ACTA IMEKO
ISSN: 2221-870X
March 2024, Volume 13, Number 1, 1 - 9

A parallel approach for ultra-fast state estimation in large power system using graph partitioning theory

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ABSTRACT
This paper introduces a novel approach for multi-area state estimation in large transmission networks through the application of graph partitioning theory. By harnessing the eigenvalues and eigenvectors of the Laplacian matrix, a large-scale transmission network is partitioned into manageable sections. Within these partitions, state estimation processes run in parallel, markedly improving efficiency compared to conventional methods. Linear state estimation is employed within each area, expediting computations and making it adaptable to large-scale networks, which traditionally pose computational challenges. The method's efficacy is demonstrated through comprehensive validation, commencing with small networks and extending to real-world applications on the IEEE 118-bus test system and the 9241-bus European high-voltage transmission network. In comparison to the integrated network method, our approach has achieved state estimation answers with reduced computation time. The partitioning of the integrated network into multi areas has effectively mitigated computational loads, showcasing its potential for enhancing operational efficiency and reliability in complex power transmission systems. This approach not only offers a robust solution for state estimation but also represents a significant stride toward advancing the field of state estimation, promising to bolster the stability and performance of modern power grids.

Section: RESEARCH PAPER

Keywords: State estimation; graph; clustering; eigenvalues; applied mathematics

Citation: B. Karim Sarmadi, A. Salehi Dobakhshari, A parallel approach for ultra-fast state estimation in large power system using graph partitioning theory, Acta IMEKO, vol. 13 (2024) no. 1, pp. 1-9. DOI: 10.21014/acta_imeko.v13i1.1704

Section Editor: Laura Fabbiano, Politecnico di Bari, Italy

Received November 5, 2023; **In final form** March 4, 2024; **Published** March 2024

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1. INTRODUCTION

Within the domain of power systems, state estimate is a critical activity that is essential to guaranteeing the efficiency and safety of power system operations [1]. This fundamental process hinges on the invaluable Supervisory Control And Data Acquisition (SCADA) system, coupled with remote terminal units, which provide an abundance of measurements on active and reactive power flows, bus voltages, current, and network injections. It is important to note that these measurements exhibit a non-linear relationship with the underlying state variables [2], Gauss-Newton techniques must be implemented in commercial software in order to address them [3]. The iterative aspect of the Gauss-Newton method, which starts from an initial value and gradually moves towards convergence, is its weakness, though, and it presents considerable challenges when used with large-scale networks. Efforts to mitigate this challenge have yielded factorized methods [4]-[5] and the utilization of bilinear formulations [6]. Nevertheless, iterative solutions are still required for such approaches, especially in heavily-loaded networks. The idea of multi-area state estimation was developed as a solution to speed up state estimation in the huge networks [7]. This innovative approach permits the exchange of state estimates and boundary measurements among the local state estimators of neighbouring areas, providing an elegant solution when complete system information sharing is unfeasible. Notably, this paradigm has found its way into the realm of smart grids [8], [9], leading to considerable speed enhancements in state estimation for large-scale systems [10]. Moreover, gradient-based techniques, often employing the Gauss-Newton method within each area, have been explored [11], along with the deployment of the Kalman filter [12]. While these advancements have undoubtedly marked progress, many existing multi-area state estimation methods rely heavily on geographical information for delineating network areas, often overlooking a distinct approach to zone partitioning. Noteworthy exceptions include the application of spectral clustering for power system network partitioning during emergency conditions [13] and the use of the k-means algorithm to expedite load flow in [14]. In reference [15], a state estimation method based on Gaussian process regression is presented, utilizing data from the SCADA unit of the New York Independent System Operator. Reference [16] employs a combination of PMU and SCADA measurement units for state estimation. However, the solution method employed in this article involves iteration, aimed at enhancing the time efficiency of state estimation through a recursive strategy (as opposed to the use of a correction method). Reference [17] delves into the impact of uncertainty factors, such as renewable resources, on state estimation, discussing proposed modelling methods. In [18], the article emphasizes the significance of state estimation, its role in blackout prevention, and factors influencing the blackout on October 14, 2003, from a state estimation perspective. [19] introduces a method for detecting bad data based on artificial intelligence, exploring its implications on state estimation. In [20], the state estimation of the distribution system with distributed production resources is evaluated and implemented using PMU measurement units, alongside a bad data detection method based on a deep neural network. Reference [21] introduces a state estimation method based on floating-point operations, aiming to reduce the computational load of distribution system state estimation and achieve faster response times. [22] presents a state estimation method based on weighted least squares and a generalized loss function. [23] introduces a multi-source mode estimator using PMU measurement units, employing a data fusion strategy to enhance estimation accuracy. A method for estimating the state of the distribution network with high penetration coefficient of solar electric energy production units is presented in [24]. Understanding the critical role of state estimation in power systems is supreme for optimizing grid operations. Accurate state estimation provides crucial information on variables such as bus voltages and power flows, ensuring the safety, reliability, and cost-effectiveness of power system operations. This paper aims to contribute to the advancement of state estimation methodologies, particularly in the context of large-scale transmission networks. This paper builds upon the premise of spectral clustering introduced in [15] and harnesses the potential of linear SCADA-based state estimation described in [25], with the objective of achieving rapid and efficient parallel state estimation. Within each area, state estimation is conducted with precision, considering the local reference bus. The pivotal role played by common buses situated at the border of neighbouring areas becomes apparent, as they serve to determine the phase angles of buses in different areas concerning the global reference angle. The distinctive contributions of this paper can be summarized as follows:

- Harnessing spectral clustering to optimize exploration of the grid's topological intricacies.
- Implementing exact linear formulations for state estimation in each area, streamlining the computational process.
- Enabling parallel state estimation across distinct areas, thereby significantly curtailing the computational time required for this critical function.

