

## ERROR SOURCES IN THE FORCE MODE OF THE “PB2” PLANCK-BALANCE

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### Abstract:

The PB2 Planck-Balance is a table-top Kibble balance, that is designed for the calibration of class E<sub>2</sub> weights in a range of 1 mg up to 100 g. This work presents typical systematic errors which have to be considered during the calibration and will show results for measurements with small masses.

**Keywords:** Kibble balance; measurement uncertainty; mass calibration; instrument adjustment

### 1. INTRODUCTION

Kibble balance experiments were originally designed for the determination of Planck constant and played an important role in the redefinition of the SI unit kilogram, which entered into force in May 2019. Today, they represent one of the two methods to achieve the lowest uncertainties for mass calibrations of 1 kg. Nonetheless, the working principle of a Kibble balance can also be used in smaller, less expensive and less complex table-top set-ups, when the user does not need to achieve higher accuracy. In the Planck-Balance (PB) project, two systems were developed in order to calibrate class E<sub>1</sub> and E<sub>2</sub> weights, where the system for the calibration of E<sub>2</sub> is named “PB2” and is depicted in Figure 1.



Figure 1: The PB2 set-up: balance and interferometer (right) and signal processing system (left).

The calibration of weights is performed in two modes: velocity mode and force mode. In the velocity mode, the coil under test is moved sinusoidally relative to the magnetic field of the actuator with a frequency of typically 4 Hz and an amplitude of 20 μm. From the ratio of coil velocity  $v$  and induced voltage  $U_{\text{ind}}$  the geometric factor  $Bl$ , which is the product of the magnetic flux density  $B$  and the wire length  $l$ , can be determined. This factor can also be derived from the ratio of the gravitational force  $F_g$  from a weight and the compensation current  $I$  that is necessary to counterbalance the weight with an equal force generated by the actuator (electromagnetic force), which corresponds to the calibration with the use of a calibrated mass (force mode). This procedure of force equilibration is similar to the common calibration process of an electromagnetic force compensation (EMFC) balance. In a Kibble balance, both modes are combined in order to determine the mass of an unknown weight. However, this work that focuses on errors sources in the force mode, the results of the force mode measurements are presented as determinations of  $Bl$  in order to be less dependent on the results of the velocity mode and to differentiate between the errors that occur in the different modes.

Similar to the existing EMFC balances, the alignment along the gravitational acceleration  $g$  and air buoyancy has to be considered in the measurement during the force mode. Due to this different calibration method compared to a calibration of the compensating actuator with mass standards, additional effects like the alignment along velocity mode trajectory, coil current effect and coil displacement need to be taken into account.

### 2. MEASUREMENT SET-UP

The PB2 system [1] consists of a commercially available EMFC balance, an interferometer, a resistor and two digital multimeters (DMMs) of type Keysight 3458A-002. The DMMs are regularly calibrated against a programmable Josephson

voltage standard that is available in PB2 laboratory, while the shunt resistor of type Vishay VHA-518 was calibrated at Physikalisch-Technische Bundesanstalt (PTB). All these individual components form the PB2 system and its measurement infrastructure are based on a modular principle, so that upon necessity the procedures of replacements or the integration of additional devices are maximally simplified. For the purpose of characterisation of the system components, additional multimeters and current sources are available in the set-up that is shown in Figure 1.

An additional voice coil actuator (measurement actuator) that is attached to the load carrier of the EMFC balance forming a mechanically rigid connection generates the compensation force in the force mode and is characterised in the velocity mode. The internal actuator (drive actuator) is utilised to excite the coil of the measurement actuator in the velocity mode and is connected to a current source of type HP 3245 in force mode in order to produce a constant force offset that is necessary for the reduction of the coil current effect that is described in section 3. Furthermore, a mirror is attached to the load carrier, similarly forming a mechanically rigid connection, in order to determine the displacement that is used to calculate the velocity of the mirror and the measurement coil. For the sake of an accurate velocity and voltage measurement, a stable time base is required. The acquisition of voltage and displacement is triggered by a signal generator which is supplied with traceable time base by a GPS-disciplined oven-stabilised quartz oscillator.

