



PLANCK-BALANCE 1 (PB1) – A TABLE-TOP KIBBLE BALANCE FOR MASSES FROM 1 mg TO 1 kg – CURRENT STATUS

C. Rothleitner¹, N. Rogge², S. Lin¹, S. Vasilyan², D. Knopf¹, F. Härtig¹, T. Fröhlich²

¹ Physikalisch-Technische Bundesanstalt (PTB), 38116 Braunschweig, Germany, christian.rothleitner@ptb.de

² Technische Universität Ilmenau, 98693 Ilmenau, Germany, norbert.rogge@tu-ilmenau.de

Abstract:

The Planck-Balance 1 (PB1) has been developed in a collaboration between the Physikalisch-Technische Bundesanstalt and the Technische Universität Ilmenau. It is a small-size Kibble balance aimed to calibrate E_1 mass standards for a mass range from 1 mg to 1 kg. This paper presents its principle and some preliminary investigations.

Keywords: Kibble balance; mass metrology; Planck-Balance; kilogram realisation; kilogram dissemination

1. INTRODUCTION

In mass metrology mass standards had been traced to the International Prototype Kilogram (IPK) for more than 130 years. This was done by comparing masses and building up the mass scale. After the re-definition of the SI unit kilogram this way of calibration can be continued. What has changed is that the IPK has been replaced by a definition based on the Planck constant and a corresponding realisation of the kilogram, e.g., a Silicon-28-Sphere [1] or a Kibble balance [2], up to now also at a nominal value of 1 kg. The redefinition, however, gives more freedom in mass metrology. Since the Planck constant is valid for any mass value the Kibble balance technology can be used to calibrate mass standards that differ from 1 kg too. Several national metrology institutes are working on such a balance **Fehler! Verweisquelle konnte nicht gefunden werden.** [4], which is sometimes called “table-top Kibble balance” or “micro Kibble balance (μ KB)”.

The Physikalisch-Technische Bundesanstalt (PTB) is developing, in collaboration with the Technische Universität Ilmenau (TUIL), two table-top sized Kibble balances, called Planck-Balance 1 (PB1) and Planck-Balance 2 (PB2), for industrial applications. Details on the PB2 have already been published elsewhere [5]. In this paper the concept

along with some technical details of PB1 will be described.

2. CONCEPT OF PLANCK-BALANCE 1

PB1 is a Kibble balance that works in two modes, as originally proposed by Bryan Kibble (see [2]). One mode, called *force mode*, where the weight of a mass under test is compared to an electromagnetic force, produced by a voice coil system. And a second mode, called *velocity mode*, where the force factor (see below) is determined. The difference of the Planck-Balance, however, with respect to its high-precision counterparts, lies in the field of application. The aimed use of PB is by industry, calibration laboratories, but also national metrology institutes that are interested in a system for the primary realisation of the new SI kilogram but cannot afford or maintain a highly sophisticated self-developed Kibble balance. Therefore, the PB1 is aimed to meet the following specifications:

- Use for mass calibrations over a mass range from 1 mg to 1 kg.
- Relative measurement uncertainties according to E_1 MPE¹ specified in OIML R 111-1 ($\rightarrow 8.4 \times 10^{-8}$ ($k=1$) @ 1 kg).
- Compact and easy to use, i.e. the mechanical part should be of comparable size to a conventional analytical mass balance. The use of the balance should be possible by staff members that do not necessarily have profound knowledge in electrical metrology.
- Modular design: parts should be easily exchangeable or adaptable in order to reach the required specifications. As an example: the load cell should be exchangeable if another nominal mass value is desired. This allows for using the, basically, same setup for the whole mass range.

As a basis a commercial load cell (Sartorius Secura 1103-1x, see Figure 1) is used, which can be found in high-precision analytical mass balances.

¹ MPE (Maximum Permissible Error). The required measurement uncertainty is calculated as 1/6 of the MPE

of the mass value under test. For 1 kg the MPE (E_1) is 0.5 mg.

The specified repeatability (standard deviation) of this load cell is 1 mg. Nevertheless, the load cell can be replaced by another one, with improved repeatability or a more adequate one for another mass range.

