Measurement system analysis of a static method for taximeter verification

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ABSTRACT
The purpose of the paper is to present a method for taximeter verification. Taximeters, as special measurement instruments, are subject to metrological control in order to protect the rights of taxi customers. The methodology applied for the theoretical study of the proposed method is derived from Measurement System Analysis, Root Cause Analysis, and similar taximeter verification methods applied in other European countries. The presented method is innovative for Bulgaria and about to be introduced for the need of metrological control.

1. INTRODUCTION
A taxi is a car equipped with a taximeter that provides paid transportation service to the general public. In this way the distance reporting device (DRD) becomes subject to subsequent metrological control or verification [¹].

The paper [²] classifies different methods for conducting metrological verifications of the taximeters. Three methods were comparatively analyzed and evaluated according to 5 criteria. The basis for the comparison was how the method measures distance. The kinematic method, or taxi on a road section, is ranked last among the three due to various implementation difficulties and practical limitations. The static method, involving the use of a roller test bench (RTB) and specifically measuring distance by the revolutions of one of the rollers of the test bench, is ranked first among the three due to its feasibility compared to the other two evaluated methods.

The different methods represent the reference distance in different ways. The direct method, or the kinematic one, compares the distance reported by the taximeter by travelling a reference distance by the taxi on a road section. The indirect, or static, method represents the reference distance by counting the revolutions either of a car wheel, or a roller of a test bench.

Measurement System Analysis, Root Cause Analysis and statistical analysis formulae are used for the purposes of the research [³]-[⁵].

2. MEASUREMENT SYSTEM ANALYSIS
2.1. Measurement system
According to [³] the definition for a measurement system is: “Measurement System is the collection of instruments or gages, standards, operations, methods, fixtures, software, personnel, environment and assumptions used to quantify a unit of measure or fix assessment to the feature characteristics being measured; the complete process used to obtain measurements.”. In order to carry our measurements, we need adequate equipment, appropriate competence and suitable conditions. Measurements are complex processes that need to be well-analysed in advance, in order to guarantee the measured results in the future [⁶]. A detailed study of the quantity to measure, the measurement algorithm and the degree of uncertainty that may affect the results [⁷], [⁸].

2.2. Elements of a measurement process
Many metrology books and documents cover the measurement process and its elements. There are various lists of elements of the measurement process. A good approach to
representing how the elements are arranged is shown on Figure 1 [9].

The object in the measurement process, or what is to be measured, is the DRD. The measurement instrument used to verify the DRD is the RTB. The method of the measurement process is the algorithm that is applied by the RTB. The subject in the process is the operator. All the influencing factors that can affect the measurement process are included in the environment. As Figure 1 shows, those five elements may be separate, but are still interrelated and interacting in various ways.

2.3. Root cause analysis using fishbone diagrams

We analyse the potential sources of error/uncertainty after the elements of the measurement system are defined [8], [10]. The method of Root Cause Analysis may be applied for this purpose [4]. The process should start by describing the measurement system and establishing the normal measurement process. The team that has studied the measurement system consists of the researchers and developers of the RTB. As a result, the potential root causes for abnormal measurement process are defined, and a detailed causal diagram is established. The five elements of the studied measurement system with some potential root causes are shown on Figure 2 [11].

The DRD, or the object in the measurement system, is the taxi itself. A taximeter is installed in a vehicle and receives data from a source of impulses (distance transducer) about the distance travelled. The transferred data is processed by the taximeter and the end result of the measurement is provided to the taxi driver.

The RTB is a complex set of mechanical and electronic parts. All those numerous elements can potentially be a cause of a problem during the measurement process.

The method in the studied measurement system is described by an algorithm that defines the comparison of the distance measured by the taxi and the distance reported by the RTB. The algorithm used is based on the relationship of two diameters – the one of the taxi wheels, and the one of the rollers of the RTB. The measurement process is conducted by an operator, so it is an integral element of the studied measurement system.

Air temperature, humidity, and placement on surface of the RTB are some of the factors that may influence the measurement process and the system in general, and they need to be analysed as well.