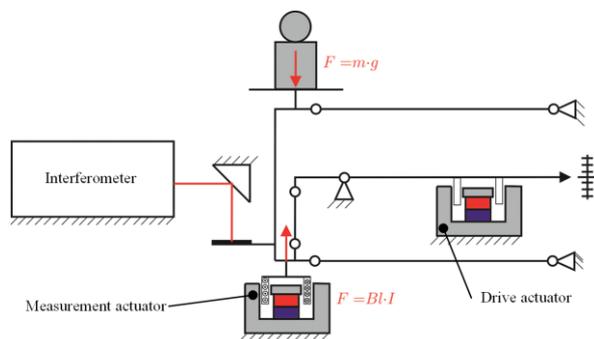


Figure 2: Schematic of the PB2 set-up.

In contrast to a normal usage of an EMFC balance, the compensation force is generated by an actuator placed on the same side of the lever as the mass under test as shown in Figure 2. In the PB2 set-up, this mass can be lifted and lowered remotely by an automated weight lifter to minimise disturbances caused by an operator of the device. Furthermore, a vacuum compatible piezo-based linear motor of type N-310 by Physik Instrumente is utilized in the weight lifter system that allows the PB2 system to operate in vacuum. Due to its working principle of a walking motion of alternately deflected piezo actuators, it does only

generate mechanical noise during the active lifting and lowering process and not during the measurement when a compensation current is applied. Additionally, it does not generate magnetic fields that could disturb the measurement actuator.

The weighing pan is designed as a fork structure as shown in Figure 3, while a fitting fork structure is used to lift the weight under test. The weight lifter is placed on a plate that can be adjusted in six degrees of freedom to allow a small gap between the forks without the risk of contact between them. This weight lifter design enables the investigation of a wide range of sizes and shapes of the mass standards including wire weights.

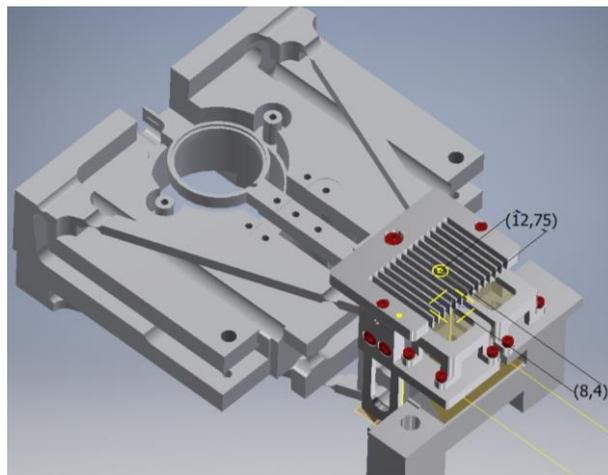


Figure 3: CAD model of the weighing cell mechanics and the weighing pan with fork structure.

The compensation current in the force mode is provided by a PID controller that is implemented on a digital signal processing system (DSP). The DSP is augmented by a power amplifier that drives the coil current of the measurement actuator in the force mode and a transimpedance amplifier circuit that converts the photo currents of the balance's internal position sensor into a voltage signal that is utilized to indicate the successful compensation of the external force. Since the DSP that is currently used, does not allow the use of an external clock signal, the closed loop frequency of the implemented control algorithm is measured by a frequency counter that is connected to the reference clock. This provides a traceable time base that is crucial for the velocity mode.

Since the measurements with the PB2 system are mostly performed in air, the air buoyancy of the weight needs to be corrected as described in OIML R 111-1. For this purpose, the density of the air has to be determined, which requires the measurement of air temperature, humidity and pressure. For this purpose, a calibrated climate module of type Sartorius YCM20MC is included in the PB2 system.