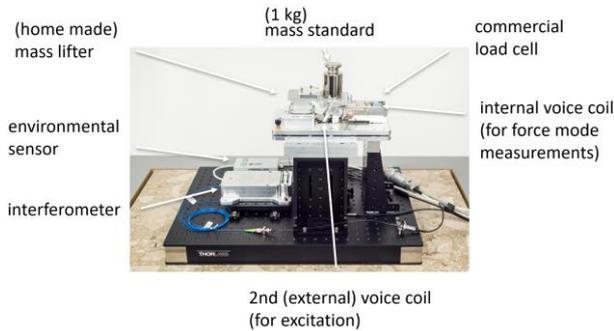


Figure 1: The setup of PB1 consists of a commercial load cell and an additional external voice coil attached to it. The motion in the velocity mode is measured with a laser interferometer. For automated weighing a mass lifter is used.

In contrast to PB2, where the load cell is used as a mechanical guiding system only, in PB1 the lever ratio is exploited additionally. As shown in the schematic of Figure 2 the internal voice coil (3), which is fixed to the longer lever arm, is used to compensate the weight (5) sitting on the shorter lever arm. The ratio of the levers is about 1:30. The velocity, however, is measured on the (short) lever arm. The external voice coil (2) is used to oscillate the lever arm, which finally induces a voltage in the internal voice coil (3).

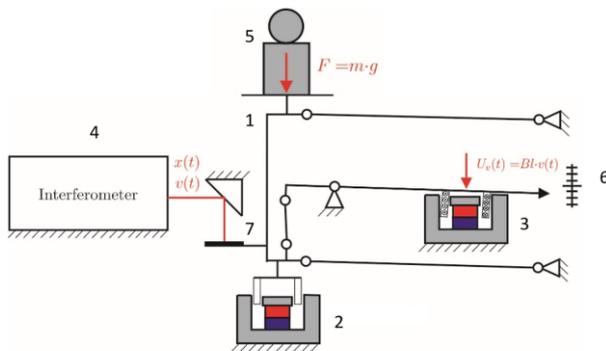


Figure 2: Schematic of PB1: 1-Load carrier, 2-External voice coil, 3-Internal voice coil, 4-Laser interferometer, 5-Weight, 6-Position sensitive device, 7-Mirror.

This allows using a smaller voice-coil system with a force factor Bl of about 27 T·m (B denotes the magnetic flux density, l the length of the coil-wire in the B -field). This lever ratio is approximately

$$Bl: \frac{u_{\text{ind}}}{v} = 1:30 \quad (1)$$

where u_{ind} is the induced voltage amplitude and v the velocity amplitude with which the coil is moved in the magnetic field. This is to be understood that the velocity is measured on the lever arm where the

weight is placed, while the voice coil is connected to the other (the longer) lever arm.

In the force mode the electrical current is measured from the voltage drop at a precision resistor. When putting a mass of a factor 10 lower then also the voltage drop will be by a factor of 10 lower. This would lead to reduced accuracy in the voltage measurement. In order to circumvent this issue, the PB1 employs resistors of values 100 Ω , 1 k Ω , 10 k Ω corresponding to the load ranges of 1 kg to 100 g, 100 g to 10 g, and 10 g to 1 mg, respectively. A custom made “switch box” has been developed in order to switch between the force and velocity mode, as well as between different resistor values.

Current measurements with PB1 are performed in air but can also be done in a high-vacuum environment as shown in Figure 3. For the sake of size scaling, on top of PB1 a weight of 1 kg is visible in Figure 3.

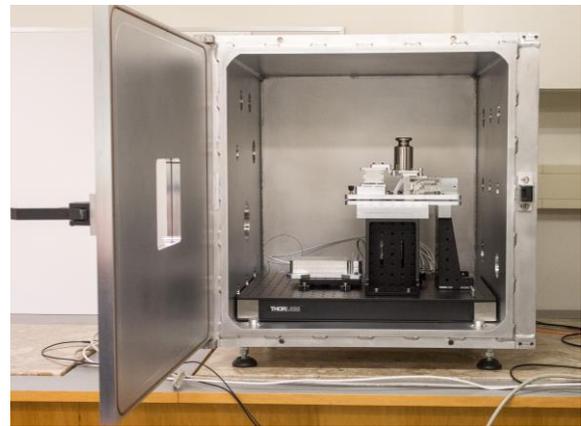


Figure 3: PB1 in a vacuum chamber. Visible on top of PB1 is a 1 kg mass standard.