This paper proceeds with an organized structure: Section 2 delves into the nuances of linear SCADA-based state estimation, Section 3 offers an in-depth exploration of power network clustering, Section 4 presents case studies to illustrate the practical application of these methodologies, and the paper concludes with Section 5.

2. LINEAR POWER SYSTEM STATE ESTIMATION

In this section, we address the idea of Linear Power System State Estimation, presenting a new formulation that represents the relationship between the system state and measurements as a linear system of equations [25]. This innovative method effectively converts state estimation problems into linear problems, eliminating the need for iterative solutions and the creation of Jacobian matrices in each iteration.

To establish the foundation of this formulation, let's begin by considering the complex electrical current flow through a network branch a-b, denoted as, $\boldsymbol{I}_{\mathrm{ab}}^{\mathrm{local}}$ which is defined as:

$$\boldsymbol{I}_{\mathrm{ab}}^{\mathrm{local}} \triangleq I_{\mathrm{ab}}\, \mathrm{e}^{\mathrm{j}\,\theta_{\mathrm{ab}}}\,. \tag{1}$$

In this equation I_{ab} represents the magnitude of the measured current phasor, and its phase angle θ_{ab} is calculated as:

$$\theta_{\mathrm{ab}} = \arctan\left(-\frac{Q_{\mathrm{ab}}}{P_{\mathrm{ab}}}\right). \tag{2}$$

This equation, according to Figure 1, allows us to express measured voltages, currents, and active and reactive powers in terms of the network's state variables, particularly with reference to the slack bus (in this case, bus number one). Complex current measurements $\boldsymbol{I}_{\mathrm{ab}}$ can be expressed as:

$$\boldsymbol{I}_{\mathrm{ab}} = \boldsymbol{I}_{\mathrm{ab}}^{\mathrm{local}}\, \mathrm{e}^{\mathrm{j}\,\delta_{\mathrm{a}}}\,. \tag{3}$$

Here, represents the unknown phase angle of the complex voltage at bus a. We further express the complex current at bus a through any branch a-b in terms of state variables as:

$$\boldsymbol{I}_{\mathrm{ab}}^{\mathrm{local}}\mathrm{e}^{\mathrm{j}\,\delta_{\mathrm{a}}} = \left(Y_{\mathrm{ab}} + \frac{1}{Z_{\mathrm{ab}}}\right)\boldsymbol{E}_{\mathbf{a}} + \left(-\frac{1}{Z_{\mathrm{ab}}}\right)\boldsymbol{E}_{\mathbf{b}}\,, \tag{4}$$

where Y_{ab} and Z_{ab} denote the transmission line shunt admittance and series impedance, respectively. The magnitude of measured voltage at each bus, represented by $\boldsymbol{E}_{\mathbf{a}}$ (assuming bus a), can be expressed in relation to state variables as:

$$E_{\mathrm{a}}\, \mathrm{e}^{\mathrm{j}\,\delta_{\mathrm{a}}} = \boldsymbol{E}_{\mathbf{a}}\,. \tag{5}$$

With these formulations in mind, we now consider E_{a} and $\mathrm{e}^{\mathrm{j}\,\delta_{\mathrm{a}}}$ as network state variables, allowing us to construct a linear system equation similar to Equation (6) for solving the state estimation problem:

$$[H]\begin{bmatrix} \boldsymbol{E}_1 \\ \boldsymbol{E}_2 \\ \vdots \\ \boldsymbol{E}_{\boldsymbol{n}} \\ \mathrm{e}^{\mathrm{j}\,\delta_2} \\ \vdots \\ \mathrm{e}^{\mathrm{j}\,\delta_n} \end{bmatrix} = Z\,. \tag{6}$$

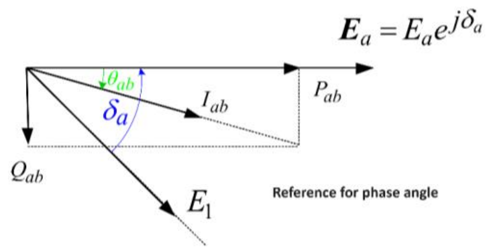

Figure 1. Relationship between measured current, voltage, reactive and active power in bus a.

Here, matrix H and vector Z are linked to network measurements and remain constant. As a result, in comparison to conventional state estimation methods, this approach facilitates faster problem solving [25]. This innovative framework sets the stage for enhanced efficiency in power system state estimation, promising accelerated computations and improved system monitoring and control. In the following sections, we will explore the application of this linear approach to real-world power system scenarios and demonstrate its advantages in achieving reliable and expedited state estimation.

