Validation of a laboratory method for the traceability of a rainfall weighing gauge

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ABSTRACT
This study aims to develop a laboratory method for the traceability of a rainfall weighing gauge, including an evaluation of the measurement uncertainty. The adopted procedure is similar to the one used for the non-automatic weighing instruments. A static approach is followed to achieve the calibration deviation of the precipitation scale. The method used to evaluate the measurement uncertainty is based on a nonlinear mathematical model. The Monte Carlo method is used to calculate uncertainties and validate estimates following the conventional Guide to the Expression of Uncertainty in Measurement (GUM) approach. Measurement uncertainty contributions of input quantities to the mathematical model used to calculate rainfall also require specific calibration procedures. Results show the accuracy level achievable with rainfall weighing gauges commonly used as a reference for meteorological monitoring networks and data modelling.

1. INTRODUCTION

Growing awareness of the impact of climate change and the United Nations sustainable development goals [1] emphasizes the need to have reliable measurements of quantities (e.g., amount of precipitation or rainfall) to support urban and water resources management.

Rainfall is measured worldwide to feed models of meteorological phenomena accounting for spatial and temporal dynamic regimes [2]. Three automatic precipitation gauge types are commonly used: weighing, tipping-bucket and floating gauges [2], [3]. The weighing gauge is the most satisfactory for measuring the different precipitation types, while the other two types provide less accurate results for rainfall [2]. Tipping buckets are the most used equipment, which is only suitable for measuring rainfall.

Advantages of weighing gauge over tipping buckets gauges are applicability in a wide range of precipitation intensity and forms of precipitation (e.g., snow, hail and mixtures of snow and rain solid precipitation) [2], while the latter underestimates rain intensities higher than the target range of the equipment.

Weighing gauges are robust equipment for site measurements, primarily because of the high sensitivity of the method, which compares positively to the sampling limitations of tipping bucket rain gauges [4]. Despite the noticeable technological benefits achieved by modern gauges and acquisition systems [5], several open issues remain regarding the interpretation of the high temporal variability of precipitation, which needs to be definitively addressed in common practice [2] to improve the traceability of these instruments.

The calibration method of rainfall weighing gauges should provide confidence regarding traceability of the results considering two aspects: the first, ensuring that the amount of weight is correct independently of the dynamic nature of the measurement; and the second, confirm that the time response of the instrument is appropriate for complying with the dynamic behaviour of rainfall. The paper presents an alternative method to provide accurate traceability for the first aspect.

The conventional approach adopted to calibrate these types of equipment is usually the volume of rainwater collected in the cylinder using a controlled system to measure the reference volume of water (e.g., calibrated automatic pipette or peristaltic
pumps). More recently, some authors propose a direct approach based on mass measurement (e.g., [2], [6]) avoiding uncertainty related to volume measurements, thus improving its accuracy. This approach allows comparing the collected water volumes with reference precision weighing instruments.

The method proposed in this paper to carry out the calibration of rainfall weighing gauges for static conditions is supported by mass measurement. However, instead of using the water volume quantities to undertake the calibration steps, reference standards weights are considered, with several advantages, namely:
- better accuracy of the reference mass;
- reduction of the effect of temperature and pressure;
- higher accuracy of density in estimating the buoyancy coefficient;
- easier conditions for repeatability and reproducibility analysis; and
- less complexity of the calibration setup, not requiring devices needed to control pumps and other components.

In this method, static measurements are made through a series of standard weights traceable to primary standards of mass (in this case, standard weights of Class E2 and F1 were used [7]). When placed in the bucket or collection container, these standard weights provide traceable reference values that can be directly related to the mass of a volume of water in the container corresponding to a certain amount of rainfall. This method allows finding estimates of the systematic deviations throughout the weighing gauge rainfall scale and the associated uncertainty.

The alternative method described also includes the calibration of the orifice rim diameter, typically not done and usually taken as a nominal reference value in the calculation. However, it can be a relevant quantity because it relates directly to the rainfall distribution per unit of area.

The mathematical relation between the output quantity (rainfall) and the input quantities is nonlinear. To calculate the measurement uncertainty in this case, the ISO Guide to the Expression of Measurement Uncertainty (GUM) in its Supplement 1, recommends the use of a Monte Carlo method (which could also validate the results obtained using a conventional GUM approach) to derive the output probability distribution function and calculate the uncertainty with a 95% of confidence interval.

This study presents the mathematical model that relates input, output quantities, the procedures to carry out the orifice rim diameter calibration of the weighing gauge and to evaluating its measurement uncertainty. The alternative is described, including calibration of a rainfall weighing gauge based on reference weights and the rainfall measurement uncertainty calculation by propagating the probability distribution functions using the mathematical model mentioned above, applying a Monte Carlo method. Finally, conclusions and future work are presented.