3. CURRENT INVESTIGATIONS OF PLANCK-BALANCE 1

A series of measurements were performed with PB1 to investigate performance and possible systematic errors in the force and velocity modes. The following gives some examples.

3.1. Force Mode

In the force mode the repeatability has been investigated by doing automated repeated measurements over several hours. Every single measurement consists of an ABBA-cycle (A-with weight; B-without weight). Figure 4 shows a series of measurements conducted with a load of 1 kg. The Bl was calculated with corrections for changes of the environmental conditions, such as temperature, pressure and humidity. A temperature correction was done to a nominal value of 23 °C, where the temperature coefficient was determined from a long-term measurement. The mass lifting has been automatized by means of the above-mentioned home-made mass lifter. The vertical translation stage is

high-vacuum compatible, has a resolution of 4 nm, and a force range of up to 20 N. It is based on a piezo actuator to avoid magnetic bias, and has a travel range of 10 mm. The obtained relative repeatability (standard deviation) for a 1 kg mass standard lies in the order of 0.6 ppm, which is better than the stated specifications of the load cell of 1 mg. For curiosity the mass change was also performed by hand. In the case the repeatability was around 1 ppm, which is still in accordance with the specifications. For lower mass values the relative standard deviation decreases. So, for example, for a 10 g mass piece the standard deviation was in the order of 2×10^{-7} kg. A complete ABBA-cycle currently takes about 80 s but can be improved further.

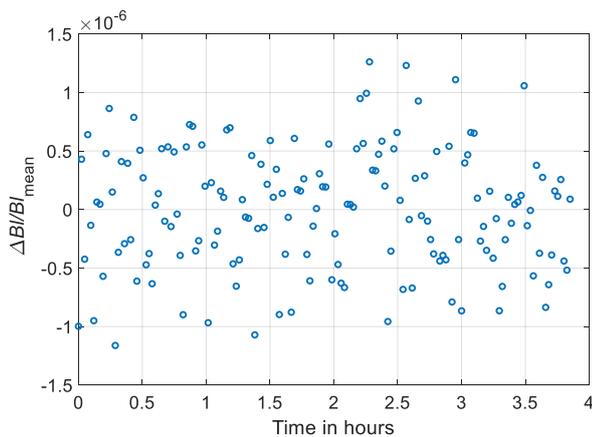


Figure 4: Repeated force mode measurement of a 1 kg mass standard. Each circle indicates a single ABBA cycle. The relative standard deviation for this measurement is 0.6×10^{-6} .

The balance allows, in principle, a measurement of masses over a range from 1 mg to 1 kg with the same load cell. Because of the limiting standard deviation, however, an accurate measurement of small masses is not expected in an appropriate time period. This is why the balance is designed in a modular way. If the focus lies on smaller masses, the load cell can be simply exchanged in order to meet the specifications. Nevertheless, measurements with calibrated E_1 mass standards were done with the same load cell (no other load cell has been used to date in this setup), which showed an increasing deviation from their calibrated value with decreasing mass value, but within the specified linearity of the balance of 2 mg. In these measurements the Bl , obtained for each nominal mass was compared to the Bl as obtained with a 1 kg mass standard. Corrections were made only for temperature (all corrected to 23 °C using the same temperature coefficient) and coil current effect [2]. The coil current effect is compensated by adjusting the tare weight for each mass value in a way that the absolute value of the required current is equal for A and B, but different in sign. This, currently, is done by putting additional physical weights on the load

carrier. For masses below 1 g this turns out to be difficult. Therefore, in the future an ‘electromagnetic’ tare will be implemented. This will be done by using the external voice-coil that is used in the velocity mode, as is already implemented in PB2. A constant current flowing through the coil will generate a constant force, which, in turn, acts like an additional weight. This second voice coil is the one that is usually used in the velocity mode to move the balance arm.

