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The determination of the volume of weights in the range of 1 g – 5 kg: a comparison of hydrostatic weighing and double weighing in air using the Monte Carlo simulation

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ABSTRACT
We have investigated two methods for the determination of the volume of weights in the range of 1 g – 5 kg: double weighing in air and hydrostatic weighing. We present the mathematical equations of both methods, showing that the Monte Carlo simulation is a suitable way of determining the measurement uncertainties and of overcoming the difficulties in dealing with correlated variables. We found that the measurement uncertainties of the two methods are comparable and that double weighing in air is an efficient method of determining the volume of weights below 1 kg.

Section: RESEARCH PAPER

Keywords: volume determination; double weighing; hydrostatic weighing; Monte Carlo

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1. INTRODUCTION

There are six accepted methods for the determination of the density of weights, which are described in the recommendation of the Organisation Internationale de Métrologie Légale (OIML) R111 and are denoted in Methods A to F [1]. These methods can broadly be classified into three categories: the hydrostatic method, the geometric measurement, and density estimation. Here, the hydrostatic method, which traces the volume/density to the reference volume weight or the water density, is considered the most accurate measurement method. It is called the reference method of volume/density determination and has been implemented by many National Metrology Institutes (NMIs) worldwide [2]. Even though hydrostatic weighing is the most accurate method among those described in OIML R111 for determining the volume of the weights, it does have some disadvantages. The method is time consuming, it is expensive, and it changes the surface of the weight. After immersion in water, it can lead to instabilities in the weight. Furthermore, the immersion of stainless steel weights in water can remove the adsorbed contaminants or the surface oxide layer, thus changing the surface properties [3]. However, immersion in water is unlikely to cause significant corrosion of stainless steel weights [3], [4]. Besides the hydrostatic method, there are some alternative methods: optical interferometry [5], which is probably the most accurate method available; weighing with a balance immersed in fluorocarbon fluid [5]; using an acoustic volumeter [7]–[9]; or double weighing in air [10]–[12]. Like hydrostatic weighing, double weighing in air is based on Archimedes' principle. However, the main difference between the two methods is the medium to which the weight is exposed. Although the double weighing in air has the advantage of being clean and efficient, its consistency with the hydrostatic method requires verification by means of experiments. Previous publications by Clarkson [10], Malengo [11], and Ueki [12] have described the principle and feasibility of determining the volume of weights by double weighing in air. Clarkson described a method of volume determination of weights of 1 kg nominal mass. Malengo adopted a modified version of Clarkson's method by considering mass and volume as two measurands that are determined simultaneously in a multivariate context. He validates the calculations experimentally by using stainless steel and platinum-

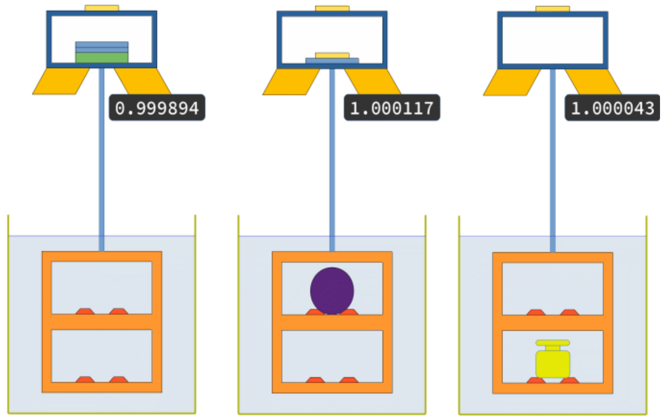

Figure 1. Principle of the volume apparatus VK1 at the Federal Institute of Metrology METAS for the determination of volume of test weights up to 1 kg by using hydrostatic weighing. Left: the weighing of the suspension only. Middle: the weighing of the reference weight (silicon sphere). Right: the weighing of the test weight.

iridium weights of a 1 kg nominal mass. Ueki investigated the simultaneous calibration of mass and volume of weights in the range of 1 kg to 20 kg. We extend the range from 5 kg down to 1 g and compare the results from double weighing to those from hydrostatic weighing. In addition, we explain how to use support weights both in double weighing and hydrostatic weighing when measuring test weights that are too small to be loaded on the weighing pan. We compare the measurement results and uncertainties of these two methods and show that the Monte Carlo simulation is a suitable and simple way to overcome the difficulties of laboriously calculating correlations between input quantities.

2. SETUP

We now describe the measurement sites that we used to perform the double weighing and hydrostatic weighing. All measurement sites are located in the mass laboratory of METAS.

2.1. Hydrostatic method

The volume apparatus VK1 at METAS consists of an AT1005 comparator from Mettler-Toledo with a maximum capacity of 1011 g and a readability of 1 µg. The suspension system is immersed in a water basin (Figure 1). The water comes from an ultrapure water system (ELGA Labwater, PURELAB). The suspension system carries both the reference weight and the test weight and loads them individually on the weighing pan in water. The weighing pan is connected with the comparator in air through a thin metal duct. The comparator measures the resulting force – the difference between the gravitational force and the buoyancy due to the water. The buoyancy is determined by the water density, which is measured experimentally by means of the reference weight. We use a silicon sphere as the reference volume. We do three weighings and measure (i) the reference volume, (ii) the test weight, and (iii) the suspension only. To compensate for the balance's linearity and for the different buoyancy forces and masses during the three weighings, we use auxiliary weights that are placed on the four-position weighing carousel of the comparator. The temperature, pressure, and relative humidity of the air are recorded during the measurement. Additionally, four thermistors from YSI Inc. measure the water temperature on the front and rear sides of the reference and test weights.