2. RAINFALL WEIGHING GAUGE DESCRIPTION

The rainfall weighing gauge tested is the OTT Pluvio®L, typically used to measure precipitation automatically and then calculate the intensity and amount of rainfall. The OTT Pluvio®L uses the weighing principle [8].

The manufacturing characteristics of the OTT Pluvio®L weighing gauge are a 400 cm² nominal caption area opening and a resolution for the water accumulation measurement equal to 0.01 mm. The model used in the present analysis is the “Bucket RT”, which shows the amount of rainfall in the bucket, an output string provided by the gauge through an SDI-12 serial communication, with a declared resolution equal to 0.01 mm.

3. MATHEMATICAL MODEL

Weighing type rain gauges measure the mass of accumulated precipitation as a function of time. This mass can be measured by weight or by volume [9]. The measurement principle of the weighing gauge relates the amount of liquid in a time interval (recording the amount of accumulated liquid), with the weight variation measured in the recipient placed over a weighing platform. The amount of precipitation per unit of time, \( P \) (usually in mm), can be expressed by the volume, \( V \), of a cylinder with an area, \( A \), of the orifice opening to collect the precipitation:

\[
P = \frac{V}{A} = \frac{m}{\rho_w \pi d^2/4}
\]

where \( \rho_w \) is the density of water \((\text{kg/m}^3)\), \( w \) is the mass \((\text{kg})\), and \( d \) is the orifice rim diameter \((\text{m})\).

Considering the weighing instrument as the central provider of results, (1) needs to be corrected using the buoyancy coefficient, \( C_B \), introduced in (2) and given by (3). This coefficient incorporates the effect of the weighing gauge calibration with standard weights while measuring the mass of water in routine operation.

\[
P_c = \frac{mC_B}{\rho_w \pi d^2/4}
\]

\[
C_B = \frac{\left(1 - \frac{\rho_{\text{air}}}{\rho_w}\right)}{\left(1 - \frac{\rho_{\text{air}}}{\rho_s}\right)}
\]

In (3), \( \rho_{\text{air}} \) is the density of air \((\text{kg/m}^3)\), \( \rho_w \) the density of water \((\text{kg/m}^3)\) and \( \rho_s \) the density of the reference standard weights \((\text{kg/m}^3)\).

The density of the air, \( \rho_{\text{air}} \), is given by [10]:

\[
\rho_{\text{air}} = \frac{0.34848 \cdot p - 0.009 RH \cdot e^{0.061t}}{273.15 + t}
\]

where \( p \) is the barometric pressure \((\text{hPa})\), \( RH \) is the relative humidity of the air \( (\%) \), and \( t \) is the air temperature \((\degree C)\).

Taking the mathematical model of (2) as representative of the evaluation of the rainfall, the relation between input, intermediate and output quantities is shown in the diagram in Figure 1.

4. CALIBRATION OF THE ORIFICE RIM DIAMETER OF A RAINFALL WEIGHING GAUGE

The method for the calibration of the orifice rim diameter of a weighing gauge is based on the determination of the orifice opening 3D coordinates using a coordinate measuring machine \((\text{CMM 3D})\), having a resolution of 0.1 \( \mu \text{m} \) on all the axes (Figure 2) and measurement uncertainties between 1 \( \mu \text{m} \) and 2 \( \mu \text{m} \) for the three axes.

Five spatial coordinate measurements were carried out (see Figure 3). In each series, 20 coordinate positions distributed around the opening orifice rim were collected.
For the calculation of repeatability and reproducibility of the measurement samples, a rotational lag of ca. 2° was applied to each series. To estimate the diameter and related uncertainty, the 2D coordinates \((X, Y)\) were used, the measured pairs, as shown in Figure 3.

Calibration results provide an accurate estimate of the diameter and its measurement uncertainty, including the dominant contributors to repeatability, reproducibility, resolution of the CMM 3D, and roundness error, as presented in Table 1.

### 5. MASS CALIBRATION OF A RAINFALL WEIGHING GAUGE

The measurement principle of the rainfall weighing gauge is similar to the non-automatic weighing instruments, allowing to adopt a method for the calibration according to the same procedure as described in [11]. In this alternative proposed method, references were given by the conventional values of the standard weights, covering the measurement interval of the weighing gauge under calibration, using standard weights of class E2 from 200 g to 500 g and of class F1 for 1 kg, 2 kg, 5 kg and 10 kg, with measurement uncertainty lower than 1 mg.