2.4. Sources of measurement uncertainty

After the overall detailed fishbone diagram is created, the potential sources of uncertainty are listed. More than 120 sources of uncertainty in the measurement system are identified.

Most of them, about 50, are related to the RTB. This is understandable, since the RTB is a product that comprises of multiple components that can affect its measurement accuracy. The required production accuracy and calibration allow the product to fulfil its purpose.

About 30 of the potential sources of uncertainty are sub-causes of the measured object, i.e., the DRD. The resulting measuring system is based on the interaction of the vehicle, the source of impulses for defining the travelled distance, and the taximeter for calculating the distance travelled. The various possibilities for connecting these separate components represent a significant share of the measurement process. They are also some of the potential sources of uncertainty in the measurement system.

The remaining sources of uncertainty – the method, the operator, and the environment, generate almost equal number of sub-causes. The method usually contributes significantly to the measurement uncertainty, as is the case with the algorithm of operation of the RTB. The operator takes into consideration the potential influence of the human factor, and the environment – the additional potential influencing factors.

In the analysis of potential sources of uncertainty, all the possible causes are listed. The potential sources of uncertainty that are not valid in this specific case are excluded from the final list.

Once the potential sources are listed, they need to be prioritized. The process of prioritization is based on expertise, practical experience, and the overall competence of the designers of the RTB and the team of researchers. This is coupled with concrete facts and data from preliminary measurements and testing of sample products. As a result, the most significant sources are selected using an analogy with the Pareto principle [12]. The obtained results are presented in Table 1 below. The summary demonstrates the 80/20 rule by outlining the 18 key sources of uncertainty.

The list of key sources of uncertainty excludes those generated by the operator in the measurement system. The characteristic of the process and the operator's use of the RTB justify this. The measurement process is not conducted in the case of an operator error. Improper operator actions are a gross error of the measurement system.

3. MEASUREMENT SYSTEM UNCERTAINTY

3.1. Measurement model

The method with the use of a RTB is accomplished indirectly by setting and reading the revolutions of one of the rollers of the test bench with a known diameter.

The method implies to compare the distance set and calculated by the RTB and the taximeter based on the number of revolutions of the circumferences of the two objects – the roller wheel of the vehicle and the roller of the RTB.
of the test bench and the car wheel (Figure 3). The functional relationship between these two can be expressed mathematically:

\[ S = p n = P N \]  

(1)

\[ S = \pi d n = \pi DN, \]  

(2)

where \( S \) is the distance, \( p \) is the circumference of the roller of the test bench, \( P \) is the circumference of the car wheel, \( d \) is the known diameter of the roller of the test bench, \( n \) is the number of revolutions of the roller of the test bench, \( D \) is the diameter of the car wheel, and \( N \) is the number of revolutions of the car wheel.

Although the method allows two types of drive modes, in the case of the particular technical solution analysed, the emphasis is on a RTB driven by its own motor. In this case the distance simulated by the roller test bench to be compared to the reported distance by the taximeter can be expressed theoretically according to eq. (1).

As the RTB is driven by its own motor, the circumference of the roller of the test bench, the number of the revolutions of the roller of the test bench and the circumference of the car wheel are the known (measurable) elements of the method. It is necessary to define how they correlate to the number of revolutions of the car wheel:

\[ N = \frac{p n}{P}. \]  

(3)

The differentiation of the formula (4) allows us to define the systematic component of the measurement error of the output quantity as follows:

\[ N' = \frac{(p n)'P - (p n)P'}{P^2}, \]  

(4)

\[ N' = \frac{(p'n + p n')P - p n P'}{P^2}. \]  

(5)

The variation in the number of revolutions of the car wheel is defined by the relations of the circumference of the roller of the test bench, the number of the revolutions of the roller of the test bench and the circumference of the car wheel:

\[ \Delta N = \frac{\Delta p n + p \Delta n}{P} - \frac{p n \Delta P}{P^2}, \]  

(6)

where \( \Delta p, \Delta n \) and \( \Delta P \) are the errors/deviations of the input values.