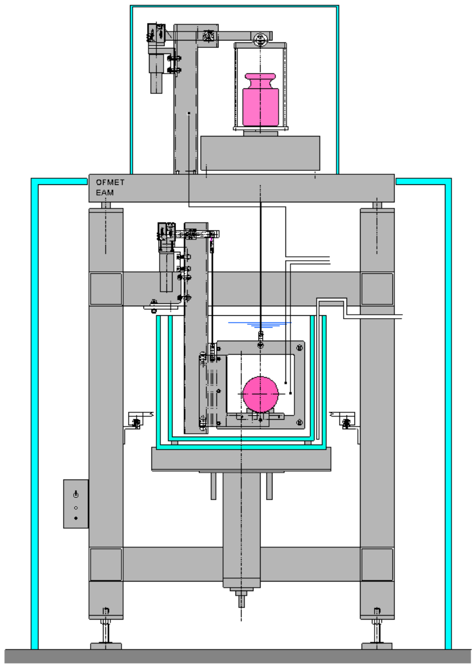

Figure 2. VK10 apparatus at METAS for volume determination of weights up to 10 kg. Only the test weight is immersed in water.

The volume apparatus VK10 consists of a PR10003 comparator from Mettler-Toledo with a maximum capacity of 10100 g and a repeatability of 2 mg (Figure 2). The operating principle is similar to that of VK1, except that the reference volume (the middle image of Figure 1) is not present. The traceability of the density is obtained by calculating the formula of Tanaka for the density of water [13].

2.2. Double weighing in air

For the double weighing measurements, we use an AT10005 comparator (Mettler-Toledo) with a repeatability of 0.02 mg for the 2 kg and 5 kg weights; an M_one (Mettler-Toledo) with a repeatability of 0.5 µg for weights from 200 g to 1 kg; and an AT106 (*Mettler-Toledo*) with a repeatability of 1.5 µg for weights from 1 g to 100 g. The comparators are in airtight enclosures, in which the air density can be varied by changing the air pressure (Figure 3). To change the air pressure, we use a membrane pump (813.3, KNF Neuberger).

3. MEASUREMENT PROCEDURES

In our experiment, we used OIML stainless steel weights with nominal mass settings of 1 g, 10 g, 20 g, 50 g, 100 g, 200 g, 500 g, 1 kg, 2 kg, and 5 kg (Figure 4). For each weight, we first carried out a double weighing measurement at 950 mbar and 750 mbar. Then, we performed a hydrostatic weighing. Thereafter, we carried out a second double weighing measurement at 950 mbar and 750 mbar to check the stability of the weight and to observe possible adverse effects after immersing the weight in water. For both methods, we calculated the air density according to the CIPM formula [14], which we verified by using air buoyancy artefacts before comparing the double weighing and hydrostatic weighing methods (CIPM = Comité international des poids et mesures, the International Committee for Weights and Measures).

Figure 3. AT106 mass comparator with an airtight enclosure in the mass laboratory at METAS.

3.1. Hydrostatic method

In the hydrostatic method, we used the silicon sphere in gravimetric weighing to determine the water density. The water density was then used to determine the volume of the test weight. Small weights from 1 g to 5 g could not be loaded directly on the weighing pan because of their geometrical dimensions. Instead, we used a 10 g stainless steel disc as a support weight (see section 4.1.1).

3.2. Double weighing method

In the double-weighing method, we compared every test weight to at least one reference weight of the same nominal mass and density. Weights smaller than 10 g could not be loaded directly on the weighing pan because of their dimensions. For this reason, we used a 100 g OIML weight as a support (see section 4.2.1).

4. CALCULATIONS

We now explain the basic mathematical formulae we used in the hydrostatic and double weighing methods. The uncertainty calculations are presented in section 5.

4.1. Hydrostatic method

The basic force equations for the three weighings of the suspension only (F_0), the suspension plus the test weight (F_1), and the suspension plus the reference weight (F_2) are

$$\left|\begin{array}{l} F_0 = G_\mathrm{O} - A_\mathrm{O} + G_\mathrm{Z} - A_\mathrm{Z} \\ F_1 = G_\mathrm{T} - A_\mathrm{T} + G_\mathrm{Z} - A_\mathrm{Z} + G_\mathrm{B} - A_\mathrm{B} \\ F_2 = G_\mathrm{R} - A_\mathrm{R} + G_\mathrm{Z} - A_\mathrm{Z} + G_\mathrm{A} - A_\mathrm{A} \end{array}\right|, \quad (1)$$

where G_i and A_i denote the gravitational force and the buoyancy, respectively. The indices represent the mass of the suspension (Z) and its auxiliary weights (O); the mass of the test weight (T) and its auxiliary weights (B); and the mass of the reference weight (R) and its auxiliary weights (A). All the auxiliary weights are in air.

Figure 4. OIML stainless steel weights used for the comparison.

