A theoretical model for uncertainty sources identification in tip-timing measurement systems

Giulio Tribbiani¹, Lorenzo Capponi², Tommaso Tocci³, Tiberio Truffarelli³, Roberto Marsili³, Gianluca Rossi³

¹ CISAS G. Colombo, University of Padova, Via Venezia 15, 35131 Padova, Italy
² Department of Aerospace Engineering, University of Illinois Urbana-Champaign, 104 S. Wright Street, 61801 Urbana, USA
³ Department of Engineering, University of Perugia, Via G. Duranti 93, 06125 Perugia, Italy

ABSTRACT
This paper presents a theoretical analysis of uncertainty sources in measurement techniques used to determine vibrations of turbomachinery blades using stationary sensors mounted on the casing of the turbomachine. A mathematical model based on fundamental physical principles is proposed, and two different measurement set-ups are evaluated. One set-up uses a reference sensor to measure the passage of an undeformed part of the blades (blade base), while the other set-up does not involve the use of a reference sensor, with both sensors facing the blades tip (deformed part). The intrinsic uncertainty of these methods and the performance of the complete measurement chain are defined. The analysis of the measurement technique leads to conclusions about the practical set-up and possible performances of these measurement techniques.

1. INTRODUCTION
In operating conditions, monitoring turbomachinery blades vibrations is necessary for improving structural health techniques and for validating the dynamics of the system [1]. In fact, uncontrolled vibrations at or close to natural frequencies, combined with high thermo-mechanical loads, can lead to damage for the machinery and increase the risk of unexpected failures [2]. Traditionally, dynamical analysis and measurement of vibrations of rotating blade have been performed using strain gauge sensors [3]–[5]. However, this technique can have impact on the monitoring procedures [6]. In fact, although strain gauges have high accuracy and their usage is well established, they have relatively limited lifetime in high-temperature conditions, and, as intrusive technique, their installation on rotating systems and their data transmission are complex steps to achieve [5]. To overcome these problematics, non-contact and non-intrusive blade vibration measurement techniques have been developed in the past years, based on vibration, temperature [7] and ultrasound approaches [5], [8]–[12]. One of the most successful and promising on-site techniques for axial turbomachinery blade dynamics measurements is called Blade Tip-Timing (BTT) [13], [14]. BTT is based on the measurement of instantaneous blade tip deflection by detecting advances or delays in the Time of Arrival (TOA) of the blade tip by means of sensors installed on the casing at fixed angular positions [15]. BTT can use different types of sensing probes, such as laser probes [16], [17], microwave sensors [4], [18], capacitive sensors [19], magneto resistive sensors [20], [21], or optical probes [6], [22]–[24]. While the measurement principle is the same for all different sensors, the optical probes are usually preferred for their performances in accuracy and resolution [6], [25]. The blade-sensor interaction generates electrical pulse signals. In ideal conditions, where no blade vibration is considered (i.e., rigid blade), the TOA is determined a priori given the geometry and dynamic of the system [26], [27]. On the other side, considering blades as vibrating flexible structures, blade deflections lead to delays or advances of the blade tip with respect to the expected TOA [13], [28], [29]. These shifts in the TOA are extracted and used for determining the amplitude of deflection of each blade [30]. For this reason, a thorough identification and definition of sources...
of uncertainties for tip timing measurement systems (i.e., uncertainty on the measurement of the TOA) is essential for obtaining a detailed information on the dynamics of the system [31]. This will lead to a reliable structural health monitoring, giving insights on instrumentation best practices and information for validating numerical models [32].

Different studies have been carried out on BTT measurement systems, focusing on the uncertainty of specific cases technique [33]–[36].

This study proposes a complete overview of the uncertainty sources of a generic BTT system, and their influence on the estimation of the TOA. With regard to this, two different probes configuration are analysed, typically employed in BTT measurement systems, either with or without a reference sensor.