3. GRAPH REPRESENTATION OF A POWER NETWORK AND NETWORK CLUSTERING: ENHANCING SYSTEM UNDERSTANDING

Within the complex domain of power system analysis, a fundamental step towards achieving a comprehensive understanding and more efficient management of power networks involves representing the system as a graph. This innovative approach entails transforming the complex web of interconnected power components into an organized, interconnected structure, which can be systematically analysed and partitioned to enhance operational efficiency. The essence of this methodology lies in the transformative notion that a power network can be conceptualized as a graph, with each bus in the network mapped to a node within the graph, and every branch of the network equated to an edge in this graph. This paradigm shift simplifies the representation of a complex power system, offering an essential framework for the application of graph clustering methods. Graph clustering techniques play a critical role here by offering an organized strategy for partitioning the connected power network into discrete areas. These methods harness the inherent patterns, connections, and dependencies within the graph to identify meaningful clusters of nodes or buses that share common characteristics or functionalities. The implications of these clusters are multi-fold, enhancing our comprehension of the network's structure, assisting in the isolation of critical regions, and facilitating more targeted analyses and actions. Graph clustering has numerous advantages, ranging from making network management easier to simplifying activities like failure detection, load flow analysis, and network optimization. Furthermore, clustering offers a framework for enhancing system resilience, enabling the identification and isolation of potential trouble spots, and facilitating the rapid deployment of corrective measures. The utility of graph representation and clustering isn't confined to traditional power systems alone; it seamlessly integrates with the evolving landscape of smart grids. In the context of smart grids, where real-time data streams and intricate interconnections are the norm, graph clustering offers a valuable tool for improving real-time monitoring, demand response, and outage management. In conclusion, the graph representation of power networks, combined with the power of graph clustering, offers a transformative approach to system analysis and management. By viewing the complex power network as an interconnected graph, we can unlock new avenues for understanding, optimizing, and enhancing the resilience of the grid. This section establishes the foundation for the subsequent exploration of how graph clustering methods can be effectively employed in practice to create distinct areas within the power system, ultimately improving its reliability and efficiency. The potential of this methodology, especially in the context of evolving smart grids, promises to revolutionize the management of power systems.

3.1. Spectral Clustering: Unveiling Network Structures

In the pursuit of understanding the underlying structures and dynamics of power networks, one powerful technique that has gained prominence is spectral clustering. This section elucidates the steps involved in the implementation of spectral clustering, a sophisticated approach that effectively reveals the intricate web of interconnected components within a power system. Spectral clustering leverages mathematical techniques and spectral graph theory to unearth latent patterns and groupings, offering a fresh perspective on the power network's topology. Spectral clustering unfolds in a series of meticulously engineered steps that we outline below:

1. Create Adjacency Matrix of the Graph: The journey begins with the creation of the adjacency matrix, a fundamental representation of the network's connections. Each element in this matrix captures the relationships between nodes, essentially encoding the structure of the graph.
2. Calculate the Laplacian Matrix: Moving onward, the Laplacian matrix emerges, a critical construct in spectral clustering. It represents the Laplacian operator of the graph and acts as a link between the topology of the network and the eigenvectors that will be examined shortly.
3. Calculate Eigenvalues and Eigenvectors: The pivotal step of spectral clustering unfolds with the calculation of eigenvalues and eigenvectors associated with the Laplacian matrix. These eigenpairs offer deep insights into the network's dynamics, effectively revealing the latent patterns concealed within the graph.
4. Identify the Second-Smallest Eigenvalue: Among the plethora of eigenvalues, it is the second-smallest eigenvalue that holds a particular significance in spectral clustering. This eigenvalue serves as a guiding light, a beacon indicating the path towards meaningful cluster formations.
5. Extract Relevant Eigenvector Rows: With the second-smallest eigenvalue identified, the corresponding eigenvector row becomes the focal point. This row, embedded with critical information, will pave the way for the creation of distinct network clusters.
6. Determine the Number of Clusters: The final, and perhaps most critical, step in the spectral clustering process is the determination of the number of clusters required to encapsulate the network's inherent structures. This decision, often guided by meticulous analysis, shapes the final outcome of the clustering process.

Application to a Small Graph: Illuminating the Process To provide an in-depth understanding of the spectral clustering process, the subsequent section applies these steps to a small graph, unravelling the intricacies and nuances involved. This practical example not only serves as an educational tool but also highlights the methodology's potential in real-world applications.

Conclusion: A Window to Unseen Power Network Structures In conclusion, spectral clustering emerges as a robust technique for uncovering hidden patterns and structures within power networks. By transforming the network into a mathematical representation and meticulously exploring the eigenvalues and eigenvectors, we gain valuable insights into the network's inherent organization. This understanding, in turn, equips us to make informed decisions regarding cluster formations, thereby enhancing our ability to manage, analyse, and optimize complex power systems. As we delve into the application of spectral clustering to a small graph, we underscore its potential to revolutionize the way we perceive and interact with power networks, promising to shed light on previously unseen structures and relationships.