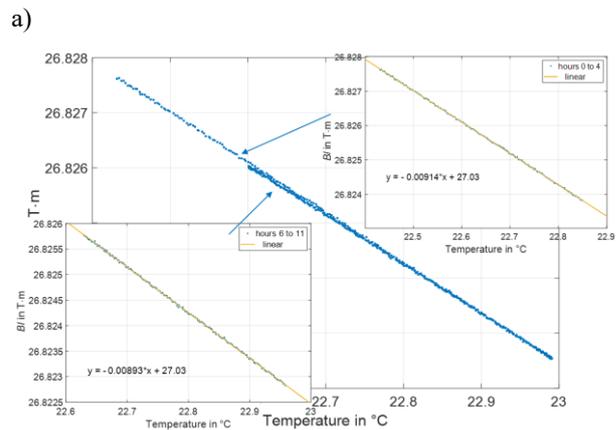
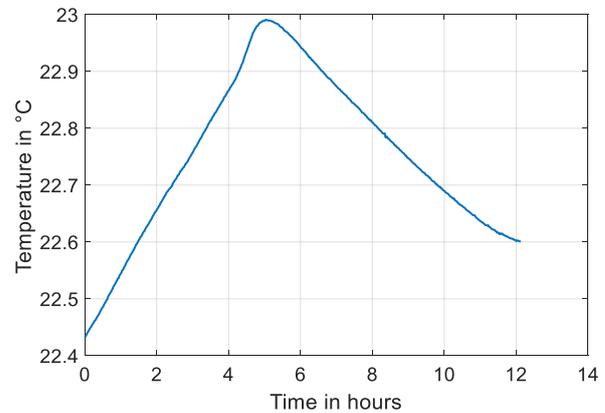


Figure 5: Measurement campaign of a 1 kg mass standard with PB1. The magnet shows a hysteresis. Depending whether the temperature is rising or falling with time, the temperature coefficient will be different by several per cent. a) Temperature measured in the magnet. b) The force factor Bl , measured during the campaign.

Further investigations were done on the stability of the magnet’s temperature coefficient. This is important since the laboratory, where the PB1 is installed, is currently not air-conditioned. The absolute temperature can vary significantly with respect to the temperature coefficient. From long term (several hours) measurements it was noticed that the temperature coefficient changed whether the temperature is increasing or decreasing, as can be seen in Figure 5 a) and b). Thus, a hysteresis-like behaviour has been observed. The temperature coefficient can vary by several per cent, while its relative value has been determined to be about $-3.3 \times 10^{-4} \text{ K}^{-1}$, which is a typical value for a SmCo magnet. This temperature change was not

forced by heating or cooling the room but was a mere coincidence that such a linear rising and falling of temperature happened during the measurement. It would be interesting to investigate this behaviour further, e.g. by placing the setup into a climate chamber and forcing temperature changes with different slopes.

The equilibrium position is monitored by means of a dual photo diode, a slit and an LED. The voltage output of the diode is measured with a 7.5 digits digital multimeter, which, after a calibration of the sensitivity curve, allows to resolve the position to several nanometres (the sensitivity factor is about $2.9 \times 10^{-3} \text{ V} \cdot \mu\text{m}^{-1}$). In order to estimate the uncertainty to repositioning, the Bl as a function of position has been measured (see Figure 6). To this end the PID control was set to maintain the equilibrium at several nominal positions, and at each position a weighing of one kilogram was performed. Due to the restoring force of the flexure hinges of the load cell, the electrical current values for mass on and mass off were not equal in their absolute value any longer. As this would give a bias in the result, for each position an adjustment of the tare weight was done.