2. MATERIALS AND METHODS

A typical BTT measurement system allows the sampling of the relative displacement law \( s(t) \), supposed periodic, between two points of the blade, expressed in a reference system rotating with the blade itself, using a non-contact sensor installed on the machine casing. The sensors are usually paired in couple, with one being the reference sensor and the other the sensor used to measure the blade deformation. The first one is typically installed at the base of the blade, where there is no deformation, the other is placed at the blade tip. By analysing the resulting signals, it is possible to obtain the instants in which the blade passes in front of the sensors and measure its deformation. If the tip deflects from the base, there will be a \( \Delta t \) in delay or in advance, between the signals obtained by the reference and the non-reference sensor. In addition to this, the pulse width of the passage signal is representative of the duration of the passage itself. In this way, a displacement and velocity value can be measured for each transition of the blade in front a pair of sensors.

The problem lies in the fact that the displacement \( s(t) \), as well as the velocity \( v(t) \), i.e., its first derivative, have generally higher frequency components than the passage frequency in front of the sensors. Hence, \( s(t) \) and \( v(t) \) are sub-sampled. Nevertheless, it is possible, under particular conditions discussed in [37], to calculate the harmonics of \( s(t) \), even if the sampling is performed in such conditions. A method is described that allows to obtain the discrete spectrums of the blade displacement and velocity, \( s(t) \) and \( v(t) \), by solving a system of \( 2M+1 \) non-linear equations, with \( M \) being the number of harmonics to be evaluated.

3. VELOCITY MEASUREMENT PRINCIPLE AND DISPLACEMENT USING REFERENCE SENSOR

In this section it will be described the working principle of a BTT measuring system using a reference sensor.

Considering a blade on a rotating drum subject to bending deformations as shown in Figure 1. Let \( \omega \) be the angular rotation velocity of the disk.

Two reference systems are considered: a fixed one \((O, \ i')\) and a second rotating one \((O, \ j)\). Both \( i \) and \( i' \) are curvilinear abscissae defined on the circumference containing the blade tips, of radius \( R \). In this reference systems, \( s \) identifies the position of the undeformed blade tip, hence \( s' \) represents the circumferential component of the blade tip displacement over time. It is assumed that this displacement changes in time according to a periodic law:

\[
s(t) = S \cdot f(t),
\]

where \( f(t) \) is a periodic function of period \( T \), fundamental frequency \( f_0 = 1/T \) and unitary amplitude, multiplied by a scalar \( S \), greater than 0. The function \( s(t) \) is the measurand, sampled by the BTT measurement technique. Deriving the displacement \( s(t) \) the velocity \( v(t) \) is obtained. Considering the relative motion between drum and casing:

\[
v(t) = v'(t) - v_0'(t)
\]

with \( v'(t) \) being the velocity in the rotating system and \( v_0'(t) \) the absolute velocity due to drum rotation.

The blade deflection can be measured by analysing the signals generated by the two sensors arranged as shown in Figure 1. Figure 2 shows the qualitative representation of these two signals. The blade passage is detected when the signal exceeds a threshold value \( \Delta S \). In conditions of undeflected blade, there will be a fixed time delay between the threshold crossing of signals from sensor 1 and 2. If the blade is deformed, the time delay \( \Delta t_{AC} \) will differ from the one obtained in the previous condition. Hence, it is possible to measure the blade deformation by measuring \( \Delta t_{AC} \).

This method is valid if some key hypotheses are met; these assumptions will inevitably lead to an increase in measurement uncertainty.

First, it is considered that the time interval \( \Delta t_{AC} \) does not vary due to the irregularity of the rotation, hence the angular speed \( \omega \) of the drum remains strictly constant. Second, the \( \Delta t_{AC} \) in the undeformed configuration is equal to the mean \( \overline{\Delta t}_{AC} \), when the blades are vibrating:

\[
\overline{\Delta t}_{AC} = \frac{1}{N} \sum_{i=1}^{N} \Delta t_{AC}(i).
\]

With these hypotheses, the time delays due to the oscillation \( s(t) \) can be calculated from:

\[
\delta(\Delta t_{AC}) = \Delta t_{AC} - \overline{\Delta t}_{AC}.
\]