3.2. Creating the Adjacency Matrix of the Graph: A Fundamental Step

The generation of the graph's adjacency matrix is a crucial step in the realm of power system analysis and network representation that lays the foundation for numerous graph-based approaches, including spectral clustering. The adjacency matrix is a mathematical representation that captures the intricate relationships between nodes and edges within the network, transforming a complex physical structure into a data-driven matrix. This section delves into the significance of the adjacency matrix and elucidates its construction through an illustrative example, offering a deeper understanding of its practical implementation. The adjacency matrix is a pivotal element in the arsenal of tools for analysing and manipulating networks, ranging from power grids to social networks. This matrix serves as a concise representation of the network's connectivity, encapsulating the relationships between nodes and edges, effectively encoding the structure of the graph. The adjacency matrix is represented mathematically as a square matrix, with each row and column representing a network node. The matrix entries are filled based on the presence or absence of edges between nodes, indicating the network connections.

Illustrative Example: Constructing an Adjacency Matrix

To shed light on the construction of the adjacency matrix, let's consider an example using Figure 2 as a reference (as shown below). The adjacency matrix, denoted as α, corresponds to the number of nodes (n) by the number of edges (m) in the graph. For Figure 2, this translates to a 6x8 matrix. The adjacency matrix α is constructed as follows:

$$\alpha = \begin{bmatrix} -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}. \qquad (7)$$

The construction of this adjacency matrix is a key preparatory step that empowers subsequent network analysis methods. This matrix provides a compact, data-driven representation of the graph's structure, setting the stage for various mathematical techniques, including spectral clustering. By employing this matrix, we gain access to a wealth of insights about the network's connectivity, patterns, and inherent structures, ultimately enhancing our ability to decipher the complexities of power systems and their underlying networks. In conclusion, the creation of the adjacency matrix of a graph serves as a fundamental gateway to network analysis, a critical step in transforming intricate power system networks into mathematically tractable entities. The adjacency matrix, with its rows and columns representing nodes and its elements encoding edge connections, offers a succinct and data-driven view of network structures. Its application extends beyond spectral clustering, facilitating various network analysis techniques that enable a deeper understanding of power system dynamics. As we move forward in our exploration of power network analysis, this adjacency matrix will remain a pivotal tool, shedding light on the intricate relationships and patterns within power networks and setting the stage for enhanced network management and optimization.

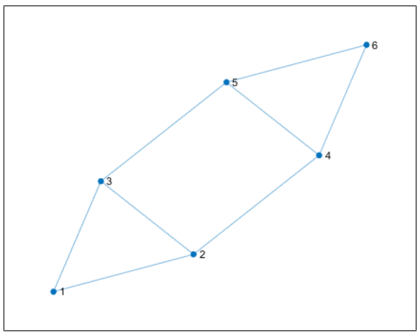

Figure 2. An illustrative graph is employed as an exemplar in this context.

3.3. Creating the Laplacian Matrix:

In the context of analysing power systems, the Laplacian matrix emerges as a critical intermediary step, bridging the gap between the established adjacency matrix and sophisticated network analysis techniques, particularly spectral clustering. This mathematical construct encapsulates the intricacies of network connectivity, serving as a vital representation of interconnections. Inspired by effective methods such as those detailed in [15], the Laplacian matrix is methodically crafted. It hinges on the computation of diagonal entries within each row, representing the sum of non-diagonal entries, thus quantifying the interconnectedness of nodes. This matrix, denoted as L, is crucial for spectral analysis, as it profoundly influences eigenvalue distributions and subsequent cluster formation. Illustrated using the graph in Figure 2, the Laplacian matrix emerges as an indispensable tool for unveiling network structures, shedding light on underlying patterns and facilitating applications ranging from fault detection to network optimization. As our exploration of power network analysis unfolds, the Laplacian matrix remains a fundamental component, empowering us to gain deeper insights into the complex web of power systems and enhancing our ability to manage and optimize these intricate networks.

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 3 & -1 & -1 & 0 & 0 \\ -1 & -1 & 3 & -1 & 0 & 0 \\ 0 & -1 & 0 & 3 & -1 & -1 \\ 0 & 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & -1 & -1 & 2 \end{bmatrix}. \qquad (8)$$

3.4. Calculating Eigenvalues and Eigenvectors:

In our quest to fathom the intricate structures of power networks, the computation of eigenvalues and eigenvectors assumes a pivotal role, particularly in the context of the Laplacian matrix derived from the graph illustrated in Figure 2. These mathematical constructs, denoted as E and V, respectively, unlock the latent patterns and inherent dynamics concealed within the network. Eigenvalues, encapsulated within matrix E, serve as insightful indicators of the network's behaviour, shedding light on its structures. Eigenvectors, found in matrix V, offer a more profound understanding of the network's underlying patterns and relationships. This meticulous process of computing eigenvalues and eigenvectors enhances our capacity to analyse and manage the complexities of power systems, with direct relevance to spectral clustering and network analysis. As our exploration of power network dynamics unfolds, these matrices remain essential tools, empowering us to gain deeper insights into network behaviour, cluster formation, and ultimately enhancing our ability to optimize these intricate systems.

