of each input quantity, etc.).

## 6. EXAMPLE. MEASURING THE RESISTANCE OF A CIRCUIT SECTION USING THE AMMETER-VOLTMETER METHOD

The electrical resistance of a circuit section is measured at direct current using the ammeter-voltmeter method. In this case, the resistance is determined by Ohm's law.

To determine the performance of the proposed method, a numerical experiment was carried out.

1. Two arrays of random numbers $V, I$, of cardinality $M = 10^6$ each were generated, normally distributed with characteristics $\mu(V)$ =1 mV, $\mu(I)$ =1 mA, and $\sigma(V)$ =0,05 mV, $\sigma(I)$ =0,05 mA.

2. From the resulting arrays, two random samples of size 7 (for voltage) and 5 (for current) were drawn, as presented in Table. 3.

For these values $\bar{V}$, $\bar{I}$ were calculated according to the formulas:

Table 3. Results of electrical voltage and current measuring.

| No. | ***Voltage*** **(mV)** | ***Current*** **(mA)** |
|---|---|---|
| 1 | 0.91289 | 0.90751 |
| 2 | 0.97870 | 1.04488 |
| 3 | 0.91655 | 0.97574 |
| 4 | 0.95428 | 0.99026 |
| 5 | 0.94955 | 0.93823 |
| 6 | 1.02396 | – |
| 7 | 0.99644 | – |

$$\bar{V} = \frac{1}{7}\sum_{q=1}^{7} V_q = 0.961768 \text{ mV},$$

$$\bar{I} = \frac{1}{5}\sum_{q=1}^{5} I_q = 0.971322 \text{ mA},$$

as well as standard uncertainties of type A $u_A(\bar{V})$, $u_A(\bar{I})$ according to the formulas:

$$u_{\text{A}}(\bar{V}) = \sqrt{\frac{1}{7 \cdot 6}\sum_{q=1}^{7}(V_{\text{q}} - \bar{V})^2} = 0.015427 \text{ mV},$$

$$u_{\text{A}}(\bar{I}) = \sqrt{\frac{1}{5 \cdot 4}\sum_{q=1}^{5}(I_{\text{q}} - \bar{I})^2} = 0.023405 \text{ mA}.$$

Using these values, the following characteristics were calculated [5]:

$$R = \frac{\bar{V}}{\bar{I}} = 0.990164\ \Omega,$$

$$c_{\text{V}} = \frac{\partial R}{\partial V} = \frac{1}{\bar{I}} = 1.029525 \text{ mA}^{-1},$$

$$c_{\text{I}} = \frac{\partial R}{\partial I} = \frac{-\bar{V}}{\bar{I}^2} = -1.019546 \frac{\text{mV}}{\text{mA}^2},$$

$$u_{\text{V}}(R) = c_{\text{V}} \cdot u_{\text{A}}(\bar{V}) = 0.015883\ \Omega,$$

$$u_{\text{I}}(R) = c_{\text{I}} \cdot u_{\text{A}}(\bar{I}) = -0.023862\ \Omega,$$

$$u_{\text{c}}(R) = \sqrt{u_{\text{V}}^2(R) + u_{\text{I}}^2(R)} = 0.028665\ \Omega,$$

$$\nu_{\text{eff}} = \frac{u_{\text{c}}^4(R)}{\frac{u_{\text{V}}^4(R)}{\nu_{\text{V}}} + \frac{u_{\text{I}}^4(R)}{\nu_{\text{I}}}} = 7.37,$$

$$k = t_{\nu_{\text{eff};0.95}} = 2.34,$$

$$U(R) = k \cdot u_{\text{c}}(R) = 0.067\ \Omega.$$

The obtained values are listed in Table 4 representing the uncertainty budget of the resistance measurement.

According to the measurement results given in Table 4, by applying transposition method data about $R_{q,p} = V_q/I_p$ ($q$ =1, 2, …, 7; $p$ = 1, 2, ..., 5) given in Table 5 were obtained.