Associating the times intervals \( \Delta t_{AB} \) and \( \Delta t_{CD} \) respectively to the distance \( \delta_{AB} \) and \( \delta_{CD} \) along which the sensors see the blade tip, defined as distances corresponding to the crossing of the threshold \( S \) as illustrated in Figure 2, it is possible to compute the average velocity respectively between A and B and between C and D by the following two equations:
Figure 2. Time history of the outputs of sensors 1 (reference on the base) and 2 (on the tip) typically in Volt: the blade is passing when the output exceeds the threshold value \( S_{threshold} \).

\[
\nu' = \frac{\delta_{AB}}{\Delta t_{AB}} \cdot \frac{R}{r} \tag{5}
\]

\[
\nu' = \frac{\delta_{CD}}{\Delta t_{CD}} \cdot \frac{R}{r} \tag{6}
\]

Being the duration of these signals very short, such quantities can be reasonably considered as the instantaneous velocity of the blade tip and base. Assuming that both \( s \) and the blade tip velocity remain constant during the passage occurred in \( \Delta t_{CD} \), the displacement can be calculated as follows:

\[
s(i) = \frac{\delta_{CD}}{\Delta t_{CD}} \cdot \delta(\Delta t_{AC}) \tag{7}
\]

The value of \( \nu' \), assuming that \( \nu(i) \) remains constant during \( \Delta t_{AC} \) using (2) can be expressed as:

\[

\nu(i) = \frac{\delta_{CD}}{\Delta t_{CD}} - \frac{\delta_{AB}}{\Delta t_{AB}} \cdot \frac{R}{r} \tag{8}
\]

The usage in (7) and (8) of the instantaneous velocities of rotation allows a considerable extension of the applicability of the BTT measurement methods on machines with high degrees of irregularity compared to the ones presented in [15], [30], [38], where velocity is calculated as \( \omega R \), hence assumed constant during the entire revolution. In the method proposed here, \( \nu(i) \) remains constant only in the \( \Delta t_{AC} \) period.

In fact, by using (7) and (8) at each instant of passage of the blade in front of a couple of fixed sensors, \( s(i) \) and \( \nu(i) \) values are calculated.

4. METHOD WITHOUT REFERENCE SENSOR

In many applications, it is not possible to install a reference sensor in a position useful to detect the passage of a not deformed part of the blade, i.e., the blade base. In these cases, it is possible to install the sensors as shown in Figure 3, facing only the blade tip. It is assumed once again that the time interval does not change due to the irregularity of the rotation between the two pulses and that the \( \Delta t \), relative to the passage of the blade in the undeformed configuration, is equal to the average value of \( \Delta t \) measured over a certain number of revolutions. The time delays \( \delta(\Delta t(i)) \) are therefore calculated with respect to these average values. Assuming a strictly constant tangential velocity equal to \( \omega R \), the measured blade deflection \( S_i \), as shown in Figure 4, can be calculated by the following relation:

\[
S_i = \omega \cdot R \cdot \delta(\Delta t(i)) \tag{9}
\]

Hence, for each couple of pulses, we can have one pair of displacement samples \( s \) and \( s' \). Average displacement can be estimated by \( (s + s')/2 \) and a velocity value \( v \) can be estimated by \( (s' - s)/t_0 \) where \( t_0 \) is the pulse width.

To estimate the vibration frequency, we can use the following considerations. If the displacement of the blade is:

\[
s = A \cdot \cos(\omega t). \tag{10}
\]

Deriving it, the blade tip velocity \( v \) is obtained and can be expressed by:

\[
v = -A \cdot \omega \cdot \sin(\omega t). \tag{11}
\]

By computing the ratio between the minimum and maximum value of \( s \) and \( v \), it is possible to estimate the vibration frequency:

\[
\hat{f} = \frac{\nu_{max} - \nu_{min}}{s_{max} - s_{min}}. \tag{12}
\]

5. VIBRATION HARMONICS CALCULATION

The sampling of blade tip vibration by BTT technique produces a series of displacement and velocity samples on the rotating reference system: one for each passage of the rotating blade in front of the couple of the sensors. At each instant of

Figure 3. Measuring technique without reference sensor.

Figure 4. Blade Vibration and Sensor Output over time; Measured blade deflection at the \( i \)-th passage, resulting from the variation \( \delta(\Delta t(i)) \).
passage, i.e., at time \( t(i) \), a deflection value \( s(i) \) and a velocity value \( v(i) \) are calculated.