As the weights are placed at different heights above the surface, we correct the force equations for the gravitational acceleration of each weight with $\alpha_i = \frac{g_\mathrm{E}}{g_i}$

$$\left|\begin{array}{l} m_\mathrm{O}\frac{g_\mathrm{E}}{\alpha_\mathrm{O}} - \rho_\mathrm{LO}V_\mathrm{O}\frac{g_\mathrm{E}}{\alpha_\mathrm{O}} + \lambda_K = M_0\frac{g_\mathrm{E}}{\alpha_\mathrm{H}}\,\Gamma \\ m_\mathrm{T}\frac{g_\mathrm{E}}{\alpha_\mathrm{T}} - \rho_\mathrm{WT}V_\mathrm{T}\frac{g_\mathrm{E}}{\alpha_\mathrm{T}} + m_\mathrm{B}\frac{g_\mathrm{E}}{\alpha_\mathrm{B}} - \rho_\mathrm{LB}V_\mathrm{B}\frac{g_\mathrm{E}}{\alpha_\mathrm{B}} + \lambda_K = M_1\frac{g_\mathrm{E}}{\alpha_\mathrm{H}}\,\Gamma \\ m_\mathrm{R}\frac{g_\mathrm{E}}{\alpha_\mathrm{R}} - \rho_\mathrm{WR}V_\mathrm{R}\frac{g_\mathrm{E}}{\alpha_R} + m_\mathrm{A}\frac{g_\mathrm{E}}{\alpha_\mathrm{A}} - \rho_\mathrm{LA}V_\mathrm{A}\frac{g_\mathrm{E}}{\alpha_\mathrm{A}} + \lambda_K = M_2\frac{g_\mathrm{E}}{\alpha_\mathrm{H}}\,\Gamma \end{array}\right|, \quad (2)$$

where g_E represents the gravitational acceleration at the load cell and λ_K equals $G_\mathrm{Z} - A_\mathrm{Z}$.

The uncorrected mass difference, i.e. the indication of the balance, is denoted as M_i and is corrected by the standard densities for air and material Γ = (1 - 1.2/8000) [14]. As the suspension is present in every weighing, its influence is denoted by λ.

$$\left|\begin{array}{l} M_0 = \left[\frac{m_\mathrm{O}}{\alpha_\mathrm{O}} - \frac{\rho_\mathrm{LO}V_\mathrm{O}}{\alpha_\mathrm{O}} + \lambda\right]\alpha_\mathrm{H}\Gamma^{-1} \\ M_1 = \left[\frac{m_\mathrm{T}}{\alpha_\mathrm{T}} - \frac{\rho_\mathrm{WT}V_\mathrm{T}}{\alpha_\mathrm{T}} + \frac{m_\mathrm{B}}{\alpha_\mathrm{B}} - \frac{\rho_\mathrm{LB}V_\mathrm{B}}{\alpha_\mathrm{B}} + \lambda\right]\alpha_\mathrm{H}\Gamma^{-1} \\ M_2 = \left[\frac{m_\mathrm{R}}{\alpha_\mathrm{R}} - \frac{\rho_\mathrm{WR}V_\mathrm{R}}{\alpha_\mathrm{R}} + \frac{m_\mathrm{A}}{\alpha_\mathrm{A}} - \frac{\rho_\mathrm{LA}V_\mathrm{A}}{\alpha_\mathrm{A}} + \lambda\right]\alpha_\mathrm{H}\Gamma^{-1} \end{array}\right| \quad (3)$$

We can now solve the volume of the test weight, V_T, at 20 °C by using the thermal expansion coefficients C_i of the weights.

$$V_\mathrm{T,water} := V_\mathrm{T} = \left[\begin{array}{c} \frac{(M_0-M_1)}{\alpha_\mathrm{H}}\Gamma + \frac{m_\mathrm{T}}{\alpha_\mathrm{T}} + \frac{m_\mathrm{B}}{\alpha_\mathrm{B}} - \frac{m_\mathrm{O}}{\alpha_\mathrm{O}} - \\ \frac{\rho_\mathrm{LB}V_\mathrm{B20}C_\mathrm{B}}{\alpha_\mathrm{B}} + \frac{\rho_\mathrm{LO}V_\mathrm{O20}C_\mathrm{O}}{\alpha_\mathrm{O}} \end{array}\right]\frac{\alpha_\mathrm{T}}{\rho_\mathrm{WT}C_\mathrm{T}} \quad (4)$$

The coefficient C_i is given by the linear thermal expansion coefficient β and the measured temperature in air or in water

$$C_i := 1 + 3\,\beta\,(T_\mathrm{meas} - 20\ °\mathrm{C}). \quad (5)$$

We must know the density of water, ρ_WT, at the test weight's position inside the water basin. To calculate ρ_WT, we determine the density of water, ρ_WR, at the position of the silicon sphere in two ways: (i) we use the well-known silicon sphere as the volume reference in gravimetric weighing

$$\rho_\mathrm{WR} = \left[\begin{array}{c} \frac{(M_0-M_2)}{\alpha_\mathrm{H}}\Gamma + \frac{m_\mathrm{R}}{\alpha_\mathrm{R}} + \frac{m_\mathrm{A}}{\alpha_\mathrm{A}} - \frac{m_\mathrm{O}}{\alpha_\mathrm{O}} - \\ \frac{\rho_\mathrm{LA}V_\mathrm{A20}C_\mathrm{A}}{\alpha_\mathrm{A}} + \frac{\rho_\mathrm{LO}V_\mathrm{O20}C_\mathrm{O}}{\alpha_\mathrm{O}} \end{array}\right]\frac{\alpha_\mathrm{R}}{V_\mathrm{R20eff}C_\mathrm{R}} \quad (6)$$

and (ii) we calculate the density $\tilde{\rho}_{\mathrm{WR}}$ by using Tanaka's formula [13]. We also calculate the water density at the test weight's position, $\tilde{\rho}_{WT}$, by using Tanaka's formula. Now, we can establish the following relation

$$\rho_{\mathrm{WT}} = \frac{\tilde{\rho}_{\mathrm{WT}}}{\tilde{\rho}_{\mathrm{WR}}} \rho_{\mathrm{WR}} \,. \tag{7}$$

The difference between the calculated ($\tilde{\rho}_{\mathrm{WR}}$) and measured ($\rho_{\mathrm{WR}}$) water densities is about 2.5 ppm, which is in line with the uncertainty contributions related to the temperature and the isotopic distribution. The volume, V_{R20eff}, of the silicon sphere was determined by optical interferometry and ellipsometry at the National Metrology Institute of Japan in 2006.