$$E=\begin{bmatrix}5&0&0&0&0&0\\0&4&0&0&0&0\\0&0&3&0&0&0\\0&0&0&3&0&0\\0&0&0&0&1&0\\0&0&0&0&0&0\end{bmatrix} \tag{9}$$

$$V=\begin{bmatrix}0&-0.4082&0.5773&-0.0040&-0.5774&0.4082\\-0.5&0.4082&-0.2921&-0.4980&-0.2887&0.4082\\0.5&0.4082&-0.2852&0.5020&-0.2887&0.4082\\0.5&-0.4082&-0.2921&-0.4980&0.2887&0.4082\\-0.5&-0.4082&-0.2852&0.5020&0.2887&0.4082\\0&0.4082&0.5773&-0.0040&0.5774&0.4082\end{bmatrix}. \tag{10}$$

3.5. Graph Clustering: Revealing Network Structures

The process of graph clustering stands as a pivotal step in unravelling network structures, particularly in the context of spectral clustering. In this section, we delineate the intricate procedure of partitioning the network into distinct clusters, underpinned by the second column of the eigenvector, corresponding to the second-smallest eigenvalue. The ensuing sorting operation is not just a mere rearrangement of values; it's a strategic endeavour that reveals underlying patterns and relationships within the network. After extracting the second column of the eigenvector, associated with the second-smallest eigenvalue, the sorting operation is executed. This meticulous process orders the values, initially ranging from the smallest to the largest, with precision. The resultant sorted eigenvalue vector, denoted as λ_2, is a crucial step in the spectral clustering methodology. It ensures that the data is prepared for the subsequent clustering procedure, illuminating relationships and patterns that might otherwise remain hidden. The sorting operation not only brings order to the data but also sets the stage for network division. The sum of entries in the sorted vector is zero, a critical characteristic that facilitates network separation. By strategically dividing the vector into two distinct components, one encompassing negative values and the other positive values, two different vectors emerge: A and B. The creation of clusters is the culmination of this process. We extract two clusters from the original graph, as shown in Figure 3, a visual representation of the partitioning effort. The clusters encapsulate nodes with shared characteristics, unveiling hidden network relationships. Figure 4 illustrates the clustering axis, offering a graphical depiction of the eigenvalues related to nodes. In conclusion, the process of graph clustering, guided by spectral analysis techniques, offers a profound means to dissect and understand network structures. The careful sorting of eigenvalues and the subsequent creation of clusters not only reveal inherent patterns but also provide insights into network relationships. As we navigate the complexities of power network analysis, graph clustering remains a valuable tool, enhancing our ability to decipher the intricate interplay of nodes and edges within the network. These insights are pivotal for applications such as fault detection, load balancing, and network optimization, making graph clustering an indispensable technique in our pursuit of enhanced power system management and analysis. In the context of our analysis, clusters A and B, distinguished through the graph clustering process, emerge as distinct entities separated from the eigenvalue λ_2. In our analysis, increasing cluster count accelerates state estimation. Yet, clustering a large network into more clusters is time-consuming. Our approach ensures equal node distribution, balancing faster estimation with computational demands.

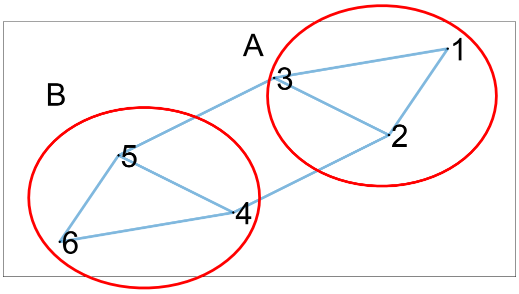

Figure 3. Cluster that separated from Figure 2.

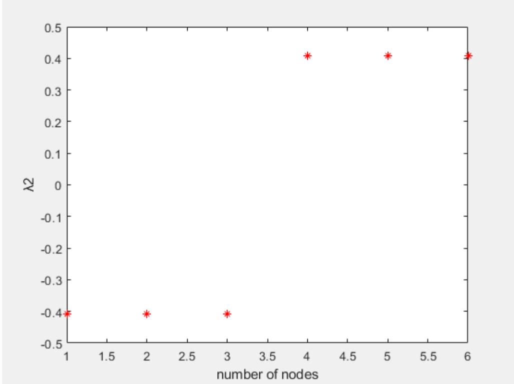

Figure 4. Eigenvalues related to nodes.

$$\lambda_2=\begin{bmatrix}-0.4082\\0.4082\\0.4082\\-0.4082\\-0.4082\\0.4082\end{bmatrix}\text{ after sorting: }\lambda_2=\begin{bmatrix}-0.4082\\-0.4082\\-0.4082\\0.4082\\0.4082\\0.4082\end{bmatrix}, \tag{11}$$

$$A=\begin{bmatrix}-0.4082\\-0.4082\\-0.4082\end{bmatrix}, B=\begin{bmatrix}0.4082\\0.4082\\0.4082\end{bmatrix}. \tag{12}$$

4. SIMULATION RESULTS

4.1. Case Study 1: IEEE-118 Bus Test Network

In this study, all voltage parameters are expressed in “per unit” (pu), the first validation we undertake delves into the IEEE-118 Bus Test System, a renowned benchmark in the realm of power network analysis. This system, portrayed as a graph in Figure 5, serves as a comprehensive testing ground for our spectral clustering approach. After applying the spectral clustering methodology, the network gracefully bifurcates into three distinct areas, as vividly depicted in Figure 6.