The relative motion of the blade tip can be in general described by a periodic function over time. As vibrations, this periodic motion can be effectively described by a relatively small number of harmonics, with the energy being almost completely contained in the first harmonics.

The two functions \( s(t) \) and \( v(t) \) can be in general approximated by a Fourier series limited to \( M \) harmonics:

\[
s(t) = \sum_{j=1}^{M} A_j \cdot \cos(2 \pi v_0 j t) + B_j \cdot \sin(2 \pi v_0 j t) \tag{13}
\]

\[
v(t) = 2 \pi v_0 \cdot \sum_{j=1}^{M} -A_j \cdot \sin(2 \pi v_0 j t) + B_j \cdot \cos(2 \pi v_0 j t). \tag{14}
\]

The values of \( s(t) \) and \( v(t) \) are measured at the instants \( t(i) \), not necessarily equally distributed in time, as the sensors could be placed on the turbine casing at not equi-spaced angles. These values \( s(i) \) and \( v(i) \) can be used to write a system of nonlinear equations with unknowns that are the fundamental frequency and the \( 2M \) coefficients \( A_j \) and \( B_j \) of the harmonics. Thus, \((2M+1)\) equations can be written; to solve this system, it is necessary to get enough samples of \( s(i) \) and \( v(i) \), at least \((2M+1)\) or more. So, in principle, by the acquisitions of the \((2M+1)\) \( s(i) \) and \( v(i) \) samples and the solution of a non-linear system of equations, it is possible to estimate the discrete spectrum of the harmonics of the vibration. With more than \((2M+1)\) samples of \( s(i) \) and \( v(i) \), a least square approach is also possible. The limits of these ideas have been discussed in [37], where a complete theoretical approach is illustrated.

6. UNCERTAINTY SOURCES

The hypotheses made for the description of the BTT measurement principle of velocity and displacements samples of a rotating blade tip, limit the applicability of the methods. The main parameters of the rotating disk with a vibrating blade are \( \omega \), \( R \), \( i \), \( v \), and \( s \). A first uncertainty source can be identified in the variation of the \( \Delta t_{AC} \) due to the irregularity rotation, expressed by:

\[
i = \frac{\omega_{\max} - \omega_{\min}}{\omega_{\text{average}}}, \tag{15}
\]

with \( \omega \) being the angular speed of the rotating drum. As a first linear approximation, this uncertainty source can be estimated as follows:

\[
E_i = i \cdot \Delta t_{AC} \tag{16}
\]

It was previously assumed that \( s(t) \) and \( v(t) \) do not change in the time period \( \Delta t_{AC} \). A linear estimation of changes of \( s(t) \) and \( v(t) \) during \( \Delta t_{AC} \) can be expressed by the following relations:

\[
\frac{ds}{dt} = 2 \pi \cdot s \cdot \Delta t_{AC} \tag{17}
\]

\[
\frac{dv}{dt} = 4 \pi^2 \cdot v^2 \cdot s \cdot \Delta t_{AC} \tag{18}
\]

Assuming the time history of \( s(t) \) is a simple sinusoidal vibration:

\[
s(t) = S \cdot \sin(2 \pi v t) \tag{19}
\]

the following relations are obtained:

\[
\frac{ds}{dt} = 2 \pi \cdot v \cdot s \tag{20}
\]

\[
\frac{dv}{dt} = 4 \pi^2 \cdot v^2 \cdot s \tag{21}
\]

By replacing these values in (17) and (18), it is possible to estimate the maximum effect on uncertainty due to changing of \( S \) and \( v \) over \( \Delta t_{AC} \):

\[
E_s = 2 \pi \cdot v \cdot s \cdot \Delta t_{AC} \tag{22}
\]

\[
E_v = 4 \pi^2 \cdot v^2 \cdot s \cdot \Delta t_{AC} \tag{23}
\]

Considering that the maximum of \( \Delta t_{AC} \) is obtained when the absolute speed of the blade tip is at minimum, (22) and (23) become:

\[
E_s = 2 \pi \cdot v \cdot s \cdot \frac{\delta_{AC}}{\omega R - 2 \pi v s} \tag{24}
\]

\[
E_v = 4 \pi^2 \cdot v^2 \cdot s \cdot \frac{\delta_{AC}}{\omega R - 2 \pi v s} \tag{25}
\]

(16), (24) and (25) estimate the uncertainty amplitude, therefore can be used to identify the conditions of applicability of the measurement methods previously described.