4.1.1. Hydrostatic method with support

Weights smaller than 10 g cannot be measured directly on our volume comparator because their geometrical dimension is too small. Instead, a support weight must be used. By doing this, we measure the total volume of the support weight and the small test weight. The mass m_{T} in Equation (4) becomes

$$m_{\mathrm{T}} = m_{\mathrm{small}} + m_{\mathrm{support}} . \tag{8}$$

The volume of the small test weight is simply the difference between the total volume and the volume of the support weight.

$$V_{\mathrm{small}} = V_{\mathrm{total}} - V_{\mathrm{support}} . \tag{9}$$

The latter was a 10 g stainless steel disc and was determined separately through hydrostatic weighing.

4.2. Double weighing in air

The weighing equation for mass determination in air on a comparator is given by

$$m_{\mathrm{T}} = m_{\mathrm{R}} + \rho_a (V_{\mathrm{T}} - V_{\mathrm{R}}) + \Delta m_w \left(1 - \frac{\rho_0}{\rho_c}\right) \tag{10}$$

where m_{R} and V_{R} are the true mass and volume of the reference weight, and V_{T} is the volume of the test weight. Δm_{w} is the uncorrected (conventional) weighing difference indicated by the balance, which needs to be corrected for the reference air density $\rho_0 = 1.2$ kg/m³ and the reference material density $\rho_{\mathrm{c}} = 8000$ kg/m³ to calculate the true mass m_{T} of the test weight [15], [16]. After a double weighing measurement at two different air densities, ρ_{a1} and ρ_{a2}, we can write the following system of equations

$$\left| \begin{aligned} m_{\mathrm{T}} &= m_{\mathrm{R}} + \rho_{\mathrm{a1}} (V_{\mathrm{T}} - V_{\mathrm{R}}) + \Delta m_{\mathrm{w1}} \left(1 - \frac{\rho_0}{\rho_{\mathrm{c}}}\right) \\ m_{\mathrm{T}} &= m_{\mathrm{R}} + \rho_{\mathrm{a2}} (V_{\mathrm{T}} - V_{\mathrm{R}}) + \Delta m_{\mathrm{w2}} \left(1 - \frac{\rho_0}{\rho_{\mathrm{c}}}\right) \end{aligned} \right| \tag{11}$$

from which we can derive the volume V_{T} of the test weight

$$V_{\mathrm{T,air}} := V_{\mathrm{T}} = V_{\mathrm{R}} + \frac{\Delta m_{\mathrm{w2}} - \Delta m_{\mathrm{w1}}}{\rho_{\mathrm{a1}} - \rho_{\mathrm{a2}}} \left(1 - \frac{\rho_0}{\rho_{\mathrm{c}}}\right). \tag{12}$$

4.2.1. Small weights with support

If the test weight is too small to be loaded on the weighing pan, a support weight is necessary. In our experiment, we used a 100 g OIML weight as support and put the small test weight (1 g or 10 g) on top of the OIML weight. This combination can be compared to another 100 g reference weight, providing that the weighing difference is still within the weighing range of the balance. Firstly, we performed a double weighing with the support weight only and calculated its volume according to Equation (12). Then, we added the small test weight, repeated the double weighing, and calculated the volume of the weight combination according to Equation (12), which is the sum of the two volumes. The volume of the small test weight is given by the total volume of the weight combination (small weight plus support weight) minus the volume of the support weight

$$V_{\mathrm{small}} = V_{\mathrm{total}} - V_{\mathrm{support}} . \tag{13}$$

The support weight can either be determined separately by double weighing or by any other method, such as hydrostatic or interferometric measurements.

5. UNCERTAINTY CALCULATIONS

We used the Monte Carlo simulations to estimate the measurement uncertainties in the hydrostatic weighing and double weighing methods. We assume that the input quantities x_i in Equations (4) and (12) are normally distributed

$$X \sim \mathcal{N}(\mu, \sigma^2). \tag{14}$$

For the mean value, μ, and the standard deviation, σ, we use the average value, $\bar{x}$, and the standard deviation, s, from the measurement or certificate. For the simulation, we used N = 10000 iterations.