Table 4. Uncertainty budget for resistance measurement.

| $X_i$ | $x_i$ | $u(x_i)$ | $\nu_i$ | $c_i$ | $u_i(y)$ |
|---|---|---|---|---|---|
| $V$ | 0.961768 mV | 0.015427 mV | 6 | 1.029525 mA$^{-1}$ | 0.015883 Ω |
| $I$ | 0.971322 mA | 0.023405 mA | 4 | -1.019546 mV/mA$^2$ | -0.023862 Ω |
| $Y$ | $y$ | $u_c(y)$ | $\nu_{\mathrm{eff}}$ | $k$ | $U$ |
| $R$ | 0.990164 Ω | 0.028665 Ω | 7.37 | 2.43 | 0.070 Ω |

Table 5. Resistance measurement results obtained by the transposition method. The results are expressed in Ohm.

| *Voltage* (mV) | *Current* (mA) | | | | |
|---|---|---|---|---|---|
| | **0.90751** | **1.04487** | **0.97573** | **0.99026** | **0.93823** |
| **0.91289** | 1.005935 | 0.873687 | 0.935596 | 0.921869 | 0.972995 |
| **0.97870** | 1.078446 | 0.936665 | 1.003036 | 0.98832 | 1.043131 |
| **0.91655** | 1.009959 | 0.877181 | 0.939338 | 0.925556 | 0.976887 |
| **0.95428** | 1.051545 | 0.913300 | 0.978015 | 0.963666 | 1.017111 |
| **0.94955** | 1.046331 | 0.908772 | 0.973166 | 0.958888 | 1.012068 |
| **1.02396** | 1.128327 | 0.979988 | 1.049429 | 1.034032 | 1.091379 |
| **0.99644** | 1.097995 | 0.953644 | 1.021218 | 1.006235 | 1.06204 |

According to Table 5

$$\bar{R} = \frac{1}{n_{\mathrm{V}} \cdot n_{\mathrm{I}}} \sum_{q=1}^{n_{\mathrm{V}}} \sum_{p=1}^{n_{\mathrm{I}}} R_{q,p} = 0.992450\ \Omega\,,$$

$$n_{\mathrm{eq}} = \frac{n_{\mathrm{V}}\, u_{\mathrm{V}}^2(R) + n_{\mathrm{I}}\, u_{\mathrm{I}}^2(R)}{u_{\mathrm{V}}^2(R) + u_{\mathrm{I}}^2(R)} = 5.61\,,$$

$$u_{\mathrm{c}}(\bar{R}) = \sqrt{\frac{1}{n_{\mathrm{eq}} \cdot (n_{\mathrm{V}}\, n_{\mathrm{I}} - 1)} \sum_{q=1}^{n_{\mathrm{V}}} \sum_{p=1}^{n_{\mathrm{I}}} (R_{q,p} - \bar{R})^2} = 0.0263\ \Omega\,,$$

$$k = t_{5.6;0.95} = 2.49\,,$$

$$U(R) = k \cdot u_c(R) = 0.065\ \Omega.$$

Thus, the unbiased estimate of the value of the measurand obtained by the transposition method eliminates the bias of the estimate of the measurand obtained by the GUM method, equal to -0.002286 **Ω**, and also reduces the type A standard uncertainty of the measured quantity from 0.028665 **Ω** to 0.026326 **Ω** and the type A expanded uncertainty of the measured quantity from 0.067 **Ω** to 0.065 **Ω**.

At the same time, the estimates of the coverage factors obtained by these methods for a probability of 0.95 differ significantly, which once again confirms the unreliability of calculating the coverage factors in terms effective number of degrees of freedom using the Welch–Satterthwaite equation [9], [10].

Unfortunately, verification of these results by the Monte Carlo method [2] will not give comparable results, since it is based on the Bayesian approach to measurement uncertainty evaluation and the type A uncertainty estimates obtained using it differ significantly from the same estimates obtained using the GUM [5].

## 7. CONCLUSIONS

The use of the transposition method improves the reliability of the result of indirect uncorrelated measurements and the reliability of the assessment of its uncertainty.

The formula for calculating the equivalent number of measurements $n_{\mathrm{eq}}$ for different numbers of observations of the input values is obtained. The analysis of this formula showed that for any ratio between the contributions of uncertainty, the value $n_{\mathrm{eq}}$ is between the minimum and maximum number of observations of input values.

It has been proven that when using the transposition method, the imaginary correlation between the initial results of observations of the input quantities does not affect the data array of possible values of the measurand obtained by the transposition method, so it can be used to process the results of indirect measurements without fear of the consequences of not taking into account the imaginary correlation between the input quantities.

Various methods for evaluation the expanded uncertainty of measurements based on the results obtained by the transposition method are proposed.

An example of measuring the resistance of a circuit section using an ammeter-voltmeter method is considered, which demonstrated the advantages of the transposition method compared to the GUM framework in terms of obtaining an unbiased estimate of the measurement result and reducing the value of the standard uncertainty of type A.

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