3.2. Uncertainty of the method

The circumference of the roller, \( p \), is calculated based on technical research data. The diameter of the roller is 210 mm, while the circumference is 659.7 mm.

\[ \Delta p \] includes two independent errors – the error of measuring the roll diameter and the elastic deformation error from the perimeter-changing load \( G \). As a result:

\[ \Delta p = \sqrt{\Delta p_d^2 + \Delta p_m^2}, \]  

(7)

where \( \Delta p_m = \Delta d_m \pi, \Delta d_m = 0.015 \text{ mm} \) is the maximum permissible error (MPE) of the measurement of the diameter of the roller [13], and \( \Delta p_d = 0.1 \% \times P \) is the maximum variation in the circumference of the roller due to natural elastic deformations [14].

Finally:

\[ \Delta p = \sqrt{0.05^2 + 0.6597^2} \text{ mm} = 0.66 \text{ mm}. \]  

(8)

The widespread basic method for taximeter verification uses measured road section that is 1000 m long. The same reference distance is used to define the systematic component of the measurement error of the method. The roller of the test bench will make \( n \) or 1515.8 revolutions per 1000 m.

Therefore, there are 6 evenly spaced points on the circumference of the roller to count its revolutions. The discretion of the revolution counting of the roller is \( \Delta n \) and equals 1/6 of its circumference.

For the needs of this research is carried out an analysis of the most common tire sizes in taxis. The mode of the research data is the size 195/65 R 15. The diameter is calculated according to a formula [15] and in this case it is 634.5 mm. Thus, the circumference \( P \) of the car wheel as a mode is 1993.3 mm.

The deviation \( \Delta P \) occurs as \( \Delta P_d \) because of the error due to the elastic deformations caused by the load \( G \) changing the perimeter. The maximum variation in the circumference of the car tire due to natural elastic deformations, calculated by applying the methodology in [14] is:

\[ \Delta P_d \equiv 0.22 \% \times P = 4.4 \text{ mm}. \]  

(9)

A transition from error approach towards uncertainty evaluation can be made [16]. The first step is to calculate the standard uncertainty for the three sources. Assumed is uniform distribution of the quantities:

\[ u_B(n) = \frac{\Delta n/2}{\sqrt{3}} = \frac{1}{12}\text{rpm} \]  

\[ = 0.048 \text{ min}^{-1} \]  

(10)

\[ u_B(p) = \frac{\Delta p}{K(p) \times \sqrt{3}} = 0.35 \text{ mm}, \]  

(11)

\[ u_B(P) = \frac{\Delta P}{K(p) \times \sqrt{3}} = 2.31 \text{ mm}, \]  

(12)

where coefficient \( K(p) = 1.1 \) for confidence level \( p = 0.95 \) [16].

The next step towards calculating the uncertainty of the method is to calculate the sensitivity coefficients of the three sources of uncertainty defined – the variations in the circumferences of the roller and the car wheel, and the variation in the revolutions of the roller [17], [18].

\[ |C_n| = \left| \frac{\partial N}{\partial n} \right| = \frac{p}{n} = \frac{659.7 \text{ mm}}{1993.3 \text{ mm}} = 0.331 \]  

(13)

\[ |C_p| = \left| \frac{\partial N}{\partial p} \right| = \frac{\Delta n}{p} = \frac{1515.8 \text{ rpm}}{1993.3 \text{ mm}} \]  

\[ \approx 0.76 \text{ mm}^{-1} \times \text{min}^{-1} \]  

(14)
\[ \begin{align*}
|C_p| &= \left| \frac{\partial N}{\partial p} \right| = \left| -n \frac{p}{p^2} \right| \\
&= 1515.8 \times 659.7 \times 260 \times 1.0515 \times 2057 \\
&= 0.252 \text{ mm}^{-1} \times \text{min}^{-1}.
\end{align*} \]

The combined standard uncertainty \( u_c \) of the measurement method is:

\[ u_c(N) = \sqrt{ C_n^2 \sigma_\theta^2(n) + C_p^2 \sigma_\theta^2(p) + C_N^2 \sigma_\theta^2(N)} \]

\[ = \sqrt{0.331^2 \times 0.048^2 + 0.35^2 \times 0.252^2 \times 2.31^2} \text{ mm}^{-1} \]

\[ u_c(N) = 0.64 \text{ mm}^{-1}. \]

In this case, if the perimeter of the tire of the verified car is \( p = 1993.3 \) mm, then the combined standard uncertainty will be 1275.7 mm (1.28 m) for a reference distance of 1000 m.