### 3. COIL CURRENT EFFECT

The necessary compensation current in the force mode is determined as the difference of the compensation currents of the loaded and the unloaded state of the balance. Since the coil current affects the magnetic field of the actuator, this current difference depends on the absolute values of both compensation currents [2]. In order to decrease the error that is caused by the coil current effect, the lever of the EMFC is balanced such, that the compensation currents in both loaded and unloaded states yield the same absolute value, but different polarity. This is ensured in the PB2 set-up by feeding a constant current  $I_i$  through the internal actuator of the EMFC balance. Therefore, the balancing of the lever can be done quickly and automatically for any arbitrary value of the test mass. To adapt this internal offset current  $I_i$  to the changing  $Bl$  of the internal drive actuator, an adjustment procedure is performed regularly. A small current  $\Delta I_i$  is added to the previously used offset current  $I_{i0}$  and for both currents through the drive actuator, the differences  $\Delta I_m$  of the absolute values of the necessary compensation current in loaded ( $I_{m+}$ ) and unloaded ( $I_{m-}$ ) state are determined:

$$\Delta I_m = |I_{m+}| - |I_{m-}|. \quad (1)$$

From these measurements the new internal current  $I_i$  is calculated by linear interpolation to obtain a vanishing  $\Delta I_m$ :

$$I_i = I_{i0} + \frac{\Delta I_m(I_{i0}) \cdot \Delta I_i}{\Delta I_m(I_{i0}) - \Delta I_m(I_{i0} + \Delta I_i)}. \quad (2)$$

The procedure is repeated before a new set of force mode measurements is taken to compensate for the temperature sensitivity and the general drift of the drive actuator.

### 4. MAGNETIC PROFILE

For the calibration of mass standards with a Kibble balance, the  $Bl$  that is determined in velocity mode is used to convert the current difference in loaded and unloaded state into a gravitational force. To achieve a low combined uncertainty, it needs to be ensured that the position of the coil in loaded and unloaded state is the same as the position for which the  $Bl$  was determined in the velocity mode. Assuming a given uncertainty of the coil positioning, the error that is caused by this effect can be reduced by adjusting the coil position to be in apex of the magnetic flux density of the magnet system. The current PB2 system does not provide the possibility to adjust the immersion depth of the coil and relies on the manufacturing tolerances of the voice coil actuator and the coil mounting. Therefore, the actual profile was acquired experimentally by determining the geometric factor  $Bl$  at different coil positions.

As shown in Figure 4, the apex was not exactly met and the  $Bl$  changes with approximately 0.5 ppm/ $\mu\text{m}$ , which is sufficient for a comparison of the  $Bl$  determined in force and velocity mode respectively, since the repeatability of the coil positioning is in the order of nanometres. The shown profile was determined in force mode while changing the set point position of the controller. To reduce a change of the profile due to the coil current effect, the symmetry of compensation currents was adjusted for each position considering the different restoring force of the differently displaced balance mechanics.

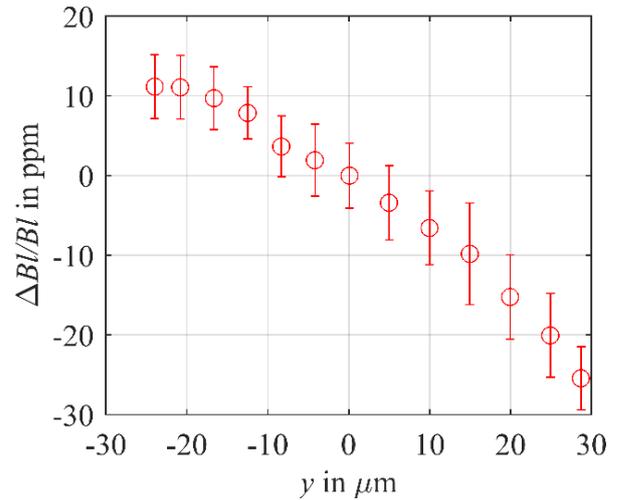


Figure 4: Relative change of  $Bl$  depending on coil position.