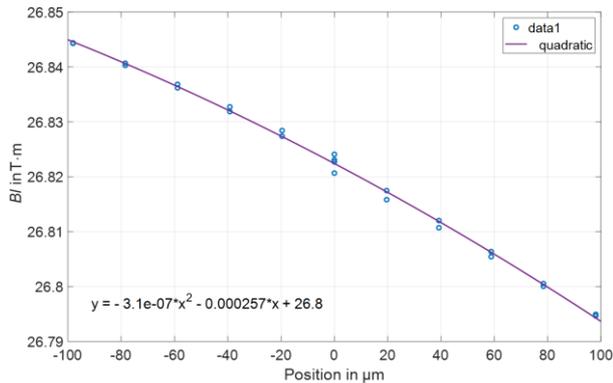


Figure 6: Measurement of the force factor Bl as a function of the coil position.

The relative gradient of Bl at the position zero was determined to be of $1.9 \times 10^{-3} \text{ V}^{-1}$ (voltage output of the position sensor). With a positioning uncertainty of, say, $150 \mu\text{V}$ (currently our PID threshold level – but can be further reduced; this voltage corresponds to about 50 nm) the possible relative error becomes in the order of about 3×10^{-7} .

3.2. Velocity Mode

As practicing in PB2 also in PB1 the voice coil is driven sinusoidally, giving a relative position over time according to

$$s(t) = S_0 + S \sin(\omega t + \varphi_s) \quad (2)$$

where S_0 denotes a constant offset and ω the angular frequency, $\omega = 2\pi f_{\text{sig}}$, with the oscillation frequency f_{sig} , and φ_s is the initial phase. The coil velocity is obtained as the derivative of equation (2) with respect to the time t as

$$v(t) = \omega S \cos(\omega t + \varphi_s) \quad (3)$$

This motion induces an ac voltage of

$$u(t) = U \cos(\omega t + \varphi_u) \quad (4)$$

across the coil ends. φ_u is again an arbitrary initial phase. Due to non-linearities, e.g. in the magnetic field and the mechanical system, higher harmonics will be excited along with the excitation frequency. By determining the amplitude of the first harmonic (fundamental note) of the induced voltage, as well as of the position signal, the force factor, Bl , can be calculated. Thus, for the sake of simplicity, assuming the force factor Bl to be a constant during the whole range of coil movement, Bl can be calculated as

$$Bl = \frac{U}{\omega S} = \frac{U}{2\pi f_{\text{sig}} S} = \frac{U}{v} \quad (5)$$

This equation, however, is still lacking the lever ratio. If this ratio is taken into account, we obtain equation (1). The signal frequency is usually around 4 Hz , which is close to the eigenfrequency of the load cell, and the amplitude is of about $4.5 \mu\text{m}$. The error in the amplitude estimation arising from higher harmonics can be compensated by either including the higher harmonics into the regression analysis, or by adjusting the data length to an integer number of sine wave cycles (coherent sampling) [6]. As the coil oscillates over a range with varying Bl (see Figure 6) the measured Bl is a weighted mean of all Bl s over this range. For an exactly linear Bl over the oscillation range, the measured Bl value corresponds to the point of zero oscillation, which is the same point to which the PID control directs (i.e. the set point) the coil during the force mode. The more non-linear the Bl becomes, the more the calculated point deviates from the set point in the force mode. This deviation, however, can be estimated by means of the shape of the Bl (see Figure 6). As the induction happens in the voice coil attached to the longer lever arm, and the lever ratio is approximately 1:30, the range of motion is about $270 \mu\text{m}$ and this gives a non-negligible bias. In our case the relative offset is in the order of -2.2×10^{-5} but can be corrected with a remaining relative uncertainty of 15 % of the correction.

Figure 7 shows the results of 19 consecutive Bl determinations in the velocity mode. The interferometer output and the digital multimeter (Keysight 3458A) were sampled with 10 kHz . Each measurement took 10 s . No temperature corrections have been applied in this case, as the temperature deviation was negligible.