Knowing this, it is possible to change the parameters of the measurement system, in order to properly choose and install the sensors in convenient locations, as well as find the optimal acquisition set up.

The resolution \( \Delta \) of the displacement measurement is given by the \( \delta(\Delta t_{AC}) \) resolution. Starting from:

\[
R_s = \frac{\Delta S}{S} \tag{26}
\]

and being \( t_s \) the resolution in the time measurements, i.e., sampling time, we can write:

\[
t_{rs} = R_s (\delta(\Delta t_{AC}))_{\max} \tag{27}
\]

and being:

\[
(\delta(\Delta t_{AC}))_{\max} = \frac{S}{\omega \cdot R}. \tag{28}
\]

\( t_s \) strictly affects the resolution on the measurement of the displacement \( s \). A useful equation can be defined to relate \( t_s \) to \( R \). This relation becomes fundamental when choosing the resolution of the time measurement, in order to achieve the target resolution of the blade tip displacement \( s \):

\[
t_{rs} = R_s \cdot \frac{S}{\omega \cdot R}. \tag{29}
\]

The resolution on blade tip velocity measurement depends essentially on \( \delta(\Delta t_{CD}) \) measurement resolution. As for the displacement measurements, the following equation is obtained:

\[
t_{rv} = R_v \cdot (\delta(\Delta t_{CD}))_{\max} \tag{30}
\]

and being
\[
(\Delta t_{CD})_{\text{min}} = \frac{\delta_{CD}}{\omega - 2 \pi \cdot v \cdot s} \\
(\Delta t_{CD})_{\text{max}} = \frac{\delta_{CD}}{\omega + 2 \pi \cdot v \cdot s}
\]

Hence

\[
(\delta(\Delta t_{CD}))_{\text{max}} = \frac{(\Delta t_{CD})_{\text{max}} - (\Delta t_{CD})_{\text{min}}}{2}
\]

Therefore, from (33), it is possible to define another useful formula to choose sampling time of sensor signals in order to have a sufficient resolution for tip time velocity measurement:

\[
t_{rv} = 2 \pi \cdot S \cdot \frac{R_v}{\omega^2 \cdot R^2 - 4 \pi^2 \cdot v^2 \cdot s^2}
\]

Further causes of uncertainty can also be due to: variations of \(\delta_{AB}\) and \(\delta_{CD}\); definition of the threshold \(\Delta z\) relative radial motion between sensor and blade tip; noise in sensors signals. This analysis has been developed in [39].

### 7. CONCLUSION

Some basic models for blade tip timing measurement systems have been defined: the first using one of the two sensors as a reference; the second with both sensors facing the blade tip. Although the second approach is more versatile and easier to install, using one sensor as a reference is needed when the analysed machineries have higher degrees of irregularity.

Uncertainty on the parameters of these models has been theoretically analysed. Some simple formulas to relate the uncertainty and resolution obtained with reference to measurement system chosen parameters and sensor installation have been proposed. These formulas could be very useful to make proper choices over measurement system components, in relation to blade tips expected vibration characteristics and turbomachine parameters. So, this work can be used for blade tip timing measurement system design and installation guidelines.

### REFERENCES


Measurements in a hard disk of an aircraft engine for rotational speed fluctuation


M. E. Mohamed et al., Experimental validation of FEM-computed stress to tip deflection ratios of aero-engine compressor blade vibration modes and quantification of associated uncertainties, Mechanical Systems and Signal Processing 178 (2022), art. No. 102257. DOI: 10.1016/j.ymssp.2022.102257

S. Catalucci, R. Marsili, M. Moretti, G. Rossi, Measurement, and undefined, Comparison between point cloud processing techniques, Measurement 127 (2018), pp. 221-226. DOI: 10.1016/j.measurement.2018.05.111