With the network segmented into these areas, we proceed to tackle the challenging task of solving the state estimation problem. The results of this endeavour are meticulously tabulated in Table 1, which offers a comparative analysis of voltage magnitude and voltage angle for four specific buses within the network. Furthermore, to provide a more comprehensive understanding, we present graphical representations comparing voltage magnitudes and angles in both integrated and multi-area forms. Figure 7 illustrates a comparison of voltage magnitudes, while Figure 8 depicts a comparison of voltage angles in integrated and multi-area forms.

Table 1. Comparison of Voltage Parameters in Multi-Area and Integrated Forms.

Bus number	Estimated voltage magnitude in multi-area form (pu)	Estimated voltage angle in multi-area form (degree)	Estimated voltage magnitude in integrated form (pu)	Estimated voltage angle in integrated form (degree)	True voltage magnitude (pu)	True voltage angle (degree)
1	0.955	10.973	0.95499	10.972	0.955	10.973
2	0.9714	11.5125	0.97139	11.512	0.97139	11.5125
3	0.967709	11.856	0.967691	11.856	0.96769	11.856
4	0.998	15.574	0.998	15.574	0.998	15.574

Due to space limitations, a comprehensive of voltage magnitudes and angles in the multi-area and integrated forms. These graphical representations unmistakably illustrate that the results obtained using the proposed clustering algorithm align seamlessly with those of the integrated approach, affirming the accuracy and efficacy of our methodology.

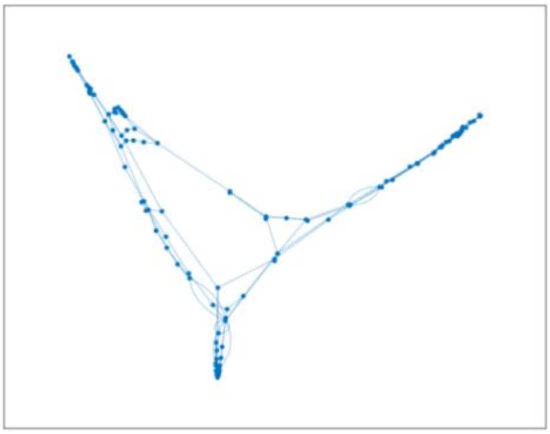

Figure 5. The IEEE-118 Bus Network in Graph Form.

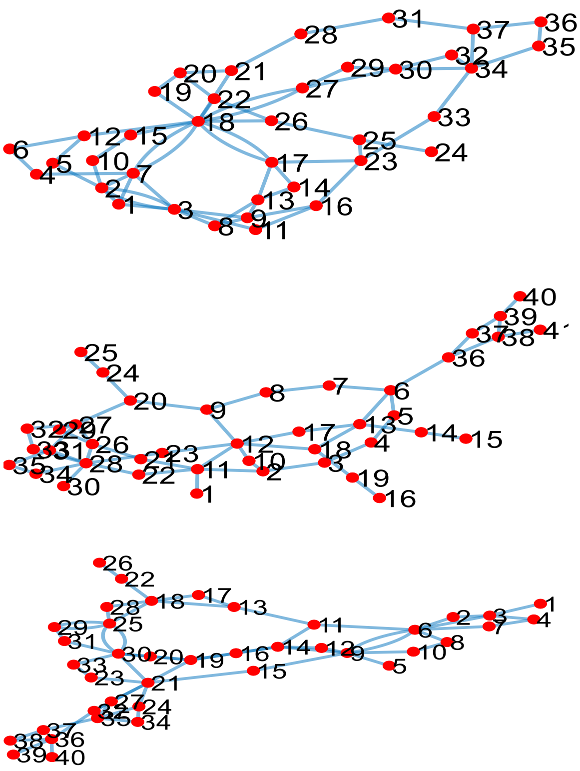

Figure 6. Three separated areas from the 118bus network.

In essence, this case study substantiates the effectiveness of our spectral clustering approach in power system state estimation. The parallel and integrated algorithms produce congruent results, reaffirming the precision of our clustering method, thus enhancing our capabilities in managing and optimizing complex power networks.

4.2. Case Study 2: 9241-Bus European Network

Our second case study embarks on a complex and extensive network, the 9241-Bus European Network [26], [27], depicted in graph form in Figure 9. This intricate system challenges our spectral clustering methodology, and the results are a testament to the approach's capabilities. The network divides into several distinct areas, each with its unique characteristics, as visually represented in Figure 9(a) through (e). These areas hold the key

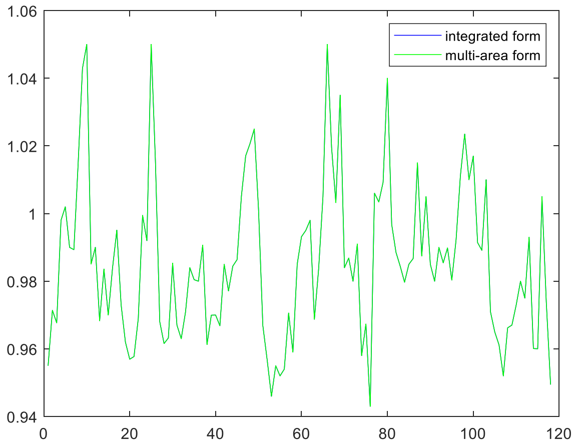

Figure 7. Comparison of voltage magnitudes in integrated and multi-area forms.