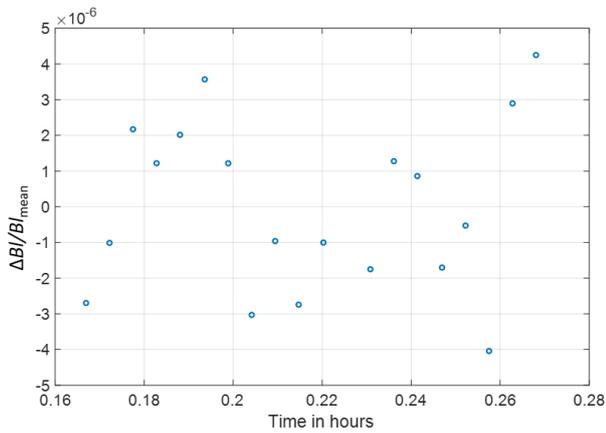


Figure 7: Repeated measurement of Bl in the velocity mode. The relative standard deviation is 2.4×10^{-6} . Each dot is the result of a 10 s measurement with a sampling rate of 10 kHz.

With PB2 it was noticed that the measured Bl was a function of the excitation frequency [7]. Based on this experience for PB1 a possible frequency dependence has been investigated. Thus, 20 different excitation frequencies were selected ranging from 1 Hz to 10.5 Hz with an increment of 0.5 Hz. For each selected excitation frequency, the induced voltage and the displacement were measured in air with a sampling period of 30 s and a sampling frequency of 10 kHz. After corrections of the air refractive index, aperture time of the 3458A, and temperature, the Bl was determined from the amplitudes of the induced voltage and the displacement as given in equation (1).

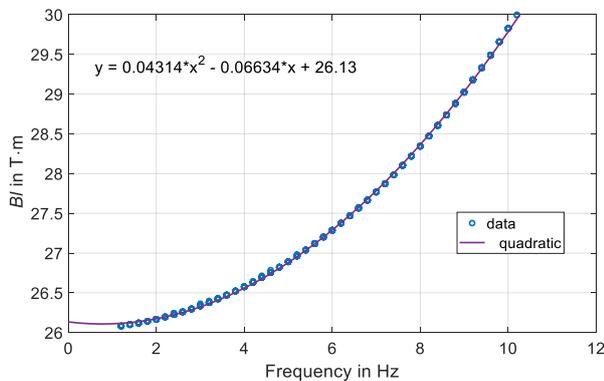


Figure 8: A measurement of the force factor Bl for different excitation frequencies.

The results shown in Figure 8 suggest a strong dependence of Bl on the excitation frequency in the order of $dBl/df = 0.28 \text{ T}\cdot\text{m}\cdot\text{Hz}^{-1}$ at 4 Hz. This effect is assumed to originate from an Abbe error, which is produced by a small tilting of the load carrier during up and down motion. As this slope is significantly higher than the one observed with PB2, the excitation coil and measuring coil of the PB1 set-up were exchanged (not physically, but the wiring, so that the internal voice coil now functioned as the actor and the external voice coil

induced the voltage), and the above measurement was performed again. Both voice coils have approximately the same parameters, hence the results allow a comparison. Compared to the former results, the variation of Bl over the excitation frequency is in the order of $dBl/df = 3.3 \times 10^{-2} \text{ T}\cdot\text{m}\cdot\text{Hz}^{-1}$ at 4 Hz in this case. This supports the conjecture that the frequency dependence comes from an Abbe error, since in the second measurement the lever ratio was not exploited. Exploiting the lever ratio thus makes the setup more sensitive to frequency variations. An explanation for this difference is given in Figure 9. The external voice coil (2) is closer situated to the mirror than the internal voice coil (3). Thus, the Abbe offset is bigger when the voltage is induced in the internal coil. The frequency dependence of Bl can be explained as follows. Due to the inertia of the lever the tilt angle of the mirror changes with changing frequency, as the lever bends different due to the inertial mass. Since the mirror axis does not overlap with the axis of the coil, both parameters, the mirror motion and the induced voltage, will be subject to different amplitudes for different frequencies. For a correction of this bias the mechanical tilt angle must be determined, as well as the Abbe offset, i.e. the distance of the measurement point of the laser interferometer to the axis of motion of the induction coil. This work is currently under investigation.