The calibration procedure (see Figure 4) includes three tests aimed at the evaluation of eccentricity, reversibility, and repeatability in a similar way to the general procedure for non-automatic weighing instruments [11], being the relevant contributions accounted for in the expanded uncertainty budget calculation.

The calibration was developed in ten steps selected according to the measurement interval of the rainfall weighing gauge, taking reading from it after applying the load and waiting for the signal stability. Repeatability was performed making three measurements in each step.

The evaluation of eccentricity test was made by selecting a load approximately equal to 1/3 of the maximum range of the

### Table 1. Calibration results

<table>
<thead>
<tr>
<th>Diameter (d)</th>
<th>Nominal value in mm</th>
<th>Calibration estimate in mm</th>
<th>Expanded Measurement Uncertainty (U_95(d))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>225.7</td>
<td>225.728</td>
<td>(2.9 \times 10^{-2})</td>
</tr>
</tbody>
</table>

**Uncertainty sources complementary information**

- Repeatability in mm: 0.005
- Roundness error in mm: 0.04
rainfall weighing gauge, being applied to four roughly equal areas on the load receiver of the weighing platform. In each defined area, three zero-load repetitions were performed.

In the reversibility test, the selected standard masses were placed on the load receptor of the weighing udometer in ascending order from the minimum permissible load (200 g) to a value close to the maximum range (15 kg). The same procedure was repeated in descending order until the minimum permissible load was reached.

The comparison uses (1) to estimate the mass, knowing the precipitation readings and considering the nominal diameter of 225.7 mm.

For the various reference mass values, \( m_i \), the relative percent error, \( \varepsilon \) (%), between the measured amount of mass, \( m \), and the reference mass value is calculated by (5).

\[
\varepsilon = \frac{m - m_i}{m_i} \times 100.
\] (5)

Calibration results allow the correction of readings directly in terms of mass (after buoyancy correction) and establish a linear regression between mass and precipitation, providing a probability distribution functions and parameters, as shown in Table 3.

**6. UNCERTAINTY ANALYSIS**

The general method used for the evaluation of measurement uncertainty [12] is presented in [13], known as the GUM, which was firstly published by ISO, IEC and other organizations in 1993. This method states that for a functional relation \( f \) of the type:

\[
y = f(x_1, \ldots, x_n),
\] (6)

where \( y \) is the output quantity calculated from \( n \) input quantities, \( x_i \), using the development of the function as a 1st order Taylor series. The formula for the measurement standard uncertainty of the output quantity, \( u(y) \), should be given by the Law of Propagation of Uncertainties:

\[
u^2(y) = \sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i)
\] + \[2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left( \frac{\partial f}{\partial x_i} \right) \left( \frac{\partial f}{\partial x_j} \right) u(x_i, x_j).
\] (7)

The first part of the second term of (7) is related to the variance of each input quantity; the second part of the second term is related to the contributions resulting from the correlation between input quantities. This approach gives an exact solution for linear functions and approximate solutions to non-linear and more complex functions. For this study, the evaluation of measurement uncertainty does not account for possible correlations between the input quantities. In this condition, the standard uncertainty of the output quantity is given by:

\[
u^2(y) = \sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i).
\] (8)

Considering (2), because of the nonlinearity of the model, the approach considered for the evaluation of uncertainty was a Monte Carlo method using the procedure described in GUM Supplement 1 [14]. In applying the method, the first step is to quantify the input quantities and related uncertainties using their probability distribution functions and parameters, as shown in Table 3.

Considering the process described in Figure 1, the uncertainty calculation of rainfall measurements is achieved through three stages:

a. the evaluation of the air density measurement uncertainty, using (4);

b. the evaluation of the buoyancy coefficient measurement uncertainty using (3), including the uncertainty resulting from in the previously stage; and

c. the evaluation of rainfall measurement uncertainty using (2), including the uncertainty resulting from in the previously stage.

For the first stage mentioned above, experimental values of temperature, relative humidity and pressure were measured using calibrated instruments, and the results are presented in Table 3.

The uncertainty evaluations are from their calibration certificates issued by an accredited laboratory. The estimates of the air density and the measurement uncertainty are presented in Table 2.