### 5. ALIGNMENT

In contrast to the calibration of a standard EMFC balance, the alignment error cannot be compensated by having the same angle of misalignment during the calibration with mass standard and weighing process. Therefore, the force measurement system needs to be aligned parallel to the vector of free-fall acceleration  $g$ . For alignment purposes, mirror cuboids that possess a reflective coating on all sides are attached to the load carrier of the balance (Figure 5). The angles between the surfaces of these optical alignment cuboids are measured and well known. In accordance to the calibration certificate they agree better than 21.8  $\mu\text{rad}$  with a right angle.

During the alignment process, a second interferometer is used to measure the orthogonal displacement of the side of the mirror cuboid, thus measuring the horizontal displacement, while the load carrier is driven up and down by an excitation of the lever. The interferometer beams need to be aligned to better than 0.4 mrad so that the interferometers work properly. From the ratio of horizontal to vertical displacement measured by the first interferometer, the trajectory of the guiding parallel lever system and its misalignment can be identified.

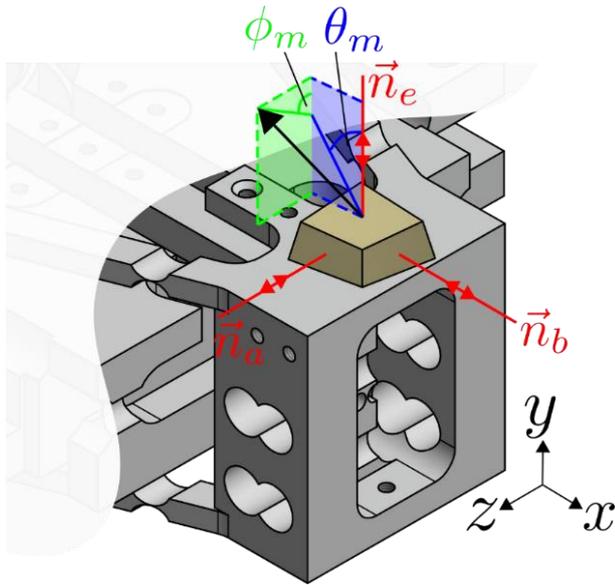


Figure 5: Schematic of the alignment axes with an alignment cuboid.

A measurement of the horizontal displacement is shown in Figure 6, which sums up a set of six repeated measurements of both horizontal axes in  $x$ - and  $z$ -direction. The shown horizontal displacements correspond to an angular misalignment of 2.8 mrad and 2.2 mrad, respectively. The resulting cosine errors of 3.9 ppm and 2.4 ppm of the determination of the displacement amplitude are therefore well known and can be corrected during the data analysis. This procedure needs to be repeated every time the angle between the trajectory of the guided coil and the normal vector changes, which occurs when the balance or the mirrors are temporarily unscrewed during redesign and readjustment of the system. Since this is done regularly in the recent devolvement process of PB2, there are no investigations on the long-term stability of these angles yet.

An additional horizontal displacement of about maximum 6 nm occurs due to the parallel lever system guides the load carrier on a circular trajectory. The horizontal velocity caused by this effect, which generates an additional induced voltage, occurs mainly at the maximum and minimum of the sinusoidal motion and is close to zero when the vertical velocity is at its maximum and minimum. Therefore, the voltage that is induced due to this horizontal motion contributes only a small error to the determination of the amplitude of the sinusoidal induced voltage caused by vertical motion and is rejected by the sine fitting of the data.