5.1. Hydrostatic weighing

Equation (4) can be written as

$$V_{\mathrm{T,water}}(j) := V_{\mathrm{T}}(j) = \left[\frac{(M_0(j) - M_1(j))}{\alpha_{\mathrm{H}}(j)} \Gamma + \frac{m_{\mathrm{T}}(j)}{\alpha_{\mathrm{T}}(j)} + \frac{m_{\mathrm{B}}(j)}{\alpha_{\mathrm{B}}(j)} - \frac{m_{\mathrm{O}}(j)}{\alpha_{\mathrm{O}}(j)} - \frac{\rho_{\mathrm{LB}}(j) V_{\mathrm{B20}}(j) C_{\mathrm{B}}(j)}{\alpha_{\mathrm{B}}(j)} + \frac{\rho_{\mathrm{LO}}(j) V_{\mathrm{O20}}(j) C_{\mathrm{O}}(j)}{\alpha_{\mathrm{O}}(j)}\right] \frac{\alpha_{\mathrm{T}}(j)}{\rho_{\mathrm{WT}}(j) C_{\mathrm{T}}(j)} . \tag{15}$$

where j represents the iteration. If a support weight is used, Equations (8)and (9) become

Table 1. Typical values of uncertainty components in the hydrostatic weighing of a 1 kg OIML weight (k = 1).

Uncertainty component	Value	Unit
$u(M_0)$	0.66239	mg
$u(M_1)$	0.82361	mg
$u(m_{\mathrm{T}})$	0.00658	mg
$u(m_{\mathrm{B}})$	0	mg
$u(m_{\mathrm{O}})$	0.16279	mg
$u(\alpha_{\mathrm{H}})$	0.032965 × 10^{-6}	(m/s²) / (m/s²)
$u(\alpha_{\mathrm{T}})$	0.017215 × 10^{-6}	(m/s²) / (m/s²)
$u(\alpha_{\mathrm{B}})$	0	(m/s²) / (m/s²)
$u(\alpha_{\mathrm{O}})$	0.032622 × 10^{-6}	(m/s²) / (m/s²)
$u(\rho_{\mathrm{LB}})$	0.002	kg/m³
$u(\rho_{\mathrm{LO}})$	0.002	kg/m³
$u(\rho_{\mathrm{WT}})$	0.00108	kg/m³
$u(V_{\mathrm{B20}})$	0	mm³
$u(V_{\mathrm{O20}})$	1.1344	mm³
$u(C_{\mathrm{T}})$	0.0223 × 10^{-6}	mm³/ mm³
$u(C_{\mathrm{B}})$	0.1187 × 10^{-6}	mm³/ mm³
$u(C_{\mathrm{O}})$	0.1187 × 10^{-6}	mm³/ mm³

Table 2. Typical values of the uncertainty components in the double weighing of a 1 kg OIML weight (k = 1).

Uncertainty component	Value	Unit
$u(V_{\mathrm{R}})$	0.20	mm³
$u(\Delta m_{\mathrm{w1}})$	12.90	ng
$u(\Delta m_{\mathrm{w2}})$	28.25	ng
$u(\rho_{\mathrm{a1}})$	0.0001	kg/m³
$u(\rho_{\mathrm{a2}})$	0.0001	kg/m³

$$m_T(j) = m_{small}(j) + m_{support}(j)$$
$$V_{small}(j) = V_{total}(j) - V_{support}(j). \qquad (16)$$

The volume of the test weight we are looking for and the estimated standard uncertainty are given by the mean and standard deviation of the simulated values $V_T(j)$ and $V_{small}(j)$. The typical values of the uncertainty components in hydrostatic weighing of a 1 kg OIML weight are listed in Table 1.

5.2. Double weighing

To estimate the measurement uncertainty in the double weighing method, we proceed in the same way as for the hydrostatic weighing. Equations (12) and (13) can be written as

$$V_{T,air}(j) := V_T(j) = V_R(j) + \frac{\Delta m_{w2}(j)-\Delta m_{w1}(j)}{\rho_{a1}(j)-\rho_{a2}(j)}\left(1-\frac{\rho_0}{\rho_c}\right) \qquad (17)$$

$$V_{small}(j) = V_{total}(j) - V_{support}(j). \qquad (18)$$

The typical values of the uncertainty components in the double weighing of a 1 kg OIML weight are listed in Table 2.

6. RESULTS

6.1. Verification of the air density

We verified our air density calculation according to the CIPM formula [14] by using air buoyancy artefacts. The pair of artefacts consisted of a tube and a hollow body with volumes of (124.80558 ± 0.00023) cm³ and (412.63374 ± 0.02563) cm³, respectively. We used the artefacts to determine the air density experimentally in the range of 650 mbar to 1060 mbar and compared the results to the air density obtained using the CIPM formula (Figure 5). The results are in good agreement. For the

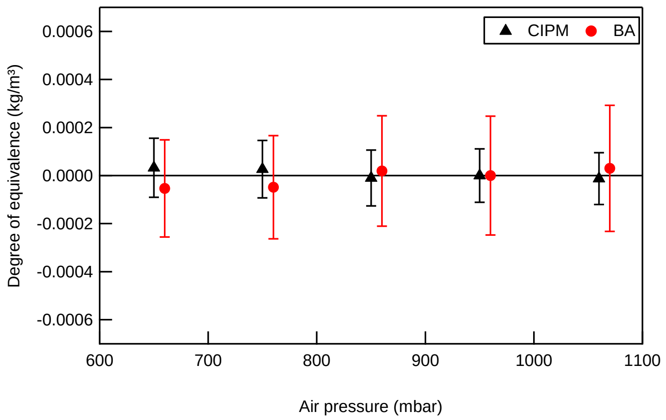

Figure 5. Air density determined experimentally by using buoyancy artefacts and calculated according to CIPM formula. Error bars represent expanded uncertainties with a coverage factor k = 2.