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**Table 2. Calibration results.**

<table>
<thead>
<tr>
<th>Reading ( m ) in g</th>
<th>Reference value ( m_s ) in g</th>
<th>Measurement error ( \varepsilon = m - m_s ) in g</th>
<th>( \varepsilon ) in %</th>
<th>Expanded Measurement Uncertainty ( U(m) ) in g</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.13</td>
<td>0.00</td>
<td>0.13</td>
<td>---</td>
<td>1.0</td>
</tr>
<tr>
<td>199.92</td>
<td>200.00</td>
<td>-0.08</td>
<td>-0.040</td>
<td>0.8</td>
</tr>
<tr>
<td>499.99</td>
<td>500.00</td>
<td>-0.01</td>
<td>-0.002</td>
<td>0.8</td>
</tr>
<tr>
<td>1 000.05</td>
<td>1 000.00</td>
<td>0.05</td>
<td>-0.005</td>
<td>0.8</td>
</tr>
<tr>
<td>1 999.92</td>
<td>2 000.00</td>
<td>-0.08</td>
<td>-0.004</td>
<td>0.8</td>
</tr>
<tr>
<td>5 000.19</td>
<td>5 000.00</td>
<td>0.19</td>
<td>0.004</td>
<td>0.9</td>
</tr>
<tr>
<td>6 999.79</td>
<td>7 000.00</td>
<td>-0.21</td>
<td>-0.003</td>
<td>1.4</td>
</tr>
<tr>
<td>9 998.99</td>
<td>10 000.00</td>
<td>-1.01</td>
<td>-0.010</td>
<td>3.1</td>
</tr>
<tr>
<td>12 001.12</td>
<td>12 000.00</td>
<td>1.12</td>
<td>0.009</td>
<td>2.3</td>
</tr>
<tr>
<td>14 999.66</td>
<td>15 000.00</td>
<td>-0.34</td>
<td>-0.002</td>
<td>3.2</td>
</tr>
</tbody>
</table>

---

**Table 3. Input quantities and output quantity estimates and measurement uncertainties related with the calculation of an air density.**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Estimate</th>
<th>Standard Uncertainty</th>
<th>Probability Distribution Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T ) in °C</td>
<td>24.0</td>
<td>0.5</td>
<td>Normal</td>
</tr>
<tr>
<td>( RH ) in %</td>
<td>57.0</td>
<td>1.0</td>
<td>Normal</td>
</tr>
<tr>
<td>( p ) in hPa</td>
<td>1026</td>
<td>1.0 \times 10^4</td>
<td>Normal</td>
</tr>
<tr>
<td>( \rho_{air} ) in kg/m³</td>
<td>1.196</td>
<td>1.2 \times 10^2</td>
<td>Normal</td>
</tr>
</tbody>
</table>
were calculated from the propagation of the probability distribution functions using numerical simulation of Monte Carlo method generating data series from 10^6 runs for each quantity of (4), using RStudio © software.

In the evaluation of measurement uncertainty related to the buoyancy coefficient, according to (4), estimates and uncertainties of the density of the standard weights, water density and air density are needed. The standard weights' density and the respective uncertainty, presented in Table 4, are given by the manufacturer and calibration certificates.

The water density and its uncertainty were determined considering the reference value and the variation from the known relations of this quantity with the testing temperature. The air density was estimated in stage 1, described above.

The estimate of the buoyancy coefficient and the measurement uncertainty were found again by the propagation of the probability distribution functions using numerical simulation of Monte Carlo method generating RStudio © software, generating data series from 10^6 runs each. The conventional GUM approach was also tested to compare the results.

To make the calculation estimates of the main input quantities and related uncertainties, the information presented in the previous sections was used and, regarding of mass estimate, a specific quantity of 5 kg was considered from the calibration steps presented in Table 2.

For other quantities of mass within the measurement interval, the same procedure should be carried out, and the overall results should allow to calculate a single measurement uncertainty or an equation to be applied to this measurement interval.

### Table 4. Input quantities and output quantity estimates and measurement uncertainties related to the evaluation of the buoyancy coefficient.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Estimate</th>
<th>Standard Uncertainty</th>
<th>Probability Distribution Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_w ) in kg/m²</td>
<td>1.196</td>
<td>1.2 \times 10^{-2}</td>
<td>Normal</td>
</tr>
<tr>
<td>( \rho_s ) in kg/m²</td>
<td>7,950</td>
<td>7 \times 10^{-5}</td>
<td>Normal</td>
</tr>
<tr>
<td>( \rho_w ) in kg/m²</td>
<td>1,000.0</td>
<td>5 \times 10^{-5}</td>
<td>Normal</td>
</tr>
<tr>
<td>( C_s )</td>
<td>1,000.047</td>
<td>1.2 \times 10^{-5}</td>
<td>Normal</td>
</tr>
</tbody>
</table>

The output probability distribution function for the corrected amount of rainfall is presented in Figure 5. The estimate of the quantity, its expanded uncertainty interval and statistical parameters were calculated from the precipitation output series and presented in Table 6.