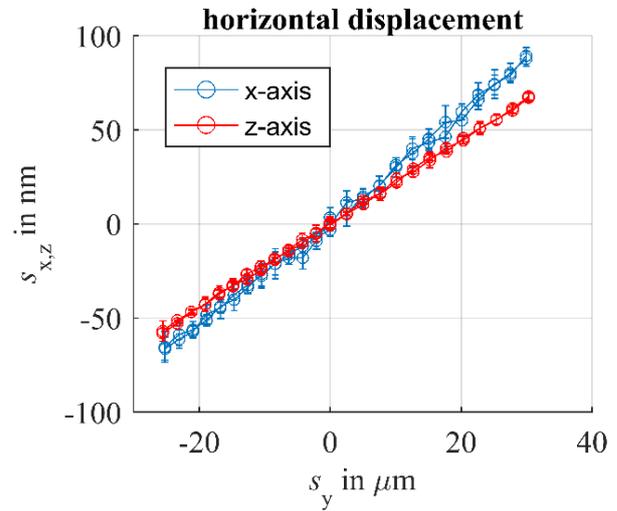


Figure 6: Horizontal displacement of the load carrier.

With an autocollimator and the reflective surface of a liquid pool, the normal vector of the top surface of the mirror cuboid can be adjusted relative to  $g$ . The water pool is only present during the adjustment procedure and regular checks of the mirror orientation. Since the orientation of the trajectory relative to the mirror surface is known from the horizontal displacement, the trajectory adjustment follows from the adjustment of the top surface.

## 6. RESULTS

The determination of  $Bl$  in force and velocity mode was performed with several different  $E_1$  mass standards in a range from 1 mg to 50 g. The systematic deviation relative to the result with a calibrated 20 g standard as well as the statistical uncertainty over time period of 8 hours are shown in Figure 7. The statistical uncertainty is composed of the relative experimental standard deviations of the mean of the determinations of  $Bl$  in velocity mode and force mode. While the contribution to the statistical uncertainty of the velocity mode experiment is independent from the weight under test, the relative standard deviation increases for smaller mass values. The maximum permissible error (MPE) of  $E_1$  and  $E_2$  standard weights according to OIML R-111 is also shown for comparison.

Compared to the MPE of  $E_2$  standards, the results are quite satisfying for the higher nominal values. However, for the smallest standards, there are deviations that need to be identified in future investigations.

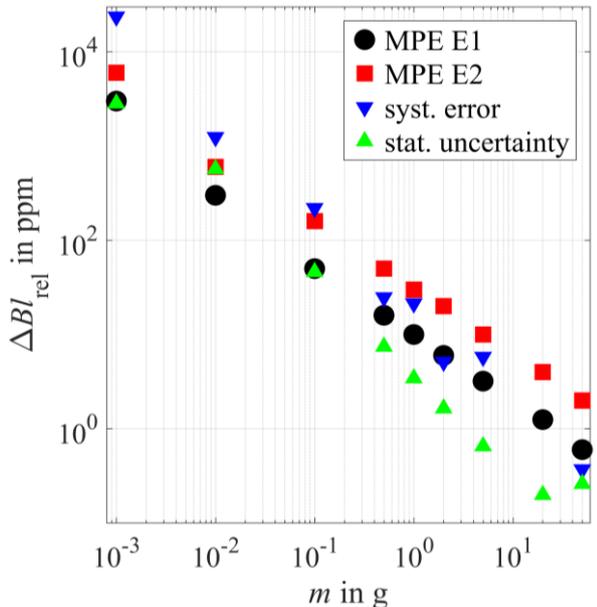


Figure 7: Deviation between force and velocity mode.

## 7. SUMMARY AND OUTLOOK

The main error sources in the force mode of the PB2 system were identified and a good agreement of force and velocity mode was achieved for most of the measurement range. Future work will aim at identifying the deviations in small mass range and improving the statistical part of the uncertainty. This can be achieved by improving the controller hardware that is generating a significant fraction of the noise on the compensation current due to its resolution of 16 bit for ADC and DAC. A signal processing system based on a combination of

microprocessors, a FPGA chip and 24 bit Delta-sigma ADC is currently under development.

In the range of masses greater than 50 g, the system performance is limited by nonlinear deformation phenomena that are subject to ongoing research. Further investigations are also planned regarding the long-term stability of the mechanical system and its adjustment status.

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