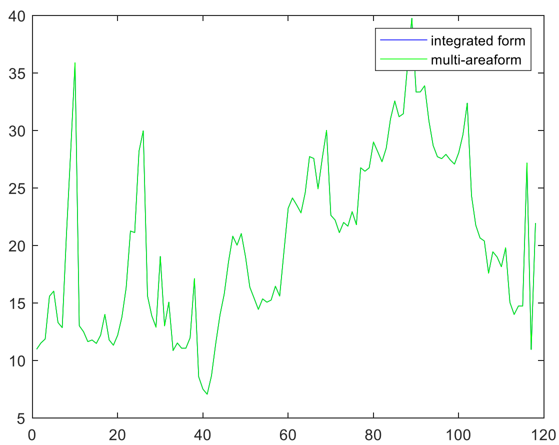

Figure 8. Comparison of voltage angles in integrated and multi-area forms.

Table 2. Magnitude and angle of estimated and measured voltages.

Bus number	Estimated voltage magnitude (pu)	Estimated voltage angle (degree)	Measured voltage magnitude (pu)	Measured voltage angle (degree)
1	1.007	-36.571	1.007	-36.571
2	1.031	-8.434	1.013	-8.434
3	1.017	-21.085	1.017	-21.085
4	1.022	-6.892	1.020	-6.892

to understanding the intricate web of interconnections within the European power network. To comprehensively validate the accuracy of our method, we meticulously examine the values of voltage magnitude and angle in three distinct forms: integrated, multi-area, and true values. Table 2 presents a detailed comparison of the measured and estimated voltage parameters for specific buses within the network. These values provide insights into the precision of our spectral clustering approach, reinforcing its value in power system state estimation. To further accentuate the efficiency of our proposed multi-area state estimator, we conduct a comparison of processing times. Table 3 presents a performance evaluation, featuring the processing time for each of the network's areas, substantiating the swiftness of our spectral clustering approach. These simulations are executed on an i7-3770 CPU, and the results are a testament to the computational efficiency of the method. In summary, our case study of the 9241-Bus European Network underscores the robustness and computational efficiency of our proposed spectral clustering-based multi-area state estimator. The segmentation of the network into distinct areas, coupled with the accuracy of voltage parameter estimation, offers a practical solution for real-world power system state estimation challenges. These findings further solidify the relevance of our methodology in enhancing power network management and optimizing complex European power systems.

4.3. Root-Mean-Square Error-Index: Quantifying Estimation Precision

To comprehensively assess the performance of our state estimation method, we employ a critical metric - the Root-Mean-Square Error (RMSE). The RMSE serves as a key benchmark in quantifying the disparity between the estimated values and the true values, providing valuable insights into the accuracy of our state estimation. The RMSE for voltage magnitude $RMSE(E)$ and voltage angle $RMSE(\delta)$ are calculated as follows:

$$RMSE(E) = \frac{1}{n}\sqrt{\sum\nolimits_{i=1}^{n}\left|E_i^{\text{est}} - E_i^{\text{true}}\right|}\,, \tag{13}$$

$$RMSE(\delta) = \frac{1}{n}\sqrt{\sum\nolimits_{i=1}^{n}\left|\delta_i^{\text{est}} - \delta_i^{\text{true}}\right|}\,. \tag{14}$$

In Equation (13), E_i^{est} represents the value of the estimated voltage magnitude, and E_i^{true} is the corresponding true value. The parameter n indicates the number of simulations performed.

Table 4. Comparative RMSE Analysis in Integrated and Multi-Area Forms.

Scenario	Type of network	Error Value σ	***RMSE*(*E*)**	***RMSE*(δ)**
1	Integrated 9241 bus network	0.002	0.10248	0.01459
2	Area number one of 9241 bus network	0.002	0.10250	0.01398
3	Area number two of 9241 bus network	0.002	0.10244	0.01337
4	Area number three of 9241 bus network	0.002	0.10239	0.01422
5	Area number four of 9241 bus network	0.002	0.10252	0.01450
6	Integrated 9241 bus network	0.005	0.4124	0.0807
7	Area number one of 9241 bus network	0.005	0.5433	0.0454
8	Area number two of 9241 bus network	0.005	0.5223	0.0793

In Equation (14), δ_i^{est} denotes the estimated voltage angle, and δ_i^{true} is the actual voltage angle. Similar to RMSE(E), 'n' signifies the number of simulations runs.

4.4. Comparative RMSE Analysis

Given space constraints, we present a summary of RMSE values exclusively for the 9241-Bus Network in Table 4. These RMSE values encompass various scenarios, each with its own standard deviation (σ) for measurement errors. This comprehensive analysis covers both integrated and multi-area forms, providing a detailed view of estimation precision in different network segments. These RMSE values offer a comprehensive understanding of estimation accuracy under different error conditions, reaffirming the robustness of our multi-area state estimator. They serve as a vital tool in assessing the precision of state estimation, ensuring the reliability of our approach in the intricate realm of power network analysis.