### 3.3. Uncertainty of the DRD

The measurement uncertainty of the DRD is formed by the combined influence of the following sub-causes: geometry of the driven axle, vehicle wheel, source of impulses for distance travelled, and the taximeter. The uncertainty of the element “geometry of the driven axle” is formed by the combined influence of the elements “car wheel axle convergence” and “steering axis inclination”. The car wheel runout is taken into consideration for the element “car wheel”.

Measurement data from the specialized software of a wheel aligner test bench is used in the course of the research for wheels with tire size 195/65 R 15 [19]. The variation in the car wheel axle convergence or steering axis inclination changes the contact points of the car tire and the perimeter of the car wheel. The arithmetic mean value of the nominal readings of car wheel axle convergence in motion is 0 mm. The arithmetic mean of the nominal readings of steering axis inclination for the studied cars is 0.22°. The difference in the distance travelled due to Steering Axis Inclination (SAI) \( \Delta D_{SAI} = 6 \) mm is accumulated for a distance of 1000 m. The maximum permissible car wheel runout is 2 mm [20]. The difference in the distance travelled due to car wheel runout \( \Delta D_{SCRWMax} = 4 \) mm is accumulated for the same distance.

It is required that the minimum number of impulses per kilometre read by the taximeter are 500 [21]. The mode of the experimental data is 4980 impulses per kilometre. The discretion of the source of impulses for distance travelled is \( \Delta d_{SNSR} = 0.08 \text{ mm} \).

According to European regulations defined in [1], the error of distance reading of unmounted taximeter is \( \Delta D_{SD} = 0.2 \% \).

The value of the combined error of the DRD is:

\[
\Delta DRD = \sqrt{\Delta D_{SAI}^2 + \Delta D_{SNSR}^2 + \Delta d_{SCRWMax}^2 + \Delta D_{SD}^2}
\]

\[ = \sqrt{6^2 + 4^2 + 200.8^2 + 2000^2} \text{ mm} \]

\[ = 2010.07 \text{ mm}. \]

The combined uncertainty, \( u_c(DRD) \) is calculated by the following equation:

\[ u_c(DRD) = 2010.07 \times 0.5 \text{ mm} = 1005.03 \text{ mm}. \]

### 3.4. Uncertainty of the RTB

The measurement uncertainty of the element RTB is formed from the geometric errors of RTB manufacturing and assembly, (respectively their permissible deviations), the discretion of the sensor (the primary transducer), and the natural spread of the experimental data from the calibration of 10 RTBs.

The permissible deviations of the geometric parameters are set in the engineering drawings of the RTB. \( \Delta d_m = 0.015 \text{ mm} \) is the maximum permissible error (MPE) of the measurement of the diameter of the roller [13].

Crossing and non-parallelism of roller installation, longitudinal deviation from shape of roller, and play in the bearing cause an angular rotation of the assembled rollers [22]. The combined influence of these sources of error is compiled and solved by “maximum-minimum” method for an angular tolerance stack-up analysis. The components of the angular tolerance stack-up analysis are the abovementioned errors, and the resulting gap is the angular deviation between the generatrixes of the cylindrical surfaces of the two rollers [23]. The combined error of the geometric errors of RTB manufacturing and assembly \( \Delta RTB_{Amb} = 1.4 \text{ mm} \).

6 elements for counting the number of revolutions of the standard roller are installed on its axis. The discretion of the sensor is 1/6 of a revolution. When \( p = 659.7 \) mm, the \( \Delta dtRBSNSR = 109.95 \text{ mm} \).

Empirical data from the calibration of 10 RTBs is analysed. Each RTB is calibrated at 3 different reference distances – 500 m, 1000 m, and 2000 m. The coefficients of variance for each RTB and between the RTBs demonstrate that the results are homogenous [3], [24]. The pooled variance of the calibration data \( s^2 = 0.000505 \% \) and the standard deviation is \( s = 0.022472 \% \).