Expanded uncertainty was given by the 2.5 and 97.5 percentiles of the output numerical series and Table 6 shows the semi-amplitude of this interval.

The results can be compared with the measurement uncertainty evaluated using the Law of Propagation of Uncertainties presented in (8), considering that there is no correlation between the input quantities, according to (9)

\[
\begin{align*}
\sigma_{Pc}^2 &= \frac{\partial P_c}{\partial m}^2 \sigma_m^2 + \frac{\partial P_c}{\partial C_B}^2 \sigma_{C_B}^2 + \frac{\partial P_c}{\partial \rho_w}^2 \sigma_{\rho_w}^2 \\
&+ \frac{\partial P_c}{\partial d}^2 \sigma_d^2
\end{align*}
\]

The determination of the sensitivity coefficients applied to the mathematical model (2) adopted requires the calculation of the partial derivatives:

\[
\frac{\partial P_c}{\partial m} = \frac{4 C_B}{\rho_w d^2 \pi} 
\]

\[
\frac{\partial P_c}{\partial C_B} = \frac{4 m}{\rho_w d^2 \pi} 
\]

\[
\frac{\partial P_c}{\partial \rho_w} = \frac{-4 C_B m}{\rho_w^2 d^2 \pi} 
\]

\[
\frac{\partial P_c}{\partial d} = \frac{-8 C_B m}{\rho_w d^3 \pi} 
\]

Using the values presented in Table 3 to Table 5 in (9) to (13), an estimate of the standard uncertainty associated with the rainfall \( P_c \) can be obtained is given by:

\[
u(P_c) = 0.076 \text{ mm}.
\]

The expanded uncertainty \( U_{95}(P_c) \) is calculated by:

\[
U_{95}(P_c) = k_{95} \cdot \nu(P_c),
\]

where \( k_{95} \) the expansion factor. Considering a value of 2.00 to this factor, the expanded uncertainty is:

\[
U_{95}(P_c) = 0.15 \text{ mm}.
\]

### Table 5. Input quantities and uncertainties related to the determination of the rainfall output quantity.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Estimate</th>
<th>Standard Uncertainty</th>
<th>Probability Distribution Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_{w} ) in kg/m²</td>
<td>1.196</td>
<td>1.2 \times 10^{-2}</td>
<td>Normal</td>
</tr>
<tr>
<td>( C_s )</td>
<td>1,000.047</td>
<td>1.2 \times 10^{-5}</td>
<td>Normal</td>
</tr>
<tr>
<td>( m ) in kg</td>
<td>5,000.19</td>
<td>4.5 \times 10^{-4}</td>
<td>Normal</td>
</tr>
<tr>
<td>( d ) in m</td>
<td>0.225 728</td>
<td>3.7 \times 10^{-3}</td>
<td>Normal</td>
</tr>
</tbody>
</table>

### Table 6. Output results for rainfall and expanded uncertainty and statistical parameters.

<table>
<thead>
<tr>
<th>Corrected amount of rainfall( P_c ) in mm</th>
<th>Expanded measurement uncertainty ( \sigma_{Pc} ) in mm</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>125.078</td>
<td>1.50 \times 10^{-1}</td>
<td>0.0035</td>
<td>3.003</td>
</tr>
</tbody>
</table>

Figure 5. Output probability distribution function of amount rainfall.
The comparison of the measurement uncertainty result using the GUM conventional approach, despite the nonlinearity of the mathematical model, does not affect the accuracy of the results.

7. CONCLUSIONS

This study evaluates the accuracy results from the calibration of a rainfall weighing gauge used as a reference in a meteorologic station. The calibration procedure has two parts related with the two main quantities of the mathematical model. The first part, is intended to calibrate the weighing system based on non-automatic weighing, using primary standard weights of Class E2 and F1. The evaluation results given as example are shown for the 5 kg step of the calibration procedure developed using the measurement uncertainty presented in Table 2.

The second part of the procedure corresponds to the metrological characterization of the orifice used to collect the rain. Measurements were based on the determination of the orifice opening 2D coordinates using a coordinate measuring machine (CMM 3D), and the results show a deviation of 0.028 mm in the orifice rim diameter. This result, from the orifice rim diameter calibration, is directly related to the catchment area of the rainfall weighing gauge (which influences the amount of rainfall that is collected).

The measurement uncertainty related to the output quantity was found using the conventional Law of Propagation of Uncertainties and a Monte Carlo method approach described in GUM Supplement 1. The comparison between both approaches was carried out based on the results presented in Table 4 together with (14) and (16), showing an agreement of the results allowing to validate the use of the GUM conventional approach in this case.

REFERENCES