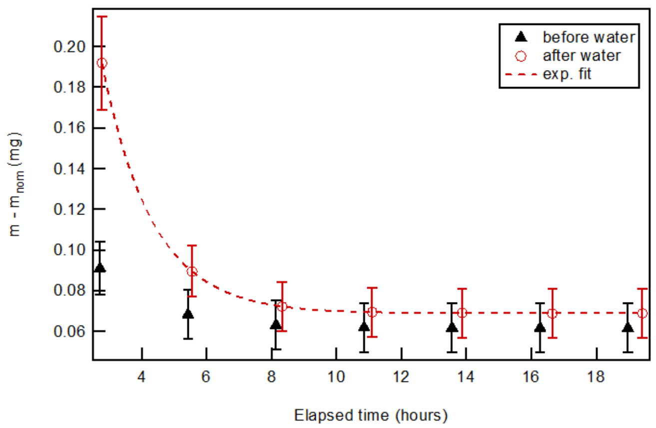

Figure 6. Stabilization time of a 1 kg OIML weight after hydrostatic weighing and the change in mass measured gravimetrically in the air before (black triangle) and after (red circle) hydrostatic weighing. The error bars represent expanded uncertainties with a coverage factor k = 2. The dashed line is an exponential fit.

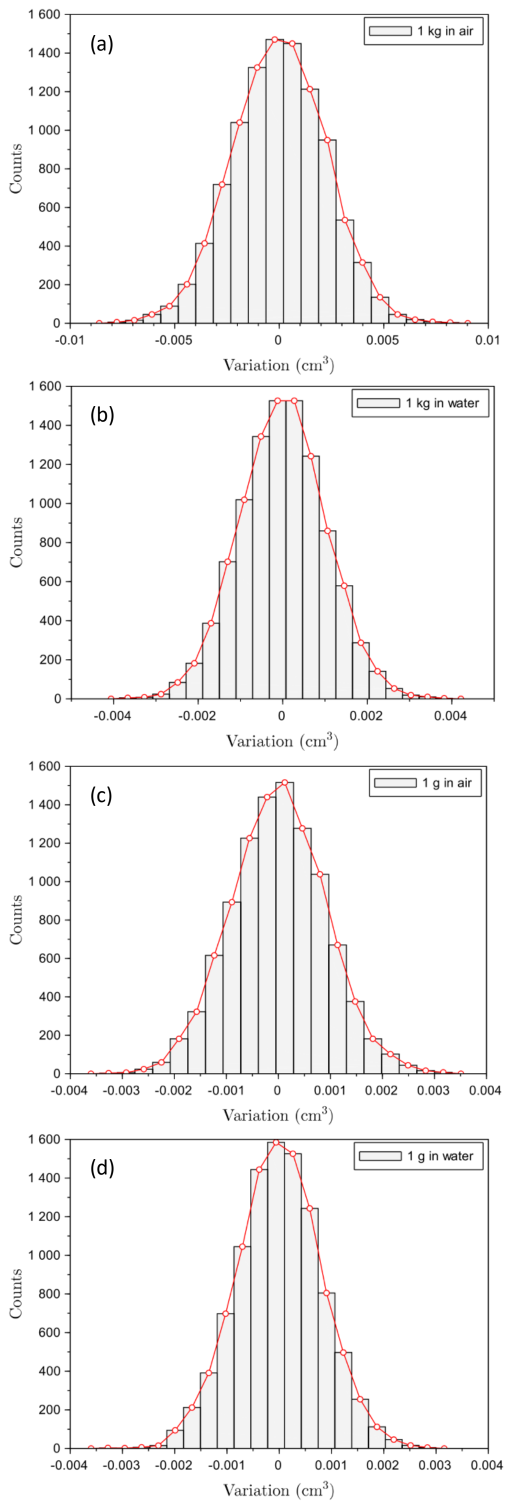

Figure 7. The simulated values of the volume are normally distributed in all four scenarios. (a) 1 kg in double weighing without support, (b) 1 kg in hydrostatic weighing without support, (c) 1 g in double weighing with support, and (d) 1 g in hydrostatic weighing with support.

subsequent calculations in the comparison of the double weighing and hydrostatic weighing, we use the CIPM formula.

6.2. Stabilization time

After we complete the hydrostatic weighing, the test weight was taken out of the water. At that moment, the surface of the weight is chemically unstable. Gravimetric measurements in the air immediately after the hydrostatic weighing show the change in mass and thus the instability of the weight very clearly (Figure 6). The weight must achieve thermal equilibrium and stable surface conditions. Therefore, a stabilization time of several hours after hydrostatic weighing is necessary (also observed by Malengo [11]).

6.3. A comparison of double weighing and hydrostatic weighing

We measured our test weights in air and in water by using double weighing and hydrostatic weighing, respectively. Then, we used Monte Carlo simulations to determine the volume and uncertainty of each test weight by varying the input quantities according to Equation (14). The output value, i.e. the volume of the test weight we are looking for, is normally distributed as well. This applies to all four scenarios: double-weighing with/without a support weight and hydrostatic weighing with/without a support weight (Figure 7).