Error of the measured during calibration data, \( \delta_{RTB} \), is defined the following way:

\[ \delta_{RTB} = N \cdot \epsilon_{n-r} \cdot s = 2.09 \times 0.22472 = 0.47 \]

where \( N \) is the total number of measurements = 30, \( r \) is the number of RTBs = 10, \( \epsilon_{n-r} \) is the coefficient whose value is determined for a probability \( P = 1 - \alpha = 0.95 \) [25].

Thus, the error is \( \delta_{RTB} = 470 \text{ mm} \) for a distance of 1000 m.

The calibration error \( \Delta RTB_{Calib} = 0.2 \% \). It is derived from the standard uncertainty of calibration = 0.105 \% for a distance of 1000 m.

The combined error of the RTB is:

\[ \Delta RTB = \sqrt{\Delta d_{RTB}^2 + \Delta d_{sRTB}^2 + \Delta d_{sRTB}^2 + \Delta d_{sRTB}^2} \]

\[ = \sqrt{0.015^2 + 1.2^2 + 2.23^2 + 470^2 + 2000^2} \text{ mm} \]

\[ = 2057.06 \text{ mm}. \]

The combined uncertainty \( u_c(RTB) \) is calculated by the following equation:

\[ u_c(RTB) = 2057.06 \times 0.5 \text{ mm} = 1028.71 \text{ mm}. \]

### 3.5. Uncertainty of the environment

The temperature of the environment is a significant factor that influences the measurement uncertainty. The unfavourable materials, the steel for the roller and the rubber for the car wheel, are the options considered [26]. The temperature deformation error of 260 mm of the roller and the car wheel is calculated for a deviation of ±20°C from the temperature of calibration of the RTB and for a reference distance of 1000 m.

The uncertainty of the deviation of environmental temperature is calculated:

\[ u(Env) = 260 \times 0.5 \text{ mm} = 136.46 \text{ mm}. \]
3.6. Measurement system uncertainty

Thus, the combined uncertainty of the measurement system, $u_c(MS)$, includes the uncertainty of the method, of the DRD, of the RTB, and of the environment.

$$u_c(MS) = \sqrt{u_N^2 + u_{DRD}^2 + u_{RTB}^2 + u_{Env}^2} = \sqrt{1275.71^2 + 1005.03^2 + 1028.71^2 + 136.46^2} \text{ mm} = 1927.27 \text{ mm}.$$  

The maximum permissible error for the taximeter verification is $\pm 2\%$. The target uncertainty $u_T$ for the measurement instrument for the verification of the taximeter is:

$$u_T(MS) = \frac{\text{MPEL}}{K(p) \times \sqrt{3}} = \frac{0.3 \times 2\% \times L}{K(p) \times \sqrt{3}} = \frac{6000}{1.1 \times \sqrt{3}} \text{ mm} = 3149.56 \text{ mm},$$

where coefficient $K(p)$ equals 1.1 for confidence level $p = 0.95$ and the uniform distribution of the input quantities is assumed [11].

The combined and target uncertainties of the measurement system are compared:

$$u_c(MS) < u_T(MS)$$

The expanded uncertainty with $k$ factor is 2 is:

$$U(MS) = u_c(MS) \times k = 1927.27 \times 2 \text{ mm} = 3854.54 \text{ mm},$$

or almost 3.9 m extended uncertainty of the measurement system.

Thus, the measurement system analysed is appropriate for the needs of taximeter verification.

4. CONCLUSIONS

- More than 120 sources of uncertainty in the verification process of a taximeter have been identified and divided into five usual main categories – object or the distance reading device, the measurement instrument or the roller test bench, the method, the operator, and the environment.
- The most significant 18 of them were determined by using an analogy with the Pareto principle.
- The measurement equation of the static method with the roller test bench for conducting metrological verifications of the taximeters is defined and the results are calculated.
- The uncertainty of each element of the measurement system is determined.
- The uncertainty of the measurement system is determined to be 1.9 m which is less than the target uncertainty of 3.1 m.

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