5. CONCLUSION

In this study, we have introduced a pioneering multi-area state estimation method, harnessing SCADA measurements for rapid analysis of large-scale power networks. By strategically partitioning the network and conducting linear state estimation in parallel, we have achieved unprecedented speed without compromising accuracy. Our approach has been rigorously tested on two substantial test systems, consistently delivering swift results. This transformative method significantly reduces the time required for solving state estimation challenges, making it a potent tool for managing complex power systems. As power networks grow in scale and intricacy, our methodology offers an efficient and accurate solution that has the potential to reshape power system management on a global scale, driving us toward a more sustainable and efficient energy future.

Table 3. Speed comparison between proposed multi-area state estimator and integrated state estimator.

Type of network	Area number one of 9241 bus network	Area number two of 9241 bus network	Area number three of 9241 bus network	Area number four of 9241 bus network	Integrated 9241 bus network
Processing time (s)	0.144	0.0917	0.420	0.205	0.538

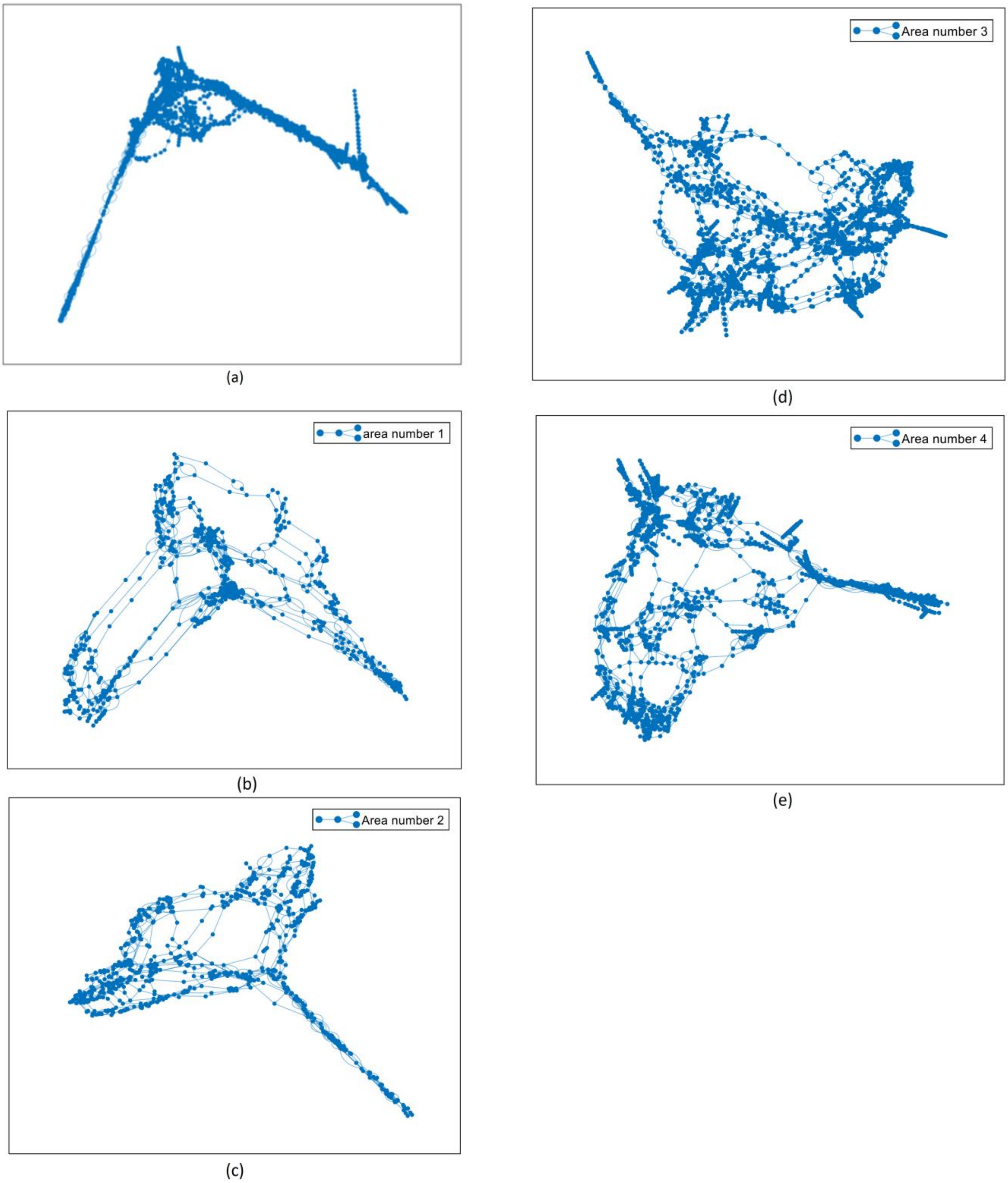

Figure 9. Visualization of the 9241-Bus European Network (a), first area (b), second area (c), third area (d) and fourth area (e)

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