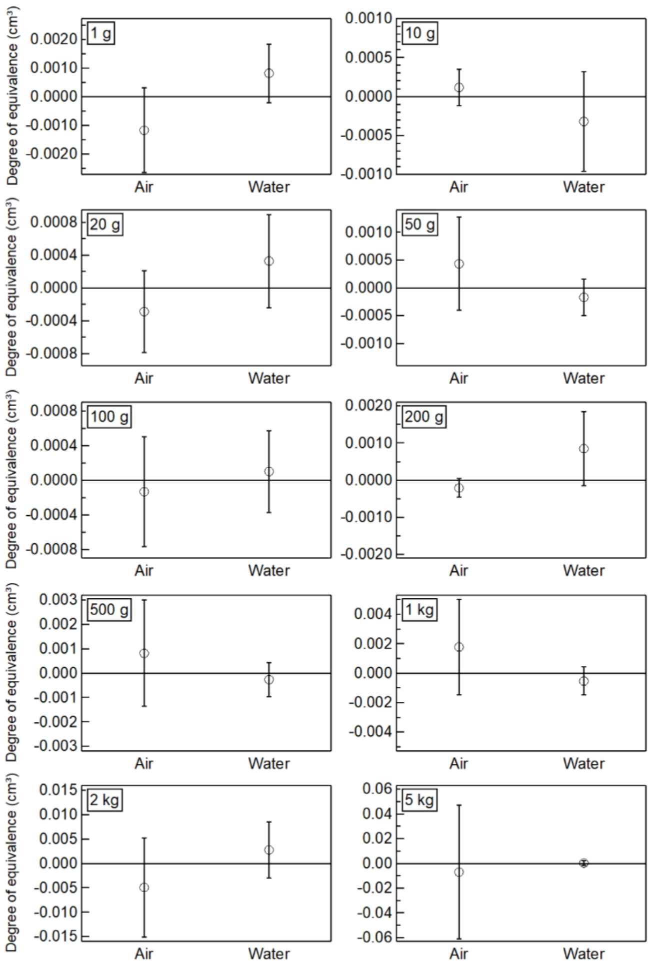

Figure 8. A direct comparison of the volumes of the test weights determined by double weighing in air and hydrostatic weighing. Error bars represent expanded standard uncertainties (k = 2).

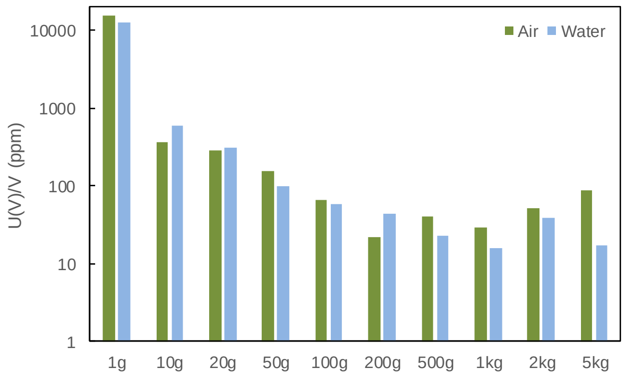

Figure 9. The relative uncertainties (k = 2) of the volume of the test weights determined in double weighing and hydrostatic weighing.

To compare the results between the two methods, we calculated the degree of equivalence and its standard uncertainty according to Cox [17].

$$d_i = V_i - V_{\mathrm{ref}} \tag{19}$$

$$u^2(d_i) = u^2(V_i) - u^2(V_{\mathrm{ref}}) \tag{20}$$

where i represents the measurement in air or in water. The comparison reference value, V_{ref}, and its standard uncertainty, $u(V_{\mathrm{ref}})$, are given by

$$V_{\mathrm{ref}} = \frac{V_{\mathrm{air}}/u^2(V_{\mathrm{air}}) + V_{\mathrm{water}}/u^2(V_{\mathrm{water}})}{1/u^2(V_{\mathrm{air}}) + 1/u^2(V_{\mathrm{water}})} \tag{21}$$

$$\frac{1}{u^2(V_{\mathrm{ref}})} = \frac{1}{u^2(V_{\mathrm{air}})} + \frac{1}{u^2(V_{\mathrm{water}})}\,, \tag{22}$$

where we use the average value of the volumes and their uncertainties from the double weighing before and after the hydrostatic measurement:

$$V_{\mathrm{air}} = \frac{V_{\mathrm{air,before}} + V_{\mathrm{air,after}}}{2} \tag{23}$$

$$u(V_{\mathrm{air}}) = \frac{u(V_{\mathrm{air,before}}) + u(V_{\mathrm{air,after}})}{2}. \tag{24}$$

The analysis shows that the volume of the test weights determined by double weighing and hydrostatic weighing are in good agreement for all the weights in the range of 1 g to 5 kg (Figure 8). The standard uncertainties are comparable between the two methods. The relative uncertainties are the largest for the 1 g weight and decrease with an increasing nominal mass up to 1 kg. Above 1 kg, the relative uncertainties increase again (Figure 9).

6.4. The influence of the different material densities

So far, we compared test weights and reference weights of the same nominal mass and density. Now, we compare the 1 kg OIML test weight with a double weighing measurement to reference weights of different material densities: an air buoyancy artefact with (2424.69 ± 1.03) kg/m³ from Mettler-Toledo, an OIML 1 kg stainless steel weight with (7914.37 ± 0.01) kg/m³ from Häfner and a platinum-iridium stack of discs with (21535.19 ± 0.07) kg/m³ from the Bureau International des Poids et Mesures. We get three values and their corresponding measurement uncertainties for the volume of the test weight. The three values are consistent. However, the measurement uncertainties are different. To compare the results, we calculate the degree of equivalence. The smallest measurement uncertainty in the volume of the test weight is achieved when the material density of the test and reference weights are the same (Table 3).

Table 3. Influence of different material densities on the determination of the volume and standard uncertainty of a 1 kg OIML test weight in double weighing measurements. d_i represents the degree of equivalence.