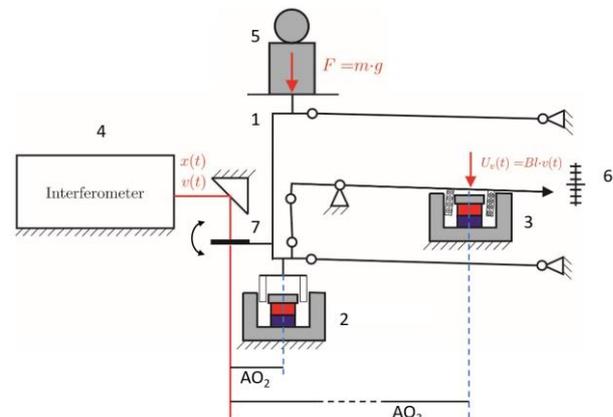


Figure 9: Explanation of the Abbe error in PB1: The external voice coil (2) has a smaller Abbe offset (AO_2) than the internal voice coil (3), which is (AO_3). The arrow at the mirror indicates that the mirror can tilt during up and down motion.

4. SUMMARY

The concept and the current focus of investigations of the PB1 are presented in this paper. These investigations include performance tests of the setup in the force and velocity mode. The relative standard deviations for the force and velocity mode were 0.6 ppm and 2.4 ppm, respectively. The temperature coefficient has been determined and a hysteresis was noticed that leads

to different values in the coefficient whether the temperature is rising or falling. The difference is in the order of several per cent in the investigated case. A frequency dependence in the velocity mode has been detected, which is orders bigger if the lever ratio is exploited. It is assumed that the reason for this effect is an Abbe error due to a tilting of the load carrier. This problem requires further investigations. In principle, measurements can be performed over the whole range from 1 mg to 1 kg with the same load cell. The standard deviation, however, is a limiting factor. To circumvent this problem, the load cell can be exchanged in order to meet the specifications for the desired load range. This will be done in future investigations.

ACKNOWLEDGEMENT

The research of this project is funded via the programme “Validierung des technologischen und gesellschaftlichen Innovationspotenzials – VIP+”, a programme of the German Federal Ministry of Education and Research (BMBF), and is managed by VDI/VDE Innovation + Technik GmbH. The authors thank also J. Kloß, L. Günther and M. Spoor for help in the measurements and J. Schleichert for fruitful discussions and technical assistance.

5. REFERENCES

- [1] D. Knopf, T. Wiedenhöfer, K. Lehrmann, F. Härtig, “A quantum of action on a scale? Dissemination of the quantum based kilogram”, *Metrologia*, vol. 56, no. 2, 024003, 2019.
- [2] I. A. Robinson, S. Schlamming, “The watt or Kibble balance: a technique for implementing the new SI definition of the unit of mass”, *Metrologia*, vol. 53, no. 5, A46, 2016.
- [3] L. Chao, F. Seifert, D. Haddad, J. Pratt, D. Newell, S. Schlamming, “The performance of the KIBB-g1 tabletop Kibble balance at NIST” *Metrologia*, vol. 57, no. 3, 035014, 2020.
- [4] I. A. Robinson, J. Berry, S. Davidson, C. Jarvis, “Towards a simplified Kibble balance to realise mass in the new SI”, *Euspen’s 17th International Conference & Exhibition*, Hannover, May 2017.
- [5] C. Rothleitner et al, “The Planck-Balance—using a fixed value of the Planck constant to calibrate E1/E2-weights”, *Measurement Science and Technology* vol. 29, no. 7, 2018.
- [6] S. Lin, C. Rothleitner, N. Rogge, T. Fröhlich, “Influences on amplitude estimation using three-parameter sine fitting algorithm in the velocity mode of the Planck-Balance” *Acta IMEKO*, ISSN: 2221-870X, accepted for publication, 2020.
- [7] N. Rogge, C. Rothleitner, S. Lin, S. Vasilyan, T. Fröhlich, F. Härtig, D. Knopf, “Error sources in the force mode of the “PB2” Planck-Balance”, in *Proc. of 24th IMEKO TC3 Int. Conf., Cavtat-Dubrovnik, Croatia, 16-18 November 2020*.