Density (kg/m³)	d_i (cm³)	$U(d_i)$ (cm³) k = 2
2424	0.07333	0.35417
21536	0.02362	0.09480
7914	-0.00001	-0.00004

7. DISCUSSION

We have demonstrated that the double weighing measurement is a suitable alternative to the hydrostatic measurement for the determination of the volume and density of weights ranging from 5 kg down to 1 g. The achieved results of the two methods are in good agreement, a result that was also found by Malengo [11]. The measurement uncertainties given by the two methods are comparable, and they are similar to those found by Clarkson [10], who performed the double weighing measurements between 950 mbar and 1050 mbar.

Double weighing offers several advantages: it is less time consuming than hydrostatic measurement to prepare and perform the measurement; it is easier to do the calculations; and it keeps the weight stable, both thermally and chemically. Furthermore, corrections for the volumetric expansion coefficient are not necessary. Firstly, the air-tight enclosure of the comparator is an isolated system that can be considered as a canonical ensemble representing a system in thermal equilibrium with the heat bath. Secondly, both the test weight and reference weight (usually) have the same nominal density and are exposed to the same environmental conditions. An expansion coefficient of $\gamma = 50 \times 10^{-6}$ C^{-1} – as suggested for stainless steel weights in OIML R-111 [1] – for the test and reference weight results in a difference of less than 0.1 ppm between the corrected and uncorrected volume.

The major contribution to the measurement uncertainty in the volume of the test weight comes from the uncertainty of the air density followed by the measured weighing difference and the volume of the reference weight. For instance, reducing the air density uncertainty by a factor of 100 from 10^{-4} to 10^{-6} kg/m³ leads to a decrease in the volume uncertainty of about 64 %. A decrease of the uncertainties of the measured weighing difference or the volume of the reference weight by a factor of 100 results in a decrease of the volume uncertainty of only 3.5 % or 1 %, respectively (Figure 10).

Thus, the only way to significantly reduce the volume uncertainty of the test weight is to reduce the uncertainty of the air density. The CIPM formula provides an uncertainty of 22 ppm [14], whereas we used a more conservative value of 100 ppm. In future, we will try to reduce the air density uncertainty by using additional air buoyancy artefacts. However, the air density uncertainty is dominated by the standard deviation of the weighing difference. So, the challenge is to improve the repeatability of the comparator.

We have also demonstrated that the Monte Carlo simulation is useful for estimating the measurement uncertainty of the measurand by varying the input quantities. In this way, we can overcome the difficulties in dealing with correlations among the variables.

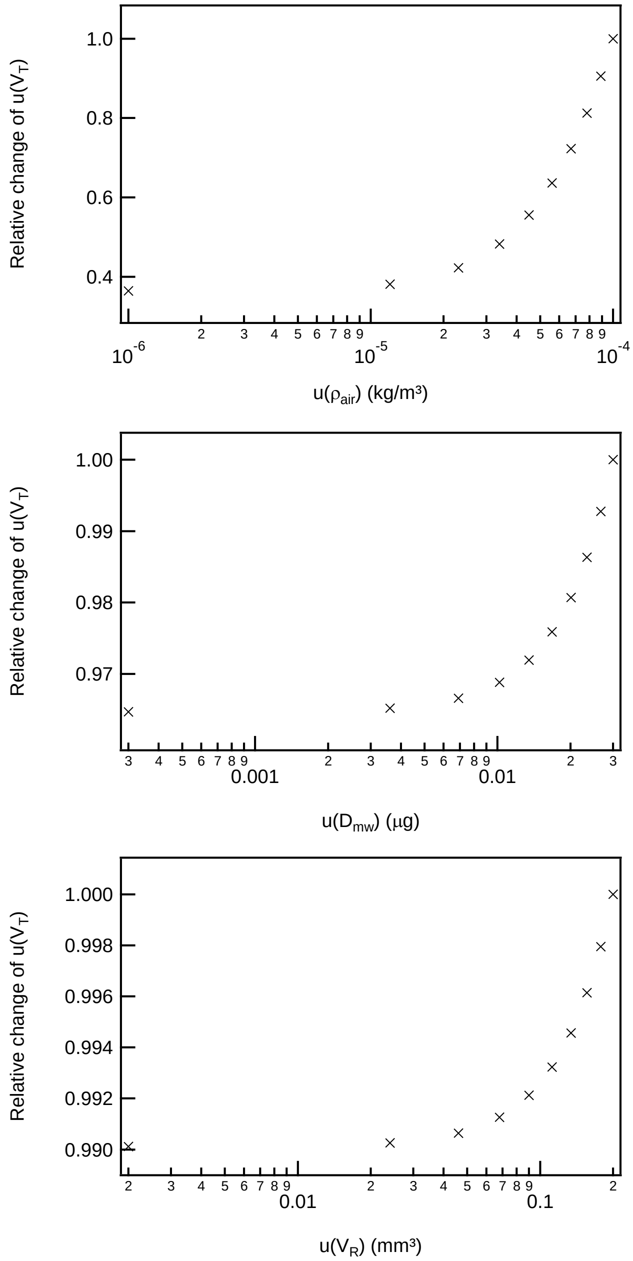

Figure 10. Major contribution to the measurement uncertainty of the test weight: (top) influence of the measurement uncertainty of the air density; (middle) influence of the measurement uncertainty of the weighing difference; (bottom) influence of the measurement uncertainty of the reference volume.

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